

Sudden Stops in a Business Cycle Model with Credit Constraints:
A Fisherian Deflation of Tobin's Q *

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Sudden Stops in emerging economies are characterized by large, sudden current account reversals, severe recessions, and corrections in relative prices. This phenomenon is a puzzle for a large class of business cycle models in which the current account is a vehicle for consumption smoothing and investment financing. Models in this class explain Sudden Stops as the result of large, unanticipated shocks to credit market access, but this explanation is at odds with the history of Sudden Stops and is grossly incomplete because the current account reversal is obtained by assumption. Why economies in which agents know that Sudden Stops can happen become vulnerable to these crises? Can equilibrium business cycle theory explain infrequent Sudden Stops nested within "normal" business cycles without recurring to large, unexpected shocks? This paper answers these questions by constructing a model in which typical realizations of the underlying exogenous shocks driving business cycles trigger credit constraints on foreign debt and working capital financing. These constraints depend on endogenous equilibrium outcomes, and they bind only when agents are highly indebted. These high debt positions are reached with positive probability in the long run. Sudden Stop dynamics are driven by endogenous financing premia on foreign debt and working capital and Fisher's debt-deflation mechanism. The latter amplifies the real effects of adverse shocks hitting the economy and increases their persistence because the price affected by the Fisherian deflation is the Tobin Q .

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1. Introduction

One important lesson from the Great Depression is that it demonstrated that market economies can experience deep recessions that differ markedly from typical business cycle downturns. The deep recessions that hit many emerging economies in the aftermath of the financial crises of the last ten years illustrates the same fact. In contrast with the Great Depression, however, the sudden loss of access to international capital markets played a key role in triggering and propagating emerging markets crises. That is, in emerging economies we observed what Calvo (1998) labeled a “Sudden Stop.”

Sudden Stops are characterized by three striking stylized facts: First, sharp, sudden reversals in the current account. Second, severe recessions with large declines in domestic output, consumption and investment. Third, large relative price swings, with collapses in domestic asset prices and in the price of nontradable goods relative to tradables. The recessions were in most cases the largest downturns that the affected countries have experienced since the Great Depression, with deviations from trend in GDP, consumption and investment well below two standard deviations of the cyclical components of the corresponding time series (see Mendoza (2002)). Moreover, these cyclical components show statistically-significant measures of skewness, kurtosis, and conditional volatility (see Calvo, Izquierdo and Mejía (2005) and Valderrama (2003)).

Explaining Sudden Stops remains a challenging task. Sudden Stops are a puzzle for a large class of business cycle models in which the current account is an efficient vehicle for consumption smoothing and investment financing, and countries never face exclusion from world credit markets. Models in this class include real business cycle (RBC) models of small open economies (SOE) as well as models with nominal rigidities. In contrast, most of the recent literature on Sudden Stops emphasizes the role of frictions in international capital markets as the main cause of Sudden Stops. Several studies propose models that predict sudden adjustments in production, absorption and the current account as a result of the adverse effects of global financial market imperfections on investment and consumption (see, for example, Calvo (1998), Gopinath (2003), Cook and Choi (2003), and Paasche (2001)). National accounts data reveals, however, that despite sharp declines in investment and employment, changes in measured capital and labor inputs account for a small fraction of the *initial* output collapse (see Bergoing, Kehoe, Kehoe and Soto (2002) and Section 2 of this paper). Thus, many of the “credit channel” transmission mechanisms explored in the Sudden Stops literature will find it hard to explain observed Sudden Stops, even if they can explain investment collapses of the magnitude observed in the data.

The fact that changes in capital and labor account for a small fraction of a Sudden Stop’s output collapse suggests that the fall in output could be caused by a drop in total factor productivity (TFP). However, Section 2 of this paper shows that large declines in use of intermediate inputs (particularly imported intermediate inputs) and in capacity utilization account for an important share of the output collapse observed in Mexico’s Sudden Stop of 1994-95. Interestingly, after adjusting for these factors, the TFP component of the output collapse is within the range that could be attributed to the standard exogenous shocks used in RBC-SOE models (see Mendoza (1995)).

The stylized facts of Sudden Stops suggest that an equilibrium business cycle framework aiming to account for this phenomenon should meet three conditions: First, it needs a transmission mechanism that can produce a stochastic stationary equilibrium in which infrequent Sudden Stops are nested together with regular business cycles (without relying on unusually large, unexpected shocks). Second, in a Sudden Stop episode, typical realizations of the same underlying exogenous shocks that drive normal business cycles in

non-Sudden Stop periods must result in a sharp reversal in the current account, a deep recession, and a fall in asset prices. Third, in a Sudden Stop, endogenous cuts in intermediate goods and capacity utilization need to be an important determinant of the output collapse.

This paper proposes an equilibrium business cycle model that aims to meet these three conditions, and explores to what extent the quantitative predictions of the model account for the observed features of Sudden Stops. The model incorporates some of the financial frictions studied in the Sudden Stops literature into an RBC-SOE setup. In particular, the model is based on the RBC-SOE setup with incomplete contingent claims markets of Mendoza (1991), enhanced to introduce demand for intermediate inputs and endogenous capacity utilization. The latter follows the analysis of Greenwood, Hercowitz and Huffman (1988), which is based in turn on the theoretical principles of Calvo (1975) -- with the difference that in this paper capacity utilization costs are expressed as a direct production cost rather than as a cost reflected in faster depreciation of physical capital.

The standard RBC-SOE model with incomplete markets, even if augmented with intermediate goods and endogenous capacity utilization, cannot produce Sudden Stops because agents in this model still have unrestricted access to a perfect international credit market. Thus, negative shocks to TFP, the world interest rate, or the world price of intermediate goods would induce the standard consumption-smoothing and investment-reducing effects described in Mendoza (1991). Large shocks to TFP could be added to this model in order to trigger large output collapses driven by cuts in intermediate goods and capacity utilization, but this would still fail to explain the Sudden Stops' reversal of the current account and the collapse of consumption. Households would borrow from abroad to smooth consumption and the current account would remain slightly counter-cyclical. These results can be altered by adding large shocks to the world interest rate or access to external financing, but this theory of Sudden Stops would hinge entirely on unexplained "large and unexpected" shocks. The shocks would need to be "large," because by definition they would need to induce recessions larger than the standard non-Sudden-Stop recessions induced by typical shocks, and they would need to be "unexpected" (in the literal sense of being outside the set of possible realizations of random shocks agents work with), because otherwise agents would self-insure to undo their real effects. Moreover, this explanation of Sudden Stops would fail to explain why Sudden Stops affect emerging economies and not industrialized countries -- except for the argument that large, unexpected exogenous shocks hit the former but not the latter.

The financial frictions that the model of this paper incorporates shift the analysis away from a theory of Sudden Stops based on large, unexpected shocks. The model deviates significantly from the frictionless credit market of typical open-economy business cycle models by introducing two credit constraints. First, a collateral constraint in the form of a margin requirement that limits foreign debt in one-period debt contracts not to exceed the liquidation value of a fraction of the economy's capital stock. Second, a constraint on firms' access to working capital financing that limits working capital not to exceed a fraction of the market value of the firms' gross sales (net of wage costs) used as guarantee. These constraints can be viewed as originating in frictions affecting contracting relationships in credit markets, such as limited enforcement or informational asymmetries. The contracting relationships are not explicitly modeled here. Instead, the emphasis is on studying the role that these constraints play in the business cycle transmission mechanism, as in the literature on business cycle effects of endogenous credit constraints in the line of Kiyotaki and Moore (1997) and Kocherlakota (2000).

The margin constraint linking debt to the liquidation value of capital follows the work of Aiyagari and Gertler (1999) and Mendoza and Smith (2005). The setup of this paper differs from theirs in that it allows for endogenous capital accumulation and a dividend stream that varies in response to the collateral constraint. The margin constraint shares some of the features of the collateral constraints proposed by Kiyotaki and Moore (1997) and Kocherlakota (2000) and similar constraints widely studied in the Sudden Stops literature (see, for example, Auenhaimer and Garcia Saltos (2000), Izquierdo (2000) and Caballero and Krishnamurty (2001)). The constraint on working capital is a modification of the working capital constraint typical of limited participation models in which firms borrow funds due at the end of the production period to finance some of their input costs. The difference is that in the model of this paper lenders require firms to secure working capital financing using the firms' sales (trade credits guaranteed on the value of shipments are one example).

The key element of the business cycle transmission mechanism that results from the above credit constraints is that the constraints only bind if agents in the economy are highly indebted. In particular, the economy's leverage ratios, the private debt-equity ratio and/or the ratio of working capital financing to gross output net of labor costs, need to be sufficiently high for the constraints to bind. If these ratios are low, adverse shocks of "standard" magnitude do not alter the economy's access to world credit markets and thus yield the responses typical of RBC-SOE models. In contrast, when the constraints bind, the same "standard" shocks set in motion a dynamic process that replicates the features of a Sudden Stop. Sudden Stops are driven by three "credit channel" effects that amplify and increase the persistence of the effects of exogenous shocks on the economy. Two of them are akin to endogenous external financing premia that arise because the effective costs of borrowing from abroad faced by households and firms increase when the collateral constraint and the constraint on working capital bind. The third is Fisher's classic debt-deflation mechanism: When the collateral constraint binds, it forces agents to liquidate capital in order to meet "margin calls." This fire sale of equity reduces the price of capital and hence tightens further the collateral constraint causing a subsequent round of margin calls. This debt-deflation spiral has important real effects because it hits the Tobin Q and affects future dividends. The deflation of Tobin's Q magnifies the investment collapse and the persistence of the output collapse.

The model explains the loss of credit market access as an endogenous outcome, instead of assuming exogenous shocks to foreign financing. In particular, the high debt levels at which the credit constraints bind and the economy is vulnerable to Sudden Stops are an endogenous feature of the stochastic stationary equilibrium of the economy. These high debt levels are reached with positive long-run probability in response to the dynamics driven by optimal plans for debt and capital accumulation under particular sequences of *typical* realizations of shocks to TFP, the world interest rate and intermediate goods prices. Households self insure to reduce the likelihood of facing a sudden loss of access to credit, but this precautionary saving does not rule out completely the possibility of hitting states with binding credit constraints in the long run. Moreover, in contrast with endowment economy models, in which agents accumulate debt to smooth consumption in response to adverse shocks, the economy of this paper accumulates debt also to finance investment and increase consumption in response to persistent favorable shocks. Hence, states of high debt that trigger Sudden Stops can be preceded by economic expansions, as observed in many emerging economies.

The rest of the paper is organized as follows. Section 2 reviews the empirical regularities of Sudden Stops, with emphasis on Mexico's 1994-95 crisis. Section 3 describes

the model economy and characterizes its competitive equilibrium. Section 4 conducts the numerical analysis based on a calibration to Mexican data. Section 5 concludes.

2. Empirical Regularities of Sudden Stops: The Mexican Case

A growing number of empirical studies document the stylized facts of Sudden Stops across emerging economies (see, for example, Calvo and Reinhart (1999), Calvo, Izquierdo and Mejía (2005), and Milesi-Ferretti and Razin (2000)). This paper focuses instead on a detailed analysis of Mexico's Sudden Stop following the December, 1994 devaluation. The objectives of this analysis are to quantify the stylized facts of the Sudden Stop, to identify potentially relevant features of the business cycle transmission mechanism that was at work at that time, and to provide information for the calibration of the model that anchors the quantitative analysis of Section 4.

Figure 1 plots the levels and the Hodrick-Prescott trend of annual per-capita GDP for the 81-year period 1920-2001. The Figure shows two striking facts. First, the Sudden Stop of 1995 produced the largest Mexican recession since the Great Depression. Second, the trend of Mexico's GDP per capita collapsed at about the same time as the 1982 debt crisis began, and its recovery was prevented first by the protracted recession that followed the debt crisis and then by the deep recession of the 1995 Sudden Stop. Since the 1982 debt crisis can also be regarded as a Sudden Stop, this second fact suggests that vulnerability to Sudden Stops has played a central role in Mexico's growth slowdown (although studying this issue is beyond the scope of this paper).

Figure 2 uses quarterly data for the annualized current-account GDP ratio to illustrate the magnitude of the sudden cutback in access to external financing during the 1982 debt crisis and in the 1995 crash. In both instances, current account deficits of about 8 percent of GDP, which were built up gradually in the years before the Sudden Stops, were fully reversed quickly. In the 1982 episode the reversal was more gradual but it was also larger and more persistent (the current account shifted into surpluses that averaged about 3 percent of GDP for nearly six years). The 1995 current account reversal was nearly immediate but the country returned to current account deficits in late 1996. In line with these observations, Figure 1 shows that the recession after 1982 was milder but more prolonged than the one during the 1995 Sudden Stop.

Figure 3 illustrates the evolution of three key relative prices: real equity prices (in units of the GDP deflator), the price of imported intermediate goods relative to export prices, and the price of nontradable goods relative to tradables (taken from Mendoza (2002)). The plot starts in 1993 because quarterly data for imported intermediate goods prices are not available before this year. The Figure shows a boom in equity prices in the two years before the 1995 Sudden Stop, followed by a collapse in 1995. The real price of imported intermediate goods rose by about 15 percent in 1995 and remained at high levels for almost six years. The relative price of nontradable goods to tradable goods fell by a little more than the increase in intermediate goods prices, but since 1996 the nontradables relative price has followed an increasing trend.

One important aspect of the above price movements is the "liability dollarization" problem identified in the Sudden Stops literature. Since foreign debt obligations of agents in emerging markets are generally denominated in units of world tradable goods (i.e. hard currencies) and this debt is leveraged on incomes and assets denominated in different units (nontradable goods or domestically-produced tradable goods), a sudden drop in the

relative price of nontradables, or a sudden surge in the relative price of imported intermediate goods, impairs the ability of domestic agents to service debts.

Table 1 summarizes key features of Mexico’s business cycles and the Sudden Stop of 1995 using quarterly data. The Table provides standard measures of business cycle variability, co-movement and persistence of macroeconomic time series (GDP, private consumption, fixed investment, the current account-GDP ratio, and equity prices) using the Hodrick-Prescott filter to isolate the cyclical components of the data. The Table also reports business cycle statistics for the three exogenous shocks present in the model of the next Section: the price of imports of intermediate goods relative to export prices, the world interest rate, and a measure of Solow residuals derived from GDP data using factor shares and factor usage data provided by Garcia (2005). These GDP Solow residuals do not match the model’s “true” TFP shock because gross production in the model uses intermediate inputs and features variable capacity utilization of installed capital. Hence, Solow residuals measured with the standard method using GDP data include the effects of changes in capacity utilization and intermediate goods prices. The GDP Solow residuals are the best proxy that can be obtained, however, because the data needed to compute a measure of TFP for gross output free from these effects are not available. Section 4 discusses an identification procedure used to address this problem in the quantitative simulations of the model.

The moments reported in Table 1 are in line with well-known business cycle facts common to most countries (except that the standard deviation of consumption is larger than that of GDP, which is explained by the fact that consumption data includes durable goods). Investment is more volatile than GDP, all variables exhibit positive first-order autocorrelations, consumption and investment are positively correlated with GDP and the current account-GDP ratio is negatively correlated with GDP. Equity prices are very volatile and pro-cyclical. Intermediate goods prices are slightly more variable than GDP and they are countercyclical, while the world interest rate and the GDP Solow residuals are about 60 percent as variable as output and the two are nearly uncorrelated with GDP.

Table 1 also reports measures of the magnitude of the Sudden Stop in each variable (defined as the lowest deviation from trend during the sample period, or the largest one in the case of the current account-GDP ratio and the price of intermediate goods) and the ratio of this measure of Sudden Stop to the standard deviation of the same variable. The latter is an indicator of the extent to which the movements observed during the Sudden Stop exceeded those of Mexico’s “typical” business cycles. The magnitude of the recessions in GDP, consumption and investment, the size of the reversal in the current account, and the collapse in asset prices are significantly larger than those of typical Mexican recessions. Except for the asset price collapse (which measured just below 2 standard deviations), the Sudden Stops in all macro aggregates are well outside the two-standard deviation threshold. With regard to the exogenous shocks at the time of the Sudden Stop, the data show an adverse world interest rate shock of typical size (of about 1 standard deviation), a larger adverse shock to intermediate goods prices (of about 2 standard deviations) and a large negative shock to the GDP-based Solow residual (of almost 3 standard deviations). The latter overestimates the true Solow residual, however, because it includes the decline in capacity utilization and the rise in intermediate goods prices.

An analysis of Mexican National Accounts data provides further insights into the characteristics of the 1995 output collapse. In particular, it shows that changes in measured capital and labor played a relatively minor role in the fall in output compared to changes in use of intermediate inputs, capacity utilization, and TFP. This analysis is

similar to the growth accounting exercise applied to Chile and Mexico by Bergoeing et al. (2002), BKKS, but modified to introduce intermediate inputs and capacity utilization.

Consider first the contribution of variations in the capital stock (k_t), labor usage (L_t) and total factor productivity (A_t) to the 1995 output collapse implied by a Cobb-Douglas technology, $A_t k_t^\beta L_t^\alpha$. The first step to measure these contributions is to construct estimates of factor income shares. Mexican national accounts indicate that average factor shares on GDP are about $\alpha = 0.35$ and $\beta = 0.65$ for the period 1988-2002, but these figures are subject of debate because of well-known problems in measurement of proprietor's income and other forms of labor income. Because of these concerns, BKKS used values of $\alpha = 0.7$ and $\beta = 0.3$, which are more in line with international evidence. Moreover, Garcia (2005) constructed alternative estimates of the factor shares using survey data and obtained shares much higher than those obtained from Mexico's National Accounts. Using the factor shares and capital stock data from BKKS, and labor data from Mexico's national accounts, a large fraction (83 percent) of the 6.2 percent drop in GDP in 1995 is assigned to TFP. Even if the change in the capital stock is ignored because of measurement problems and adjustment costs, the TFP contribution to the output collapse is still large at 68 percent. Using the factor shares calculated directly from the data, the TFP contributions with or without the change in k are even higher.

The results are very different if we consider instead a Cobb-Douglas production function for gross output, $A_t k_t^\beta L_t^\alpha v_t^\eta$, where v_t are intermediate goods. The estimate of η implied by the 1988-2002 average share of intermediate goods in gross output is about 0.43. Given the GDP shares of capital and labor from BKKS and this estimate of η , the implied factor shares of capital and labor in gross output are $\alpha = 0.4$ and $\beta = 0.17$. Combining national accounts data on labor and intermediate inputs, the BKKS capital stock estimates, and these factor shares, the contribution of TFP to the output collapse falls to 53 percent. Moreover, in this gross output specification, using the factor shares of k and L obtained directly from the data instead of the BKKS estimates makes little difference (the TFP contribution based on factor shares from the data is 58 percent).

The contribution of changes in capacity utilization is more difficult to gauge because capacity utilization is ignored in National Accounts. It is relatively straightforward to show, however, that changes in capacity utilization need not be large to play an important role in the fall in output. This can be shown by rewriting the production function of gross output as $A_t (m_t k_t)^\beta L_t^\alpha v_t^\eta$, to allow for variable capacity utilization at rate m_t . Using the factor shares from the previous calculations, if capacity utilization fell 9 percent in 1995, the residual contribution of TFP would fall to 0.27. If capacity utilization fell only half of that, the contribution of TFP falls to 0.4.¹

This analysis of the contributions of inputs to the fall in output indicates that if, as the Sudden Stops literature argues, financial frictions are at the core of the transmission mechanism that trigger Sudden Stops, their effects on investment and employment are of little relevance to explain the *initial* output collapse. Sudden Stops models that focus only on the investment or labor effects of financial frictions could mimic perfectly the observed falls in investment and employment and yet they would explain less than 1/5 of the fall in output. Changes in intermediate inputs and capacity utilization play a much bigger role. Thus, models of Sudden Stops need to model gross production explicitly and need to link financial frictions with intermediate inputs and capacity utilization.

¹ Alternatively, survey data on capacity utilization could be used to measure m , but utilization surveys in Mexico started only in November, 1996 and cover only the manufacturing sector.

The findings of this TFP accounting exercise should not be taken as suggesting that financial factors are irrelevant. The sudden, sharp reversal of Mexico's current account in 1995 clearly shows that during Sudden Stops access to external financing is severely restricted. There is also evidence from firm-level data showing that (a) corporate leverage ratios rise in the buildup phase to a Sudden Stop, and (b) when the Sudden Stop hits, leverage ratios collapse. Chapter II of IMF (2002) reports country and regional aggregates of leverage ratios generated with firm-level data from *Worldscope*, including ratios of debt to assets and debt to market value of equity.² Unfortunately, data limitations prevent computing these ratios before 1992. Still, the median debt-to-assets ratio of Mexican corporations rose by 5 percentage points in the two years before the Sudden Stop, and fell by more than 5 percentage points in the fiscal year 1995-1996. The changes in leverage are even more striking in East Asia. For the aggregate of emerging markets in Asia, the median ratio of debt to market value of equity rose from 0.4 to 1.2 between 1996 and 1998, and then fell by 20 percentage points in 1998-1999.

3. An Equilibrium Business Cycle Model with Financial Frictions

The model economy is a variation of the standard RBC-SOE model with incomplete insurance markets and capital adjustment costs proposed by Mendoza (1991). Two important modifications are introduced here. First, the supply-side of the model is modified to introduce demand for intermediate goods and endogenous capacity utilization. Second, the assumption of perfect credit markets is relaxed to introduce credit constraints. Households face a constraint that limits their access to external debt to the market value of the fraction of the physical capital that they offer as collateral. Firms face a constraint that limits their access to working capital financing to a fraction of their gross sales net of labor costs (as if working capital loans were guaranteed with the firms' sales).

Households

The small open economy is inhabited by a large set of identical, infinitely lived households. The preferences of the representative household are defined over stochastic sequences of consumption c_t and labor supply L_t , for $t=0, \dots, \infty$. As in Mendoza (1991), preferences are modeled using Epstein's (1983) Stationary Cardinal Utility (SCU) function, which features an endogenous rate of time preference, so as to ensure that the small open economy attains a unique, invariant limiting distribution of foreign assets.³ Epstein showed that SCU requires weaker axioms on the preference order over stochastic consumption streams than those that support the standard utility function with a constant rate of time preference. The standard setup requires preferences over future allocations to be risk-independent from past allocations and past allocations to be risk-independent from future allocations, while preferences with endogenous time preference only require the latter. Epstein proved that a preference order consistent with these

²The database *Worldscope* contains annual data on a fiscal-year basis for the main balance sheet items of all publicly-traded firms of a large list of countries, including emerging markets.

³ Since agents face non-insurable income uncertainty and the world interest rate is exogenous, the long-run distribution of foreign assets is not well defined with the standard assumption of a constant rate of time preference equal to the interest rate. Precautionary saving leads foreign assets to diverge to infinity in this case.

axioms has a von Neumann-Morgenstern representation if and only if it takes the form of the SCU function:

$$E_0 \left[\sum_{t=0}^{\infty} \exp \left\{ - \sum_{\tau=0}^{t-1} \rho(c_\tau - N(L_\tau)) \right\} u(c_t - N(L_t)) \right] \quad (1)$$

In the above expression, $u(\cdot)$ is a standard twice-continuously-differentiable and concave period utility function and $\rho(\cdot)$ is an increasing, concave and twice-continuously-differentiable time preference function. Following Greenwood et al. (1988), utility is defined in terms of the excess of consumption relative to the disutility of labor, with the latter given by the twice-continuously-differentiable, convex function $N(\cdot)$. This assumption eliminates the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor independent of consumption. SCU also imposes restrictions linking $u(\cdot)$ and $\rho(\cdot)$ that effectively set an upper bound on the elasticity of the rate of time preference with respect to $c-N(L)$. These restrictions ensure that c_t is a normal good for all t and that the model supports a well-defined unique, invariant limiting distribution (see Epstein (1983)).

There are other methods that yield well-defined stochastic stationary equilibria in SOE models. These methods include setting a fixed rate of time preference arbitrarily higher than the interest rate, modeling households with stochastically finite lives, or introducing ad-hoc functions making the world interest rate depend on foreign assets or making asset trading costly to undertake (see Arellano and Mendoza (2003) and Schmitt-Grohe and Uribe (2002)). The method based on the SCU function has the advantage that it is consistent with two standard features of RBC models: agents are infinitely-lived and the rate of time preference and the rate of interest are equal in the long run. Moreover, this method is in line with the aim to model economic behavior from first principles, rather than imposing exogenous functional forms linking foreign assets to their prices.

In the context of models with credit constraints, SCU has the extra advantage that it supports stationary equilibria in which these constraints can be binding. This is because a binding credit constraint drives a wedge between the intertemporal marginal rate of substitution in consumption and the rate of interest. In a stationary state with a binding credit constraint, the rate of time preference adjusts endogenously to accommodate this wedge. In contrast, in models with an exogenous discount factor credit constraints never bind in the long run (if the rate of time preference is set greater or equal than the world interest rate) or always bind at steady state (if the rate of time preference is fixed below the interest rate).

Households choose sequences of consumption, labor supply, investment in domestic capital, k_{t+1} , and foreign borrowing or lending in one-period international bonds, b_{t+1} , so as to maximize SCU subject to the following period budget constraint:

$$c_t = (d_t + q_t(1 - \delta))k_t - q_t k_{t+1} + w_t L_t - b_{t+1} + b_t R \exp(\varepsilon_t^R) \quad (2)$$

Capital depreciates at a constant rate δ . Households take as given the dividend rate on capital holdings, d_t , the market price of capital, q_t , the wage rate, w_t , and the stochastic gross world real interest rate on foreign assets, $R \exp(\varepsilon_t^R)$. ε_t^R is an interest rate shock that follows a Markov process joint with the other shocks defined later in this section.

The world credit market is imperfect. In particular, lenders require households to guarantee their debt by offering physical capital as collateral. The collateral constraint

takes the form of the margin requirement examined in the equilibrium asset pricing models of Aiyagari and Gertler (1999) and Mendoza and Smith (2005):

$$b_{t+1} \geq -\kappa q_t k_{t+1} \quad (3)$$

Thus, households can borrow up to a fraction κ of the market value of their capital. This constraint resembles a debt contract with a margin clause. Margin clauses require borrowers to surrender the control of collateral assets when the debt contract is entered and give creditors the right to sell the assets when their market value falls below the contract value. There are also other arrangements in financial markets that operate in a similar way as a margin constraint without explicit margin clauses. These include value-at-risk strategies of portfolio risk management used by investment banks and capital requirements set by regulators on financial institutions and institutional investors. For example, if an aggregate shock hits emerging markets, value-at-risk estimates increase and lead investment banks to reduce their exposure in these markets, but since the shock is aggregate, the resulting sale of assets increases price volatility and leads value-at-risk models to require further portfolio adjustments. Dunbar (2000) provides a detailed account of the central role that these mechanisms played in propagating the Russian crisis of 1998 across the financial markets of emerging and industrial economies.

The margin constraint is not derived from an optimal contract between borrowers and lenders. Instead, the constraint is imposed directly as in the models with endogenous credit constraints examined in the literature on credit and business cycles by Kiyotaki and Moore (1997) and Kocherlakota (2000). Still, a credit relationship with a constraint like (3) could result, for example, from an environment in which limited enforcement prevents lenders to collect more than a fraction κ of the value of a defaulting debtor's assets. In states of nature in which (3) binds, the model produces an endogenous, market-determined premium over the world interest rate at which borrowers would agree to contracts in which b_{t+1} satisfies (3).⁴ Note also that the margin constraint differs from the Kiyotaki-Moore constraint, which limits debt to the discounted expected one-period-ahead liquidation value of collateral assets. The two constraints differ on the timing of the valuation of collateral assets and on institutional assumptions that enforce them (i.e., the Kiyotaki-Moore constraint assumes that lenders confiscate the assets of debtors if they default, while margin clauses assume that lenders already control the fraction of a borrower's assets used as collateral). Yet, Mendoza and Smith (2005) showed that the qualitative effects of the two constraints on asset prices are similar, while the time-recursive nature of the margin constraint is significantly more tractable.

Firms

Firms are owned by households and hence discount future profits taking as given the representative agent's stochastic discount factors (i.e., the intertemporal marginal rates of substitution in consumption, the reciprocal of which are denoted by \tilde{R}_{t+1}^t , for $t=0, \dots, \infty$ with $\tilde{R}_0^{-1} = 1$). Firms operate a CRS technology to produce a tradable commodity that

⁴Arellano (2004), Cook and Devereux (2005), Neumeyer and Perri (2005), Oviedo (2004) and Uribe and Yue (2005) study the quantitative implications of country risk for business cycles of small open economies. Arellano endogenizes country risk in a setup of strategic default. Neumeyer and Perri, Oviedo, and Uribe and Yue study the effects of introducing exogenous risk premia of the magnitude observed in the data in models with working capital constraints, while Cook and Devereux conduct a similar experiment in a model with sticky prices.

sells at a world-determined price (normalized to unity without loss of generality). They make plans for factor demands and physical investment, facing unitary investment costs determined by the function $\Psi(i_t/k_t)$, which is linearly homogeneous in i_t and k_t . They also need working capital to pay for a fraction ϕ of their purchases of intermediate goods and capacity utilization costs in advance of sales. Creditors require firms to guarantee working capital loans with their sales net of labor costs, so working capital financing cannot exceed the fraction κ^f of net sales. Capacity utilization entails a direct cost per unit of capital determined by the continuously-differentiable, increasing function $h(m_t)$.⁵ Intermediate goods are tradable in world markets at an exogenous, stochastic relative price $p \exp(\varepsilon_t^P)$, where $\exp(\varepsilon_t^P)$ represents a shock to the world price of intermediate goods. TFP is also subject to random shocks $\exp(\varepsilon_t^A)$.

The firms' problem is to choose labor demand, investment, intermediate inputs, and the rate of utilization of the capital stock so as to maximize their value:

$$E_0 \left[\sum_{t=0}^{\infty} \left(\prod_{j=0}^t (\tilde{R}_j^{j-1})^{-1} \right) \left(\exp(\varepsilon_t^A) F(m_t k_t, L_t, v_t) - w_t L_t - (1 + \phi r_t) (p \exp(\varepsilon_t^P) v_t + h(m_t) k_t) - i_t \left[1 + \Psi \left(\frac{i_t}{k_t} \right) \right] \right) \right], \quad (4)$$

where $r_t \equiv R \exp(\varepsilon_t^R) - 1$, subject to the law of motion for capital,

$$i_t = k_{t+1} - k_t (1 - \delta), \quad (5)$$

and the credit constraint on working capital financing:

$$R \exp(\varepsilon_t^R) \phi (p \exp(\varepsilon_t^P) v_t + h(m_t) k_t) \leq \kappa^f (\exp(\varepsilon_t^A) F(m_t k_t, L_t, v_t) - w_t L_t). \quad (6)$$

Working capital is a within-period loan contracted prior to production at the beginning of each period and paid off after the current output is sold at the end of each period. Hence, lenders set the limit on working capital considering interest and principal and do not lend more than the discounted value of the fraction of sales net of wage costs used to guarantee these loans. The margin constraint faced by households differs in this regard because it applies to a loan paid off one period after it is contracted.

The Competitive Equilibrium & The Credit Channel Effects of The Credit Constraints

A competitive equilibrium for the small open economy is defined by stochastic sequences of allocations $[c_t, L_t, k_{t+1}, b_{t+1}, m_t, v_t, i_t]_0^\infty$ and prices $[q_t, d_t, w_t, \tilde{R}_{t+1}]_0^\infty$ such that: (a) households maximize SCU subject to (2) and (3), taking as given dividends, wages, equity prices, the world interest rate, and the initial conditions (k_0, b_0) , (b) firms maximize their value subject to (5) and (6), taking as given wages, the price of intermediate goods, the world interest rate, the household discount factors and the initial condition k_0 , and (c) the capital, labor and goods markets clear.

In the absence of credit constraints, the competitive equilibrium is the same as in a standard RBC-SOE model. The credit constraints distort this equilibrium by introducing

⁵ This assumption is in line with the view that m is kin to an intermediate input with a direct cost to the firm, but it deviates from the standard capacity utilization setup in which the cost of a higher utilization rate is faster depreciation of capital (see Calvo (1975)). The direct cost is easier to deal with when solving the firms' problem and preserves Hayashi's (1982) results regarding the conditions to equate the marginal and average Tobin Q in models with adjustment costs.

three credit-channel effects. Two of them are external financing premia affecting the cost of borrowing for households and firms and the third is the Fisherian debt-deflation process. These credit-channel effects can be analyzed using the optimality conditions of the competitive equilibrium.

The household's optimality conditions yield the following Euler equation for b_{t+1} :

$$0 < 1 - \frac{\mu_t}{\lambda_t} = E_t \left[\frac{\lambda_{t+1}}{\lambda_t} R \exp(\varepsilon_{t+1}^R) \right] \leq 1 \quad (7)$$

where λ_t is the non-negative Lagrange multiplier on the date- t budget constraint (2), which equals also the lifetime marginal utility of c_t , and μ_t is the non-negative Lagrange multiplier on the collateral constraint (3). It follows from (7) that, when the collateral constraint binds, households face an endogenous external financing premium on the effective real interest rate at which they borrow (R_{t+1}^h) relative to the world interest rate. The expected premium is given by:

$$E_t [R_{t+1}^h - R \exp(\varepsilon_{t+1}^R)] = \frac{\mu_t + \text{cov}(\lambda_{t+1}, \varepsilon_{t+1}^R)}{E_t [\lambda_{t+1}]}, \quad R_{t+1}^h \equiv \frac{\lambda_t}{E_t [\lambda_{t+1}]} \quad (8)$$

In the canonical RBC-SOE model, international bonds are a risk-free asset and $\mu_t=0$ for all t , so there is no premium. If the world interest rate is stochastic, the premium can be positive or negative depending on the sign of the covariance term in (8). If the covariance is positive (negative), foreign assets are a good (bad) hedge to help smooth consumption, and hence the premium would be positive (negative). If the collateral constraint binds, there is a direct effect by which the multiplier μ_t increases the external financing premium. In addition, there is an indirect effect that pushes in the same direction because a binding credit constraint makes it harder to smooth consumption, and hence the covariance between marginal utility and the world interest rate is likely to increase. This external financing premium can be viewed as the premium at which domestic agents would choose debt contracts with loan amounts that satisfy the collateral constraint with equality in a credit market in which the constraint is not imposed directly.

The effects of the external financing premium on the asset price valuation of domestic households can be derived from the Euler equation for capital. Solving forward this equation yields the following expression:

$$(1 - \delta)q_t = E_t \left[\sum_{j=0}^{\infty} \left(\prod_{i=0}^j \left(\frac{(1 - \delta)}{\tilde{R}_{t+i+1}^{t+i}} \right) \right) d_{t+1+j} \right], \quad \tilde{R}_{t+i+1}^{t+i} \equiv \frac{\lambda_{t+i} - \kappa \mu_{t+i}}{\lambda_{t+i+1}} \quad (9)$$

where \tilde{R}_{t+i+1}^{t+i} is the reciprocal of the households' stochastic discount factor. Given the financing premium in (8), it can be shown that a collateral constraint binding at t or expected to bind at any future date, increases the rate at which future dividends are discounted and thus lowers the date- t price of equity. In particular, the Euler equations for bonds and equity yield the following expression for the equity premium (the excess return on equity, $R_{t+1}^q \equiv (d_{t+1} + (1 - \delta)d_{t+1})/q_t$, relative to the gross world interest rate):

$$\begin{aligned} E_t [R_{t+1}^q - R \exp(\varepsilon_{t+1}^R)] &= \frac{\mu_t(1 - \kappa) + \text{COV}_t(\lambda_{t+1}, \varepsilon_{t+1}^R) - \text{COV}_t(\lambda_{t+1}, R_{t+1}^q)}{E_t[\lambda_{t+1}]} \\ &= E_t [R_{t+1}^h - R \exp(\varepsilon_{t+1}^R)] - \frac{\mu_t \kappa + \text{COV}_t(\lambda_{t+1}, R_{t+1}^q)}{E_t[\lambda_{t+1}]} \end{aligned} \quad (10)$$

The above expression collapses to the standard equity premium result from asset pricing theory if the collateral constraint does not bind and the world interest rate is deterministic. As Mendoza and Smith (2005) showed, when the collateral constraint binds it induces direct and indirect effects on the equity premium similar to those affecting the external financing premium. In fact, the equity premium increases one-to-one with the financing premium. The direct effect of the binding collateral constraint is reduced by the term $\frac{\kappa\mu_t}{E_t[\lambda_{t+1}]}$, which measures the marginal benefit of being able to borrow more by holding an additional unit of capital. There is also a new element in the indirect effect that is not present in the external financing premium and is implicit in the covariance between λ_{t+1} and R_{t+1}^q : since a binding collateral constraint makes it harder for households to smooth consumption and self-insure, this covariance term is likely to become more negative when the constraint binds, thereby increasing the equity premium.

Given the sequence of expected equity returns from (10), the forward solution for the households' valuation of equity can be re-written as:

$$q_t(1-\delta) = E_t \left[\sum_{j=0}^{\infty} \left[\prod_{i=0}^j \left(\frac{(1-\delta)}{E_t[R_{t+1+i}^q]} \right) \right] d_{t+1+i} \right] \quad (11)$$

It follows then from (10) and (11) that higher expected returns when the collateral constraint binds, or is expected to bind in the future, increase the discount rate of future dividends and lower equity prices in the present.

The above results reflect the optimality conditions that characterize the borrowing and investment choices of the household. In general equilibrium, the equity market clears and hence asset prices adjust so that the investment plans of households are consistent with those formulated by firms. On the side of firms, the optimality conditions for k_{t+1} and i_t produce familiar results:

$$\left(1 + \Psi \left(\frac{i_t}{k_t} \right) + \left[\frac{i_t}{k_t} \right] \Psi' \left(\frac{i_t}{k_t} \right) \right) = \zeta_t \quad (12)$$

$$\begin{aligned} E_t \left[\left(\tilde{R}_{t+1}^t \right)^{-1} \left(d_{t+1} + (1-\delta)\zeta_{t+1} \right) \right] &= \zeta_t \\ d_{t+1} \equiv \exp(\varepsilon_{t+1}^A) m_{t+1} F_1(m_{t+1} k_{t+1}, L_{t+1}, v_{t+1}) & \\ - h(m_{t+1}) \left(1 + \phi(r_{t+1} + \chi_{t+1} R \exp(\varepsilon_{t+1}^R)) \right) &+ \left[\frac{i_{t+1}}{k_{t+1}} \right]^2 \Psi' \left(\frac{i_{t+1}}{k_{t+1}} \right) \end{aligned} \quad (13)$$

where ζ and χ are the Lagrange multipliers on the investment equation (5) and the working capital constraint (6) respectively.

Notice that, since firms discount at the households' stochastic discount factors, the forward solution of (13) yields asset prices consistent with those from the households' forward solution (9) only when the equilibrium condition $\zeta_t = q_t$ holds. Hence, since the production function and the adjustment cost function satisfy the conditions from Hayashi (1982), the marginal Tobin Q (ζ_t) equals the average Tobin Q (q_t) at equilibrium. This also implies that at equilibrium the resource constraint simplifies to:

$$\begin{aligned} c_t = \exp(\varepsilon_t^A) F(m_t k_t, L_t, v_t) - (1 + \phi r_t) \left(p \exp(\varepsilon_t^P) v_t + h(m_t) k_t \right) & \\ - i_t \left(1 + \Psi \left(\frac{i_t}{k_t} \right) \right) - b_{t+1} + b_t R \exp(\varepsilon_t^R) & \end{aligned} \quad (14)$$

The optimal choice for i_{t+1} (given k_t and q_t) implied by the firm's optimal investment condition (12) represents the firms' demand for investment resources (i.e., its equity supply function). Since (12) is a standard Tobin Q relationship, the fact that $\Psi(\cdot)$ is increasing and convex implies that there is a positive relationship between investment demand and the equity price, or that the firms' equity supply function is upward sloping. This is because adjustment costs prevent firms from instantaneously adjusting the stock of capital to its long-run desired level.⁶ Hence, when a margin call causes a negative shock to the households' demand for equity, firms can only reduce gradually the capital stock and the fall in equity demand is therefore accommodated partly with a reduction in firm investment and partly with a fall in the price of equity.

The second external financing premium triggered by the model's credit constraints is induced by the constraint limiting working capital financing. Firms observe the date- t realizations of the shocks and, since the date- t capital stock is predetermined, they set factor demands and capacity utilization according to these marginal productivity rules:

$$\exp(\varepsilon_t^A)F_2(m_t k_t, L_t, v_t) = w_t \quad (15)$$

$$\exp(\varepsilon_t^A)F_1(m_t k_t, L_t, v_t)[1 + \kappa^f \chi_t] = h'(m_t)[1 + \phi(r_t + \chi_t R \exp(\varepsilon_t^R))] \quad (16)$$

$$\exp(\varepsilon_t^A)F_3(m_t k_t, L_t, v_t)[1 + \kappa^f \chi_t] = p \exp(\varepsilon_t^P)[1 + \phi(r_t + \chi_t R \exp(\varepsilon_t^R))] \quad (17)$$

The labor demand condition in (15) is standard. Labor demand has neutral effects on the working capital constraint because wage costs are not paid in advance and the constraint is set in terms of gross sales net of wage costs. In contrast, the conditions setting demand for intermediate goods and capacity utilization include the distortions induced by the working capital constraint. The terms in square brackets in the left-hand-side of (16) and (17) show the rise in the effective marginal products of v_t and m_t resulting from the extra sales that these factors generate, which relaxes the constraint by the amount $\kappa^f \chi_t$. The term $\chi_t R \exp(\varepsilon_t^R)$ in the right-hand-side of the same expressions reflects the increase in the effective financing cost of working capital when the constraint binds. The distortions on the left- and right-hand-side of (17) combine to yield the firms' external financing premium. This is the premium over r_t at which firms in a competitive market of working capital loans would find it optimal to agree to loan contracts that satisfy constraint (6) with equality even if this constraint is not imposed directly.

It is important to note that, with a CRS production function, the ratio of output net of wage costs to costs of intermediate inputs and capacity utilization is independent of productivity and price shocks, and is also independent of the levels of factor demands. Hence, in this environment the working capital constraint can only be triggered by sufficiently large interest rate shocks and, for a given interest rate shock, the constraint is equally tight for all levels of k_t , m_t , v_t and L_t .

The third credit channel effect present in the model, the Fisherian debt-deflation mechanism, is harder to illustrate than the two external financing premia because the model lacks closed-form solutions for equilibrium equity prices and investment allocations. However, this mechanism can be described intuitively. When the households' collateral constraint binds, they respond to "margin calls" from lenders by rushing to fire-sale equity

⁶ With the standard functional form $\Psi(i/k) = (a/2)[(i/k) - \delta]$, the elasticity of investment with respect to q is $(kq)/(ai)$ so without adjustment costs investment demand would be infinitely elastic.

in order to satisfy the constraint. However, when they do this they meet with firms that feature an upward-sloping supply of equity because of Tobin's Q . These firms thus find it optimal to lower investment given the reduced demand for equity and higher discounting of future dividends, and hence at equilibrium equity prices fall. But if the collateral constraint was binding at the initial equity prices and equity holdings, it must be more binding with the reduced prices and investment levels, so another round of margin calls takes place and Fisher's debt-deflation mechanism is set in motion.

4. Quantitative Analysis

This section studies the quantitative implications of the model by analyzing the results of numerical simulations calibrated to Mexican data.

Functional Forms and Numerical Solution Method

The functional forms of preferences and technology are specified as follows:

$$u(c_t - N(L_t)) = \frac{\left[c_t - \frac{L_t^\omega}{\omega} \right]^{1-\sigma} - 1}{1-\sigma}, \quad \sigma, \omega > 1, \quad (18)$$

$$v(c_t - N(L_t)) = \gamma \left[\text{Ln} \left(1 + c_t - \frac{L_t^\omega}{\omega} \right) \right], \quad 0 < \gamma \leq \sigma, \quad (19)$$

$$F(m_t k_t, L_t, v_t) = A(m_t k_t)^\beta L_t^\alpha v_t^\eta, \quad 0 \leq \alpha, \beta, \eta \leq 1, \quad \alpha + \beta + \eta = 1, \quad A > 0, \quad (20)$$

$$\Psi \left(\frac{i_t}{k_t} \right) = \frac{a}{2} \left(\frac{i_t}{k_t} - \delta \right), \quad a, \delta \geq 0 \quad (21)$$

$$h(m_t) = \frac{m_t^\theta}{\theta}, \quad \theta \geq 1. \quad (22)$$

The period utility and time preference functions in (18) and (19) are standard from RBC-SOE models. The parameter σ is the coefficient of relative risk aversion, ω determines the wage elasticity of labor supply, which is given by $1/(\omega - 1)$, and γ is the semi-elasticity of the rate of time preference with respect to composite good $c - N(L)$. The parameter restriction $\gamma \leq \sigma$ is one of the conditions required to ensure that the SCU function can support a unique, invariant limiting distribution of foreign assets (see Epstein (1983)). The Cobb-Douglas production function (20) is the same used in the TFP accounting exercises of Section 2. Equation (21) describes the investment-adjustment cost function. The parameter a determines the marginal adjustment cost relative to the investment-capital ratio. The depreciation rate is subtracted from the investment-capital ratio to ensure that the steady-state adjustment cost is zero. Given this adjustment cost formulation, it follows from the total differential of the firms' investment optimality condition in (12) that the elasticity of investment with respect to q_t is given by $q_t/a(i_t/k_t)$. Following Hayashi (1982), the production function and the adjustment cost function are linearly homogeneous in their arguments. The isoelastic capacity utilization cost in (22) is standard from the capacity utilization literature, with the caveat mentioned before that

this cost is modeled here as a direct production cost rather than as an indirect cost resulting from faster depreciation of the capital stock.

The model is solved numerically by representing the competitive equilibrium in recursive form. Mendoza and Smith (2005) and Arellano and Mendoza (2003) describe different algorithms for solving small open economy models with asset trading and credit constraints. This paper uses Mendoza and Smith's social planner's method which involves solving a single Bellman equation via value function iteration. The endogenous state variables of the planner's problem are k and b . These are chosen from discrete grids of NK non-negative values of the capital stock, $K=\{k_1 < k_2 < \dots < k_{NK}\}$, and NB values of bond positions, $B=\{b_1 < b_2 < \dots < b_{NB}\}$. The exogenous states are the realizations of the shocks given by the triple $e=(\varepsilon^A, \varepsilon^R, \varepsilon^P)$. The shocks follow a joint Markov process, which defines the set E of all triples of possible realizations of the shocks and their one-step transition probability matrix π . Hence, the state space of the problem is defined by all the combinations (k, b, e) in the set $K \times B \times E$.

The Bellman equation of the planner's problem is the following:

$$V(k, b, e) = \max_{(k', b') \in K \times B, c, L, m, v} \left\{ \frac{\left(c - \frac{L^\omega}{\omega} \right)^{1-\sigma}}{1-\sigma} + \exp\left(-\gamma \left[\text{Ln} \left(1 + c - \frac{L^\omega}{\omega} \right) \right] E[V(k', b', e')]\right) \right\}$$

s.t.

$$c = \exp(\varepsilon^A) A(mk)^\beta L^\alpha v^\eta - (k' - (1-\delta)k) \left(1 + \frac{a}{2} \left(\frac{k' - (1-\delta)k}{k} - \delta \right) \right) \quad (23)$$

$$- (1 + \phi(R \exp(\varepsilon^R) - 1)) \left(\frac{m^\theta}{\theta} k + p \exp(\varepsilon^P) v \right) - b' + bR \exp(\varepsilon^R)$$

$$b' \geq -\kappa \zeta k'$$

$$\phi \left(\frac{m^\theta}{\theta} k + p \exp(\varepsilon^P) v \right) \leq \frac{\kappa^f [(1-\alpha) \exp(\varepsilon^A) A(mk)^\beta L^\alpha v^\eta]}{R \exp(\varepsilon^R)}$$

If the collateral and working capital constraints never bind, this Bellman equation reduces to the standard representation of the competitive equilibrium of an RBC-SOE model as a planner's problem. With the credit constraints, the planner's problem includes terms representing equilibrium asset prices and labor costs, which are taken from the firm's optimality conditions.

Calibration to Mexican Data

The calibration exercise matches long-run empirical regularities of Mexican data with properties of the model's deterministic stationary equilibrium. The Mexican data are taken from the same set of 1988-2001 National Accounts data used in Section 2. In the deterministic stationary state, the stochastic shocks are set to zero so that TFP, intermediate goods prices and the world interest rate take their mean values. The mean price of intermediate goods is set to 1. The mean annual gross real interest rate is set at 1.065, which is the standard value of the real interest rate in the RBC literature. The mean of TFP is derived from the set of calibration conditions described below. The calibration also assumes that the credit constraint coefficients, κ and κ^f , are low enough so that the constraints do not bind at the deterministic stationary state.

The average share of intermediate inputs in gross output is 0.43. Assuming that firms finance 30 percent of purchases of intermediate inputs and utilization costs with working capital (i.e., $\phi=0.3$), the firms' optimality condition for demand of intermediate inputs (equation (17)) implies $\eta=0.43(1+\phi(R-1))=0.439$.⁷ This factor share, combined with the labor and capital shares on GDP from Bergoeing et al. (2002), which are 0.7 and 0.3 respectively, implies the following factor shares in the production function of gross output:

$\alpha = 0.7 \left(1 - \frac{\eta}{1 + \phi(R-1)} \right) = 0.398$ and $\beta=1-\alpha-\eta=0.162$. The values of η , ϕ and R also imply that the ratio of gross output to GDP is $(1+\phi(R-1))/[(1+\phi(R-1))-\eta]=1.757$. Given this ratio, and assuming that capacity utilization costs are just 3 percent of GDP, the optimality condition for capacity utilization (eq. (16)) implies that $\theta=\beta(1.757)/0.03=9.508$.

The Euler equation for capital accumulation (eq. (13)) evaluated at steady state yields an equation that determines the depreciation rate as a function of the investment-GDP ratio, the ratio of gross output to GDP and the values of β , θ , a , and R . The values of β , θ , R and the gross output-GDP ratio were determined above, and the average share of fixed investment in GDP in the data is about 16 percent. The value of a is set later in the stochastic simulations to match the business cycle variability of investment. However, this parameter has only a second order effect on the steady state investment Euler equation. Setting $a=0$, the Euler equation implies $\delta=0.109$. Matching the cyclical variability of investment will require setting $a=0.165$, and with this the investment Euler equation yields $\delta=0.11$.

The households' optimality condition for labor supply equates the marginal disutility of labor with the real wage, which at equilibrium is equal to the marginal product of labor. Given (18) and (20), this equilibrium condition reduces to: $L_t^\omega = \alpha \exp(\varepsilon_t^A) F(\cdot)$. Using the logarithm of this expression and Mexican data on gross output and employment growth, the implied value of the exponent of labor supply in utility is $\omega=2.12$.

Since demand for GDP in the data includes government expenditures, the model needs an adjustment to consider government purchases in order for the deterministic steady state calibration to match the average consumption-GDP ratio in the data (0.67). This adjustment is done by setting the deterministic steady state to match the average ratio of government purchases to GDP in the data (0.10), assuming that these government purchases are unproductive and paid out of a time-invariant, ad-valorem consumption tax. The tax is equal to the ratio of the GDP shares of government and private consumption, $0.10/0.67=0.15$, which is very close to the statutory value-added tax rate in Mexico. Since this tax is time invariant, it does not distort the households' intertemporal decision margins and any distortion on the consumption-leisure margin does not vary over the business cycle.

Once the preference and technology parameters set in the previous paragraphs are determined, the equilibrium conditions for factor demands for m , L and v , the firms' steady-state Euler equation for capital accumulation, and the condition that the ratio of gross output per unit of labor matches the average ratio observed in the data (14.3), are solved as a five-equation, nonlinear simultaneous equation system in the steady state values of k , m , L , v and A . The solutions are: $k=49.35$, $m=0.84$, $L=4.17$, $v=25.51$ and $A=4.45$. Given these, the values of gross output and GDP are $Y \equiv F(mk, L, v) = 59.7$ and $GDP = 39.98$.

⁷ These calculations reflect the fact that the National Accounts definition of the cost of intermediate inputs does not include financing costs of working capital.

The steady-state *level* of consumption is set by the product of GDP times the average consumption-GDP ratio taken from the data, $c=0.67 \times GDP=22.74$. The steady-state foreign asset position follows then from the household budget constraint (eq. (2)) evaluated at steady state. The solution is $b=-12.24$. This implies a ratio of external debt to GDP of about 36 percent, in line with the estimates produced for Mexico by Lane and Milesi-Ferretti (2001). Finally, the value of γ (the semi-elasticity of the rate of time preference with respect to the argument of utility) follows from the steady-state consumption Euler equation, which implies $\gamma = \frac{\ln(R)}{\ln(1 + c - \omega^{-1}L^\omega)} = 0.024$. As is typical in calibration exercises for RBC-SOE models with SCU preferences (see Mendoza (1991)), the value of the time preference coefficient needed to match the observed debt ratio is very small, suggesting that the “impatience effects” introduced by the endogenous rate of time preference have negligible quantitative implications on business cycle dynamics.

Markov Process of Exogenous Shocks

Price, TFP, and world interest rate shocks follow a joint Markov process. This Markov process is designed to represent a discrete approximation to a vector-autoregression of the cyclical components of TFP, the gross world interest rate and the relative price of intermediate inputs as defined in Section 2. The approximation is produced using Tauchen’s (1991) quadrature method. Given that the cyclical components of these variables have zero mean, the VAR representation of the system is the following:

$$e_{t+1} = RHO \cdot e_t + u_t, \quad (24)$$

$$e_t \equiv \begin{bmatrix} \varepsilon_t^R \\ \varepsilon_t^P \\ \varepsilon_t^A \end{bmatrix}, \quad RHO \equiv \begin{bmatrix} \rho_R & \rho_{R,P} & \rho_{R,A} \\ \rho_{P,R} & \rho_P & \rho_{P,A} \\ \rho_{A,R} & \rho_{A,P} & \rho_A \end{bmatrix}, \quad u \equiv \begin{bmatrix} u_t^R \\ u_t^P \\ u_t^A \end{bmatrix}$$

The errors in the vector u are independently and identically distributed over time, with zero mean and a known stationary variance-covariance matrix $cova(u)$.

Tauchen’s algorithm takes as input the VAR estimates of RHO and $cova(u)$ and the desired number of elements of the vector of Markov realizations of the shocks, and it returns as output the vectors of realizations, the transition probability matrix π of moving across states of the Markov process, and the associated vector Π of long-run probabilities of hitting each state.

The ideal approach is to estimate the VAR using actual data for all three shocks and then apply Tauchen’s algorithm. As explained in Section 2, however, limitations of the Mexican data make it unfeasible to construct a reliable series of TFP for gross output. Hence, the approach adopted here follows an “identification” procedure that begins with a VAR estimated using the actual data for intermediate goods prices and the world interest rate and the GDP-based Solow residuals from Section 2, and then adjusts the elements of RHO and $cova(u)$ related to TFP shocks so that the model matches the relevant moments for GDP (its standard deviation, first-order autocorrelation, correlation with interest rate shocks and correlation with price shocks). This procedure is similar to an approach used often in RBC analysis to set the moments of productivity shocks so that the models mimic the cyclical moments of GDP (see Greenwood et al. (1988)).

The VAR estimation in the first step of the identification procedure yields these results (listing only statistically-significant values):

$$RHO \equiv \begin{bmatrix} .205933 & 0 & 0 \\ 0 & .584573 & 0 \\ 0 & 0 & .671174 \end{bmatrix}, \quad cov(u) \equiv \begin{bmatrix} .00012 & 0 & 0 \\ 0 & .000531 & 0 \\ 0 & 0 & .000123 \end{bmatrix} \quad (25)$$

These results suggest that the shocks could be modeled as independent AR(1) processes. However, the solutions that the model yields using the Markov process produced by Tauchen's algorithm with these VAR results and 2 realizations per shock overestimate the standard deviation of GDP (4.7 percent v. 2.6 percent in the data) and underestimate the correlation of GDP with intermediate goods prices (-0.65 v. -0.56 in the data). The second step of the identification procedure shows that the model matches these two moments by reducing $cova(u)_{AA}$ from 0.000123 to 0.00006 and increasing $cova(u)_{PA}$ and $cova(u)_{AP}$ from 0 to 0.000105. This specification also matches the observed standard deviation of investment with the adjustment cost parameter set to $a=0.165$.

The reduction in the variance of TFP innovations reflects the fact that the GDP-based Solow residuals used in the VAR estimation overestimate the variability of the "true" TFP shocks that the model requires to mimic the observed standard deviation of output. This is the case because the GDP-based Solow residuals include changes in capacity utilization and intermediate goods prices. The increase in the covariance of the innovations of the price and TFP processes is needed to prevent the correlation between GDP and intermediate goods prices from falling sharply as the variance of TFP innovations falls. Without this correction, the reduction in $cova(u)_{AA}$ lowers the GDP-intermediate goods prices correlation to -0.9.

After the identification procedure is completed, Tauchen's algorithm yields the following Markov realizations of the shocks and associated long-run probabilities (the 8×8 transition matrix is omitted to save space but is available from the author on request):

$$(\varepsilon^R, \varepsilon^P, \varepsilon^A) = \begin{bmatrix} -0.010954 & -0.023043 & -0.010821 \\ 0.010954 & -0.023043 & -0.010821 \\ -0.010954 & 0.023043 & -0.001707 \\ 0.010954 & 0.023043 & -0.001707 \\ -0.010954 & -0.023043 & 0.001707 \\ 0.010954 & -0.023043 & 0.001707 \\ -0.010954 & 0.023043 & 0.010821 \\ 0.010954 & 0.023043 & 0.010821 \end{bmatrix}, \quad \Pi = \begin{bmatrix} 0.12889 \\ 0.12889 \\ 0.12111 \\ 0.12111 \\ 0.12111 \\ 0.12111 \\ 0.12889 \\ 0.12889 \end{bmatrix}. \quad (26)$$

Tauchen's method yields 8 combinations of the shocks. Productivity and interest rate shocks have one symmetric pair of realizations each, but the quadrature approximation needs *two* symmetric pairs of TFP realizations in order to approximate the non-zero elements off the main diagonal of $cova(u)$.

The unconditional moments of the shocks in the Markov process are as follows. Standard deviations: $\sigma(\varepsilon^R)=1.09$ percent, $\sigma(\varepsilon^P)=2.3$ percent and $\sigma(\varepsilon^A)=0.78$ percent. First-order autocorrelations: $\rho(\varepsilon^R)=0.203$, $\rho(\varepsilon^P)=0.526$ and $\rho(\varepsilon^A)=0.584$. Contemporaneous correlations: $\rho(\varepsilon^P, \varepsilon^A)=0.605$, $\rho(\varepsilon^P, \varepsilon^R)=\rho(\varepsilon^R, \varepsilon^A)=0$. The absolute values of the Markov realizations of ε^R and ε^P measure one standard deviation of the corresponding shock, while those for ε^A measure 0.22 and 1.38 standard deviations of the TFP process. The moments of interest rate and price shocks are close to the corresponding moments from

the data in Table 1, but they are not a perfect match because of the quadrature approximation. The approximation improves by increasing the number of realizations of each shock in the Markov process but the cost in computing time is high due to the curse of dimensionality.

Baseline Simulation Results

The baseline experiment compares the results of a simulation in which the collateral constraint and the working capital constraint never bind (the “frictionless model”) with those from simulations in which the credit constraints bind in some states of nature. The latter include three scenarios, one with the collateral constraint, one with the working capital constraint, and one with the two constraints together.

The simulations use a grid of capital stocks with 52 evenly-spaced nodes spanning the interval [44.24,51.49], and a grid of bond positions with 100 evenly-spaced nodes spanning the interval [-41.07,18.78]. Since there are eight triples of realization of the shocks, the discrete state space has $52 \times 100 \times 8$ coordinates that represent all possible combinations of k , b and e .⁸ The B grid has twice the number of elements of the K grid so that when the margin constraint binds the algorithm can find points in the B grid that approximate closely the values implied by the collateral constraint.

The statistical moments that characterize business cycles in the stochastic steady state of the frictionless economy, which are computed using the economy’s limiting distribution of k , b , and e , are reported in Panel I of Table 2. These moments are consistent with standard results from RBC-SOE models: Consumption is less volatile than GDP, investment is more volatile than GDP, consumption and investment are procyclical and they display positive persistence. However, some of the other moments are not a close match to the moments from Mexican data in Table 1.

As indicated earlier, the frictionless model cannot produce Sudden Stops because it assumes that credit markets are perfect. In particular, as the quantitative results that follow show, the economy responds to adverse shocks with a relatively smooth adjustment of the current account and “regular” recessions in absorption and production, regardless of the size of household debt and working capital financing or the leverage ratios of these obligations relative to the value of assets or net sales.

Panels II-IV of Table 2 lists business cycle moments for the simulations with credit constraints. Consider first the economy with margin constraints only. Panel II reports results for a simulation that sets $\kappa = 0.4$, so that the “capital leverage ratio” (i.e. the ratio $-b'/qk'$) cannot exceed 40 percent. In the frictionless economy, the average capital leverage ratio is about 20 percent with a standard deviation of 0.1, which is nearly 4 times larger than the variability of GDP. Thus, the value of κ was set 2 standard deviations higher than the average capital leverage ratio of the frictionless economy. This setup of the margin constraints allows the model to approximate the observed frequency of Sudden Stops in the 1980:1-2002:4 sample of quarterly data for Mexico. This probability is 1.09 percent counting only the 1995 crisis as a Sudden Stop, or 2.17 if the 1982 debt crisis is also counted as a Sudden Stop. Panel II shows that, with $\kappa=0.4$, the model reaches states of nature in which the collateral constraint binds with 1.7 percent probability.

⁸ A simulation with 100 nodes in the K grid and 200 nodes in the B grid showed negligible differences in the first and second long-run moments of the endogenous variables, but CPU time increased by a factor of 30 (from 18 to 551 minutes using a Pentium Xeon 2.4Mhz processor).

The high leverage states at which the economy is vulnerable to Sudden Stops are reached after sequences of realizations of the shocks lead to capital leverage ratios approaching 40 percent. Because of the curvature of the constant-relative-risk-aversion period utility function, households are highly averse to the drastic consumption adjustments implied by Sudden Stops, and hence they have an incentive to engage in precautionary saving to minimize the risk of Sudden Stops.⁹ In the long run, this eliminates from the stochastic stationary equilibrium states of nature with levels of capital and debt in which Sudden Stops would trigger excessive consumption collapses. As a result, the long-run business cycle indicators reported in Panel II of Table 2 show negligible differences compared with those of the frictionless economy in Panel I.

The fact that Panels I and II show similar business cycle moments indicates that “regular” business cycles have similar features in the model with collateral constraints and in the frictionless economy. The next task is to show that, in the economy with collateral constraints, Sudden Stops with unusually large recessions coexist with regular cycles. To show that this is the case, Figures 4.a and 4.b plot the differences in the forecasting functions of the equilibrium Markov processes of macroeconomic aggregates between the simulation with collateral constraints and the frictionless model, in response to combined one-standard-deviation shocks to the world interest rate, intermediate goods prices and TFP at $t=1$. These forecasting functions are Markovian impulse response functions that preserve all the non-linearity of the decision rules of the recursive equilibrium of the model. The forecasting functions are conditional on an initial condition of high debt (or high initial leverage defined as the b/qk ratio) inside the “Sudden Stop region” of the state space in which adverse shocks trigger a Sudden Stop with positive long-run probability. Since initial conditions of this class are distant from the long-run averages of k and b , the figures show the difference between the forecasting functions of the economies with and without credit constraints. This removes the low-frequency transitional dynamics driving the variables to converge to their long-run means in the two economies regardless of the existence of credit constraints (as Table 2 showed, these long-run means are very similar in the two economies). The economy with collateral constraints hits the initial state used in the forecasting functions of Figures 4a-4b with 0.0012 percent probability in the long run. The plots show the first 20 quarters of the forecasting functions as percent deviations of the corresponding long-run means in the frictionless economy.

Figure 4a shows that, when adverse shocks trigger the collateral constraint, there is no difference on the impact effects at $t=1$ on output and factor demands in the economies with and without collateral constraints. This is because the initial capital stock is predetermined and identical in both simulations, and because the realizations of the shocks are identical. In contrast, the impact effects on consumption, investment, the Tobin Q and the external accounts shown in Figure 4.b are strikingly different in the economy with collateral constraints. The decline in consumption is 2 percentage points larger, investment declines nearly 35 percent more, and the Tobin Q falls by nearly 0.9 percent more. The current account- and trade balance-GDP ratios experience reversals of about 6 percentage points of GDP.

Output and factor utilization display larger and more persistent declines in the economy with collateral constraints starting one period after the shocks hit. As Figure 4.a

⁹ This quantitative feature of the model is in line with Carroll’s (2001) results about the role of precautionary saving in explaining U.S. private consumption behavior. He found that the effects of precautionary saving and liquidity constraints are almost indistinguishable because the precautionary saving motive is akin to a self-imposed reluctance to borrow.

shows, GDP, intermediate inputs, labor and capital are 1 to 4.75 percentage points lower 4 periods after the shocks hit the economy, and it takes over 20 periods for these variables to converge to the levels of the frictionless economy (working capital financing in Figure 4.b shows the same pattern). The shocks induce firms to substitute away from factor demands into more intensive use of existing capital by increasing the rate of capacity utilization, by about a quarter of a percentage point, and this increase also takes about 20 periods to be reversed. The increase in utilization reduces the adverse effects of the shocks on output, and hence (in the absence of the constraint on working capital financing) it is actually a force that weakens Sudden Stops.

Figure 4.b shows that a Sudden Stop induces a tilt of the time profile of consumption, with lower consumption than in the frictionless economy for the first 10 periods after the shocks hit, converging thereafter to a slightly higher consumption level (as indicated by the small difference in mean consumption levels in Panels I and II of Table 2). The Figure also shows that the large, immediate changes in investment, the Tobin Q and the external accounts are relatively short lived, compared with the persistent effects on output and factor usage.

Kocherlakota (2000) argued that, in examining the business cycle implications of credit constraints, it is important to separate persistence, as illustrated in Figures 4.a-4.b, from amplification. Amplification measures how much larger are the economic fluctuations driven by binding credit constraints than those obtained with perfect credit markets.¹⁰ He measured output amplification as the absolute value of the difference between output in the period after a once-and-for-all, linear income shock hits a deterministic small open economy, minus its steady-state output, relative to the size of the shock. Fixing the labor share at 60 percent and varying the shares of capital (land) from 10 to 30 (30 to 10) percent in a setup in which labor and land are inelastic, he found small amplification effects (at most 34.9 percent for output and 0.8 percent for the price of land).

Kocherlakota's measure of amplification is adjusted here to consider the stochastic nature of the model. Amplification is measured as the difference between the values of the variables in the frictionless economy and those pertaining to the economies with credit frictions, relative to the standard deviation of each variable in the frictionless economy. Hence, this amplification coefficient measures how much larger are the recessions of a Sudden Stop in units of the standard deviations that measure "normal" business cycles. The top charts of Figures 5-8 plot the amplification effects on GDP, consumption, the current account-GDP ratio and Tobin's Q for the first 20 periods after the same adverse shocks considered in Figures 4a-4b hit the economy (with the same initial conditions for k and b). These amplification effects are larger than Kocherlakota's. The initial amplification effects are 40 percent for GDP (Figure 5) and 250 percent for the price of capital (Figure 8), and there are also sizable initial amplification effects on consumption (Figure 6) and the current account-GDP ratio (Figure 7). These estimates imply that GDP (the price of capital) is lower in the economy with collateral constraints than in the frictionless economy by a deviation from the mean that is 0.4 (2.5) times larger than the standard deviation of output (the price of capital) in the frictionless economy. Moreover, the initial amplification effects are not the largest ones. The amplification effect on GDP peaks on date 4 at about 75 percent, which is more than twice the largest estimate of amplification obtained by Kocherlakota.

¹⁰ Kocherlakota also noted that the effects of the credit constraints are asymmetric, because there are larger and more persistent recessions in response to adverse shocks but the opposite does not occur with favorable shocks.

The Fisherian debt-deflation mechanism plays an important role in driving the amplification effects of the collateral constraint. The surface plots in Figures 9-10 illustrate this point. Figure 9 shows consumption impact effects (i.e., deviations from the long-run mean of consumption on the initial date that adverse, one-standard-deviation shocks hit the economy) for all pairs (k,b) in the state space. The plot on the left is for the economy with perfect credit markets and the plot on the right is for the economy with collateral constraints. Figure 10 shows similar plots for the impact effects on Tobin's Q . The region for the lowest values of b in the plots of the economy with collateral constraints cannot support equilibrium allocations because, for the corresponding (k,b) pairs, there is no pair (k',b') in the state space that can keep consumption positive and at the same time satisfy the collateral constraint.

Figure 9 shows large drops in consumption in response to adverse shocks of standard size in the high-leverage area of pairs (k,b) where the collateral constraint binds. These consumption collapses are much larger than the “mild” recessions that these shocks cause in the same economy outside of this area, or in the same area for the economy with perfect credit markets. In contrast, the impact effects on consumption are nearly identical in the two economies for high values of b that are outside the region in which collateral constraints bind. Note that, because of the precautionary saving effect, the area of large consumption collapses in the economy with collateral constraints includes many states that have zero probability in the long run. These states can be interpreted as representing outcomes that the model would produce for “large, unexpected” shocks (i.e., shocks that could move the economy to states with leverage ratios that otherwise would have zero probability in the long run). In response to shocks of this type, the model can predict massive consumption collapses of up to 60 percent!

Figure 10 shows the effects of the Fisherian deflation. Adverse shocks cause small declines in the price of capital when credit markets are perfect, or outside the area of binding margin calls in the economy with collateral constraints (i.e., when the capital leverage ratio is low or b is sufficiently high). On the other hand, when debt is sufficiently high (or b is low), the same adverse shocks trigger the collateral constraint and this results in agents fire-selling capital. The price of capital thus sinks below the value that would have prevailed in the economy with perfect credit markets, but this tightens further the collateral constraint, inducing agents to reduce further their capital holdings and cause further price declines. Figure 10 shows the end result of this process. The downward spiral in the price of capital is hampered by the future output loss that the implied reduction in investment would cause, but even so there are still states of nature at t in which agents may seek to reduce their capital holdings for $t+1$ down to the lowest feasible value of k (i.e., the first value in the capital grid). This can be inferred from the area of the plot for the economy with collateral constraints in Figure 10 in which the price impact effect behaves like a smooth linear, negative function of k . For all these coordinates, k' falls to the lowest value of the K grid when the adverse shocks hit. The resulting price decline is larger the larger the distance between k and the lowest value of capital in the K grid. This lower bound in capital can be interpreted as if agents were hitting a constraint on equity sales. However, this constraint is not binding in the long run, because in the limiting probability distribution of the economy with collateral constraints the lowest value in the K grid with positive long-run probability is the 6th coordinate. Thus, the states of nature with the largest equity price collapses and largest downward investment responses are again ruled out in the long run by precautionary saving.

The simulation results for the economy with collateral constraints reproduce several features of Sudden Stops. They show that unusually large and persistent recessions can

take place in response to shocks of “standard” magnitude when the economy is highly leveraged, and that these Sudden Stops are nested within smoother “normal” business cycles. The results also show that the economy arrives with positive long-run probability at these high-leverage states, and that persistence and amplification effects on business cycles induced by credit constraints are large. However, there are also some important flaws. In particular, the model produces two counterfactual outcomes. First, output, factor demands, capacity utilization and working capital financing do not respond on impact when a Sudden Stop starts. Second, capacity utilization rises above its long-run mean during the Sudden Stop. The results of simulations that activate the working capital constraint show that this constraint allows the model to partially address these problems.

Panels III and IV of Table 2 report the business cycle moments for simulations with the working capital constraint alone and with both the collateral constraint and the working capital constraint. The middle and bottom charts of Figures 5-8 show the corresponding amplification effects, and Figure 11 shows the conditional forecasting functions for the same initial conditions as Figures 4a-4b. Despite the potential for general equilibrium feedback effects between the collateral and working capital constraints, the forecasting functions of foreign assets, capital, investment, the Tobin Q and the external accounts are nearly invariant to the addition of the working capital constraint (i.e., the simulation with the working capital constraint alone yields almost the same forecasting functions for these variables as the frictionless model, and the simulation with both working capital and collateral constraints yields nearly the same results as the simulation with collateral constraints alone). Hence, Figure 11 only shows plots for those variables that displayed significant changes in forecasting functions as a result of the introduction of the working capital constraint: consumption, GDP, factor demands, capacity utilization and working capital.

A comparison of Panels III and IV with Panel I in Table 1 shows again that the credit constraints have small effects on the long-run business cycle statistics. However, the two simulations with the working capital constraint display slightly lower long-run averages for consumption, output, factor demands, capacity utilization, and working capital, which explains why the differences in forecasting functions for the simulations with working capital constraint in Figure 11 do not converge to zero over time. These changes in long-run averages are the result of assumptions about the production technology and the nature of the working capital constraint that affect the mechanism that triggers this constraint. The working capital constraint sets a limit on the ratio of working capital financing as a fraction of sales net of wage costs. However, with a Cobb-Douglas technology and competitive factor pricing, this ratio depends only on the realization of the interest rate shock and a subset of parameter values (the factor shares of the production function, the curvature parameter of the cost of capital utilization, and the fraction of costs of intermediate inputs and costs of utilization paid in advance of production). Thus, by construction, shocks to productivity or intermediate goods prices, as well as the values of k and b , cannot affect the working capital constraint. Given the two-point structure of the Markov vector of interest rate shocks, the simulations with working capital constraints yield a binding (non-binding) working capital constraint when the interest rate is high (low). This implies that the working capital constraint binds 50 percent of the time in the limiting distribution, and since it binds often its effects are more noticeable in the long-run moments. Adding more points to the Markov vector of interest rate shocks can reduce the probability of a binding working capital constraint in the long run, but the fact that the constraint is triggered only by interest rate shocks, regardless of the values of the other state variables, would be unaltered.

Figure 11 shows that the working capital constraint yields sizable contemporaneous effects on GDP and factor demands in response to one-standard-deviation, adverse shocks. GDP and labor fall by 0.8 to 1 percentage points more than in the frictionless model when the shocks first hit the economy, while intermediate goods drop 3 percentage points and capacity utilization falls by 0.3 percent. Thus, the working capital constraint enables the model to produce a fall in output contemporaneous with the current account reversal as a Sudden Stop begins, and it also eliminates the counterfactual response of capacity utilization found in the simulation that only used the collateral constraint. In the simulation with working capital constraint only, utilization recovers from the large initial hit but remains always below the mean of the frictionless model. In the model with the two constraints, utilization takes a similar initial hit and it only rises temporarily above the long-run mean in the model with perfect credit markets (utilization is permanently below the mean of the frictionless model by about 0.1 percent). The impact effect on consumption is almost 1 percent in the economy with the working capital constraint alone. After that, consumption converges slowly to its long run average. In contrast, the simulation with the two constraints yields an initial decline in consumption of nearly 3 percent, and the recession in consumption displays much more persistence.

5. Conclusions

This paper proposes an equilibrium business cycle model with imperfect credit markets that accounts for key features of the Sudden Stop phenomenon. The assumption of a perfect international credit market, typical of real-business-cycle models of the small open economy, is replaced with a credit market that features two credit constraints: A collateral constraint that limits debt not to exceed a fraction of the market value of the economy’s physical assets, and a working capital constraint that limits working capital financing not to exceed a fraction of sales net of labor costs. These constraints only bind in states in which the economy’s leverage ratios (the ratio of debt to market value of capital and the ratio of working capital financing to net sales) are sufficiently high. These states are an endogenous outcome of the economy’s stochastic competitive equilibrium dynamics in response to exogenous random shocks to productivity, the world interest rate, and the price of intermediate goods. Despite incentives for precautionary saving, these high-leverage states in which Sudden Stops are possible remain a feature of the economy even in the stochastic stationary state.

The motivation to introduce collateral and working capital constraints simultaneously comes from observations from emerging markets data, particularly the Mexican Sudden Stop of 1995. These data suggests that explanations of the Sudden Stop phenomenon need to reconcile two sets of stylized facts. First, the large, sudden reversal of the current account, the loss of access to credit markets, and the collapse of equity prices indicate that frictions in financial markets are an important element of Sudden Stops. Second, changes in use of intermediate inputs, capacity utilization and TFP account for the bulk of the large initial output collapse that occurs at the same time world credit market access is lost. In the model of this paper, the working capital constraint provides a link between financial frictions and factor demands and capacity utilization, and the collateral constraint introduces a mechanism for amplification and propagation of the business cycle response of the economy to adverse exogenous shocks.

The business cycle model developed in this paper produces Sudden Stops as infrequent events nested within “regular” business cycles, and those Sudden Stops are triggered by real shocks of the same magnitude as those that drive “regular” cycles. In

simulations calibrated to Mexican data, the long-run business cycle moments of model economies with and without credit constraints differ by small margins, while the predicted mean responses to one-standard-deviation shocks starting from an initial condition of high leverage differ sharply and reproduce several of the observed features of Sudden Stops. Amplification effects on the responses of macroeconomic aggregates to exogenous shocks are significantly larger than those that previous studies of the role of credit constraints in creating business cycles suggest (see Kocherlakota (2000)). Thus, this paper shows that explanations of Sudden Stops need not rely on large and/or unexpected shocks, and that credit constraints can be used to integrate a theory of “regular” business cycles with a theory of Sudden Stops within the same dynamic, stochastic general equilibrium framework. This is done without relying on multiplicity of equilibria, nominal rigidities, or large, unexpected shocks to the real side of the economy or its access to credit.

The collateral constraint and the working capital constraint introduce three credit channel effects. Two of these effects are in the form of endogenous external financing premia that emerge when the credit constraints bind. These premia reflect the effective (or shadow) real interest rates that lead households and firms to choose levels of debt and working capital that satisfy their credit constraints as levels that are also consistent with their optimal plans. Thus, even though the credit constraints are not derived as features of optimal contracts, these external financing premia are analogous to endogenous risk premia that lenders would charge in an environment in which limited enforcement allows them to confiscate only the fractions of the market value of physical assets or sales net of labor costs specified by the credit constraints.

The third credit channel effect is the Fisherian debt-deflation mechanism associated with the collateral constraint. This mechanism plays a key role in the ability of the model to amplify the real effects of exogenous shocks and increase their persistence. In a high-leverage state of the economy with collateral constraints, adverse shocks that would result in a small equity price decline with perfect credit markets trigger the collateral constraint causing a fall in physical investment and equity prices. The latter tightens further the credit constraint inducing a new round of “margin calls.” The end result of this deflationary spiral is a much larger drop in equity prices and physical investment than in the economy with perfect credit markets.

Further research on this subject can go in several directions. The model has important limitations in accounting for some quantitative features of the data. In particular, the asset price collapses, even though much larger with the credit constraints than without them, are still small compared to actual asset price collapses. Another limitation is that the model does not feature the “liability dollarization effect,” caused by the fact that the foreign debt of emerging economies is denominated in hard currencies (i.e. tradables goods units) but largely leveraged on assets and incomes in domestic currencies and generated by non-tradables industries. As a result, a fall in the relative price of nontradables increases the effective real interest rate, and a sharp relative price collapse can trigger financial collapse. One option to introduce this feature into the model would be to incorporate fully consumption and production decisions for nontradables. However, since the key aspects of the “liability dollarization” effect are (1) the difference in the units in which debt contracts and incomes and assets securing those contracts are denominated, and (2) sharp movements in the relative price of those two units, the “liability dollarization” effect could be introduced via changes in prices of intermediate goods imports. Adding this feature to the model requires denominating foreign debt in units of a world tradable good that is the same numeraire as for intermediate goods imports. In this case, a sharp increase in intermediate goods prices will not only increase

the direct input cost of intermediate goods but also the financing cost of all financial contracts.

The findings of this paper suggest that the key to reducing the probability of Sudden Stops in emerging economies is in promoting the attainment of levels of financial and institutional development that lower collateral and working capital constraints. Taking as given the underlying uncertainty driving business cycles in the form of aggregate, non-insurable shocks to TFP, world interest rates and relative prices, tighter credit limits designed to manage exposure to idiosyncratic risk can be counterproductive and lead to a higher probability of observing Sudden Stops.

A second policy conclusion derived from this paper relates to financial contagion. In the setup of this paper, an emerging economy can have solid domestic policies and competitive, open markets, and still reach a point of high leverage at which a Sudden Stop is caused by an increase in the world interest rate, or in the relative price of imported intermediate goods, induced by developments elsewhere in the world. If waiting for financial development is a costly alternative and tighter credit limits can make things worse, an alternative to explore is the use of mechanisms set by international financial organizations to help emerging markets maintain access to credit in these situations of financial contagion (see, for example, Calvo's (2002) proposal).

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Figure 1. Mexico: Real GDP Per Capita 1920-2001
(logarithms with Hodrick-Prescott trend)

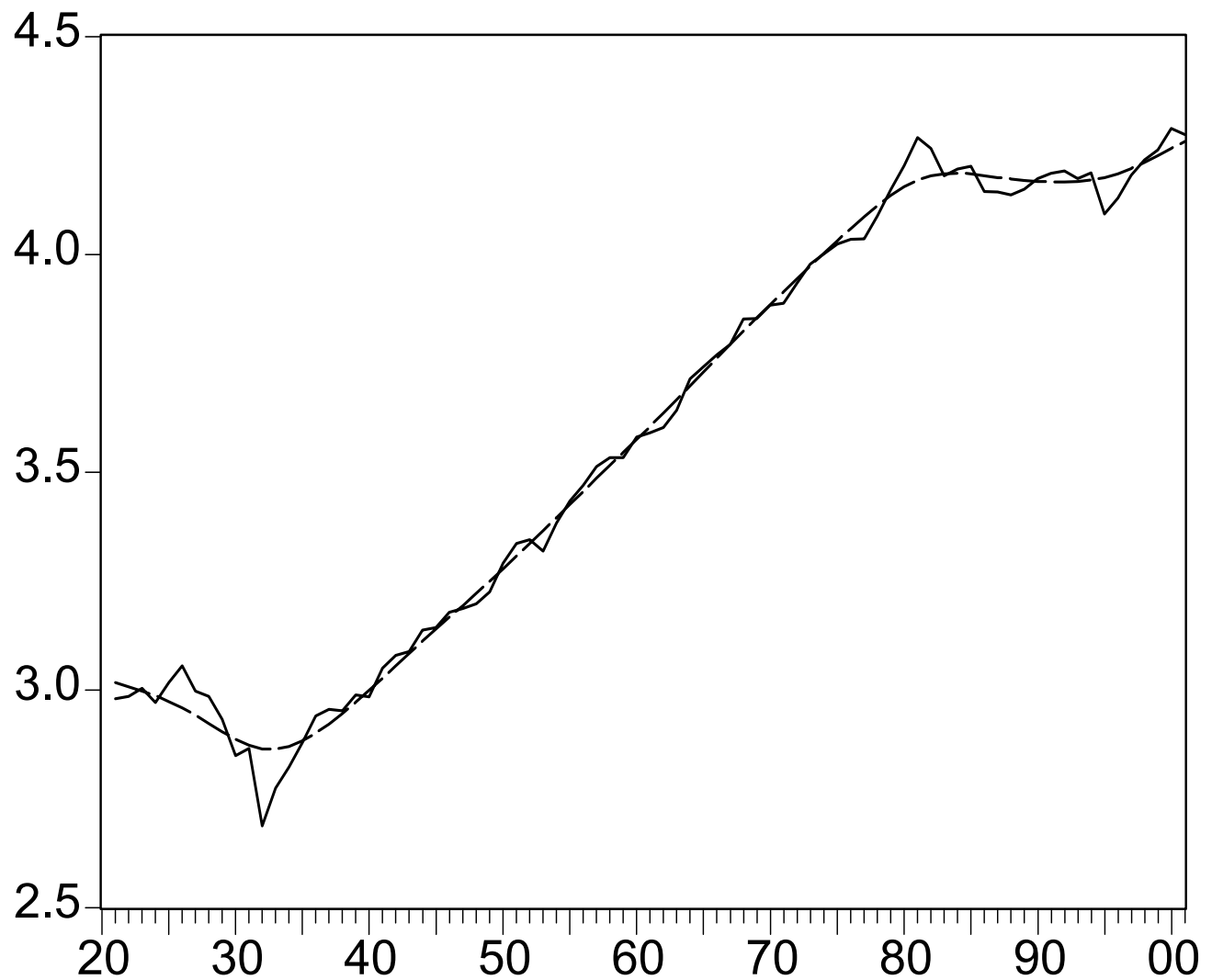


Figure 2. Mexico: Current Account-GDP Ratio
(quarterly data at annual rates)

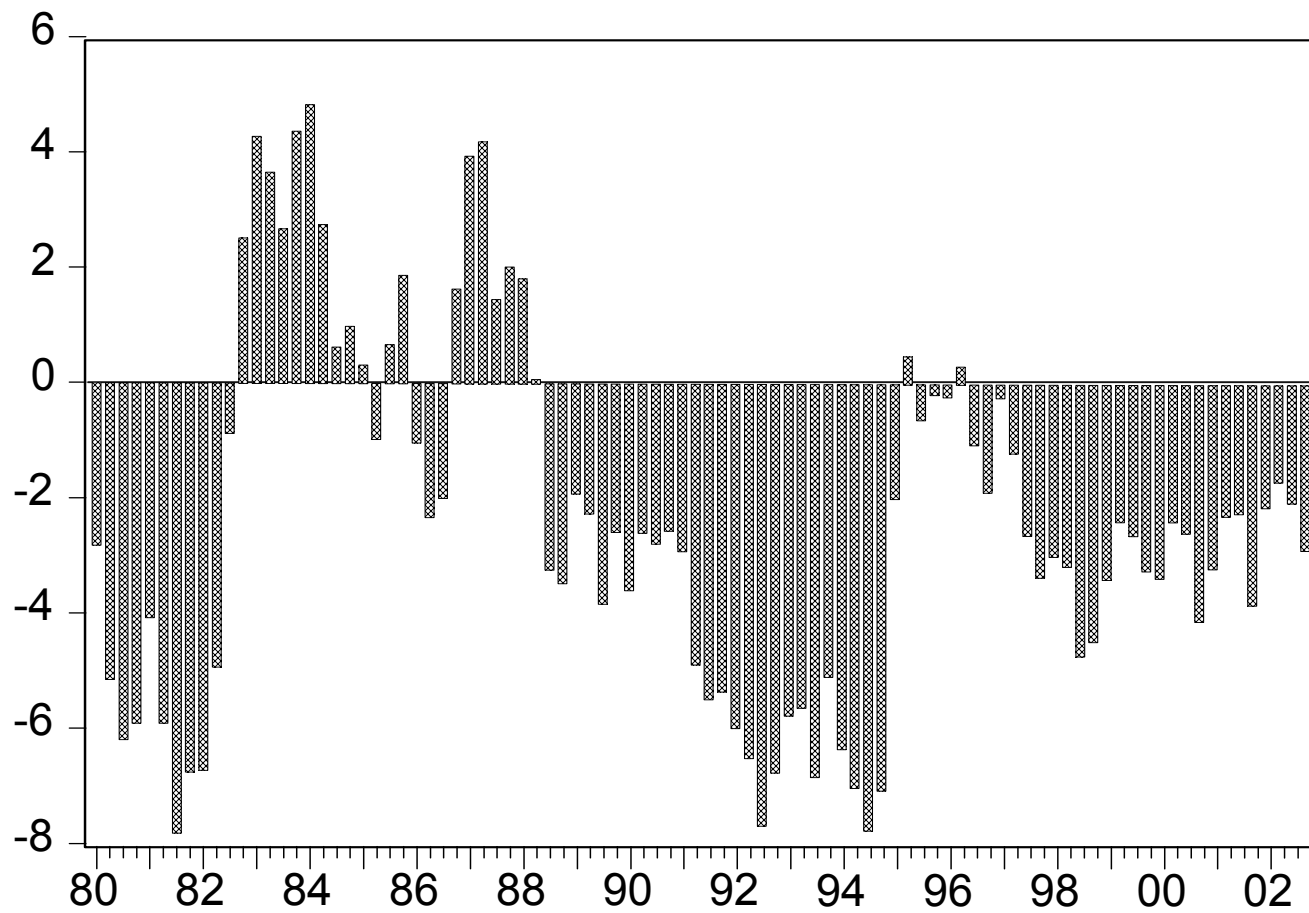


Figure 3. Mexico: Real Prices of Equity, Imports of Intermediate Goods and Nontradable Goods

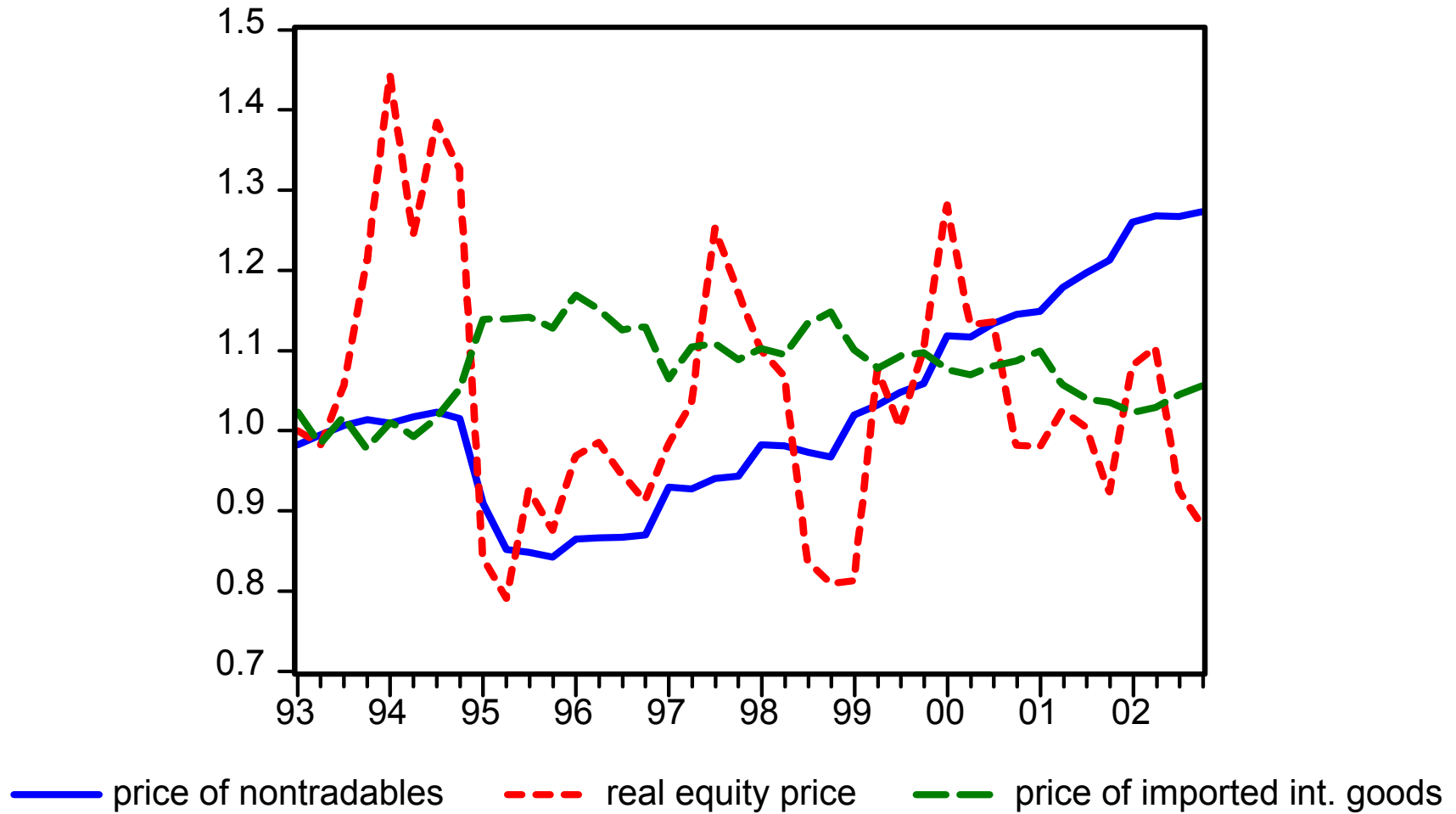
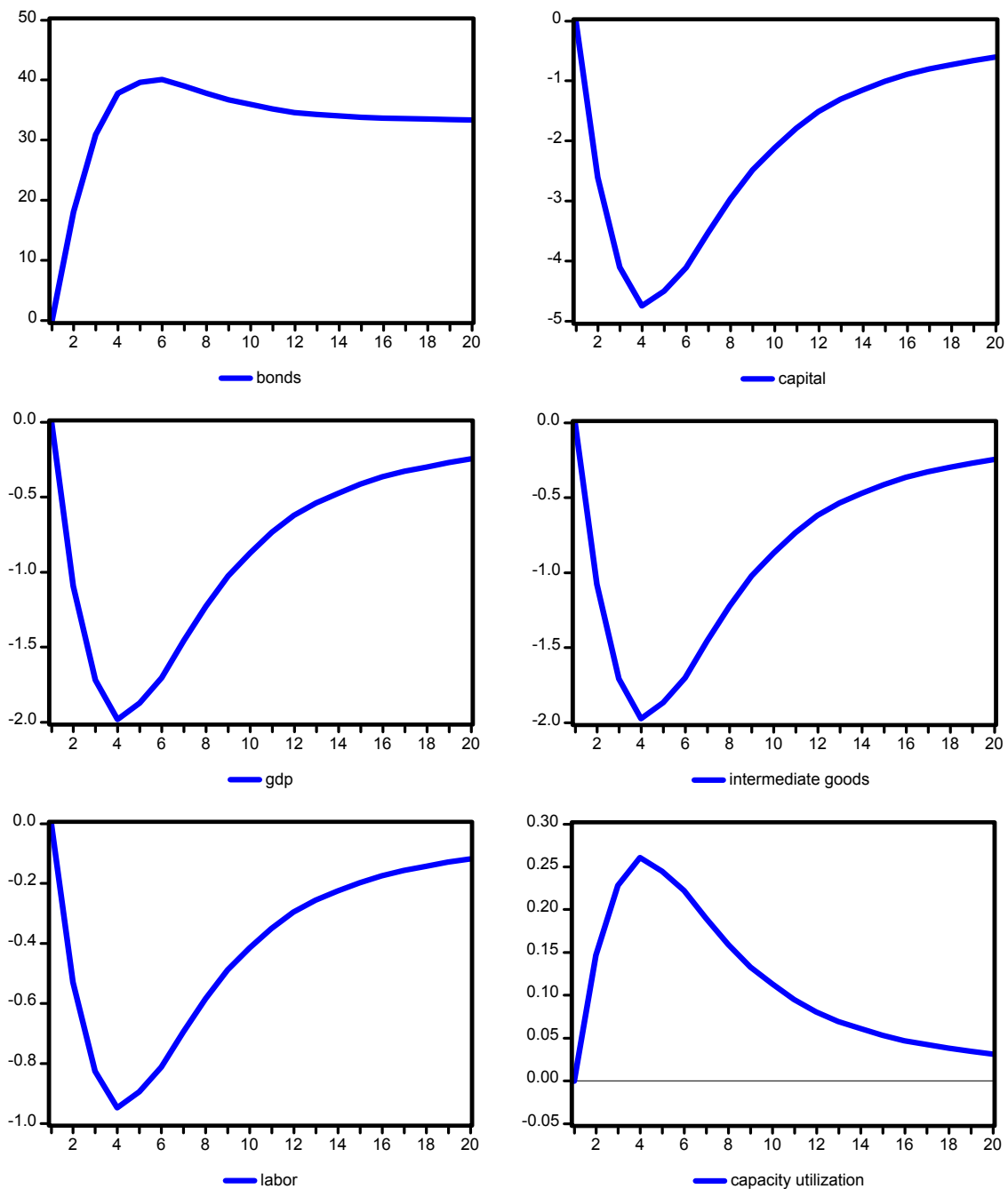
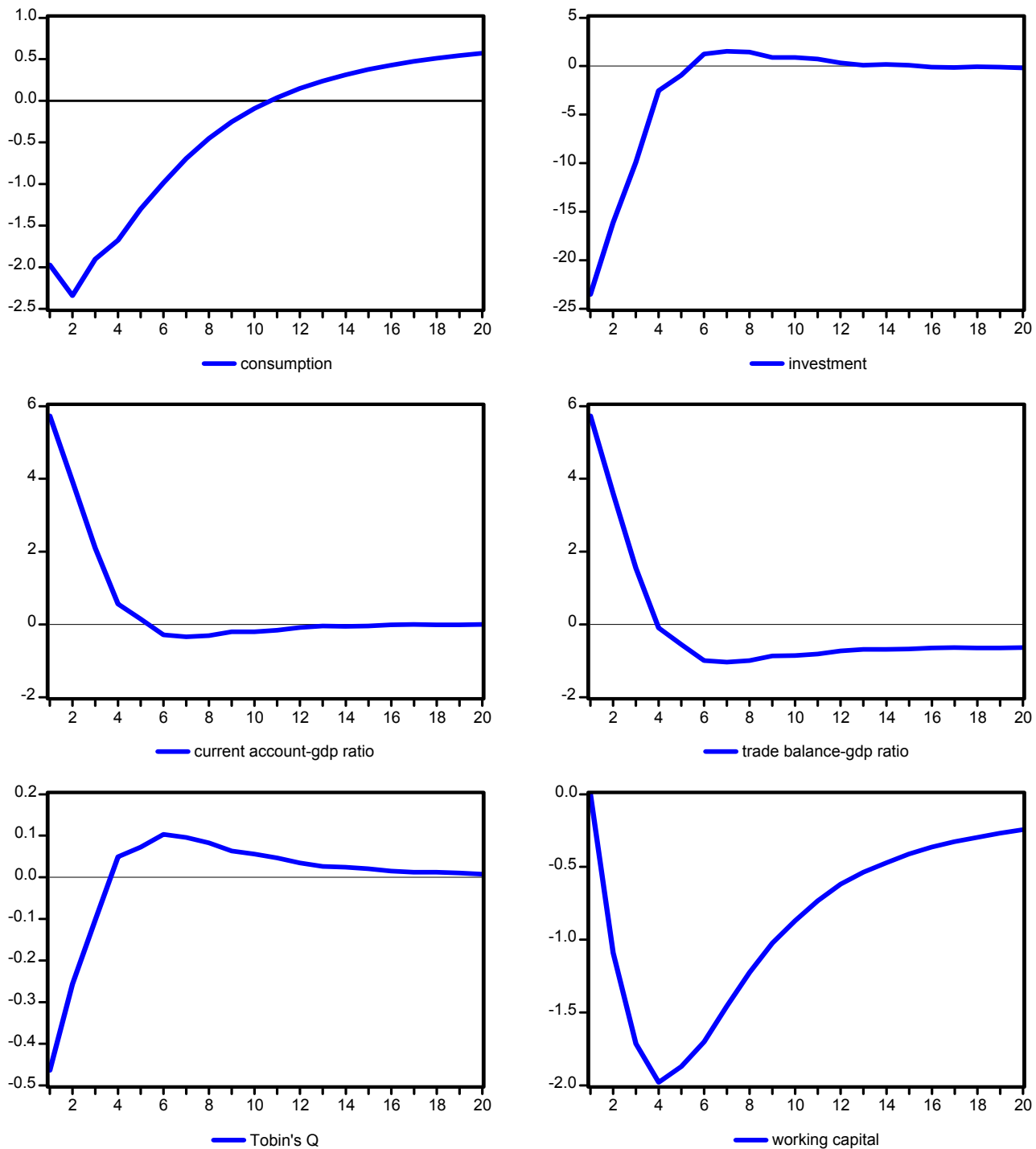


Figure 4.a Differences in Forecasting Functions of Economies with Perfect Credit Markets and Collateral Constraints in Response to Adverse One-Standard-Deviation Shocks to Productivity, Intermediate Goods Prices and World Interest Rate (percent deviations from long-run average)



Note: Forecasting functions are conditional on the initial conditions $K=45.09$, $B=-18.09$, which imply a debt/GDP ratio of 57.2 percent and a debt-to-value-of-capital ratio of 39.6 percent.

Figure 4.b Differences in Forecasting Functions of Economies with Perfect Credit Markets and Collateral Constraints in Response to Adverse One-Standard-Deviation Shocks to Productivity, Intermediate Goods Prices and World Interest Rate (percent deviations from long-run average)



Note: Forecasting functions are conditional on the initial conditions $K=45.09$, $B=-18.09$, which imply a debt/GDP ratio of 57.2 percent and a debt-to-value-of-capital ratio of 39.6 percent.

Figure 5. Output Magnification Effects of Credit Constraints
(in percent of standard deviation in frictionless economy)

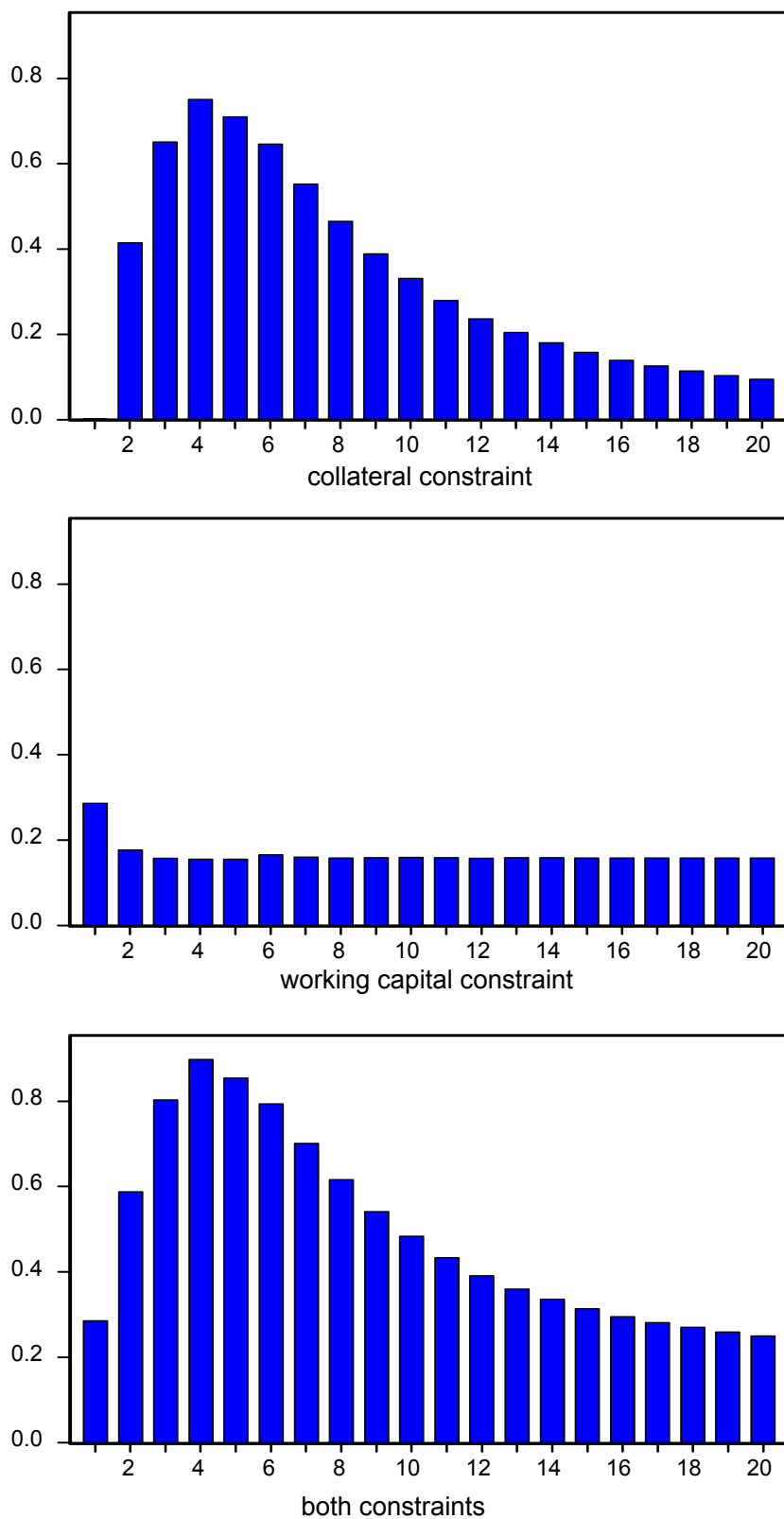


Figure 6. Consumption Magnification Effects of Credit Constraints
(in percent of standard deviation in frictionless economy)

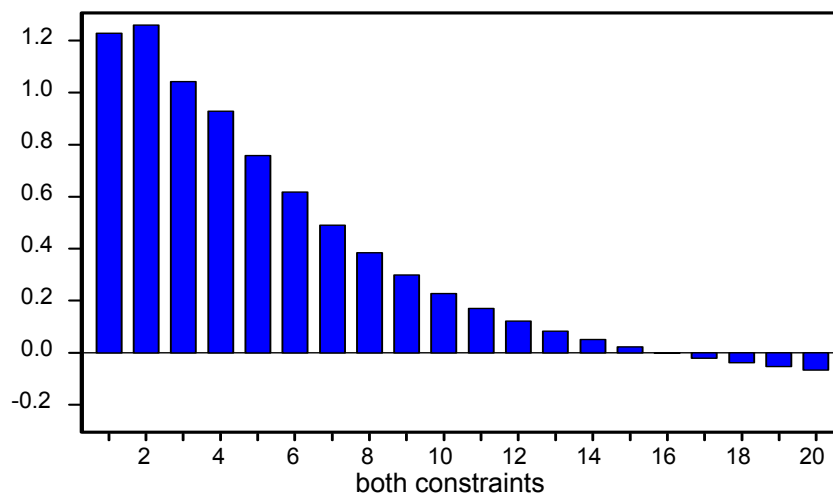
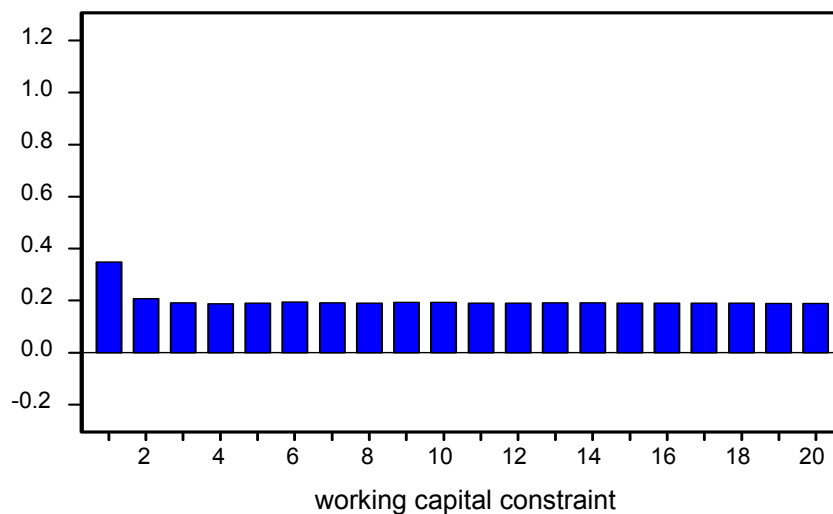
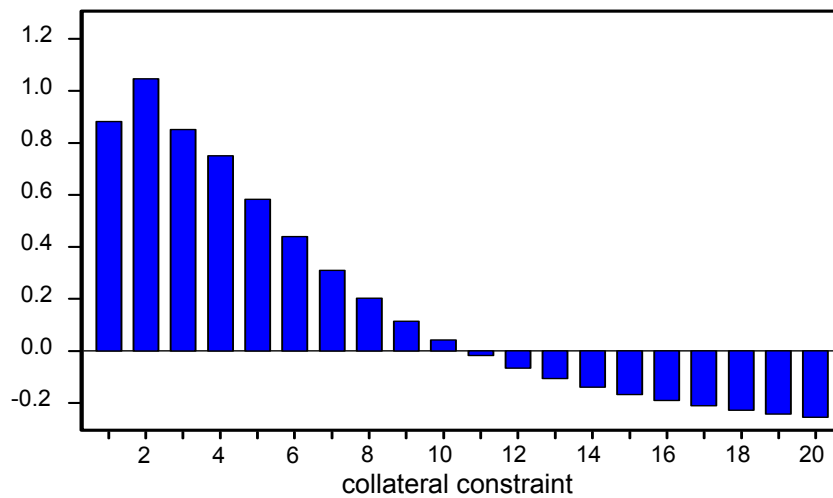


Figure 7. Current Account-GDP Ratio Magnification Effects of Credit Constraints
 (in percent of standard deviation in frictionless economy)

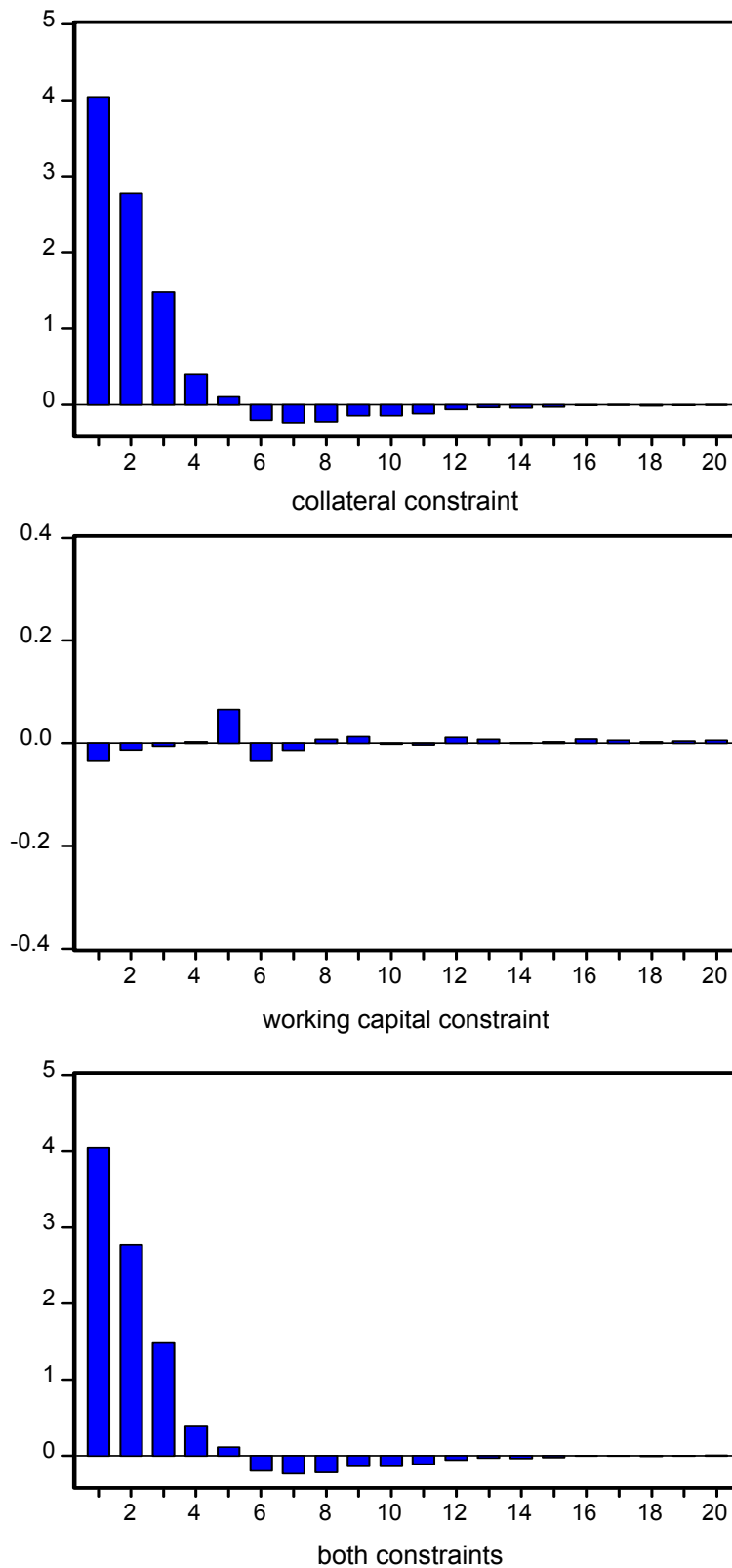


Figure 8. Equity Price Magnification Effects of Credit Constraints
(in percent of standard deviation in frictionless economy)

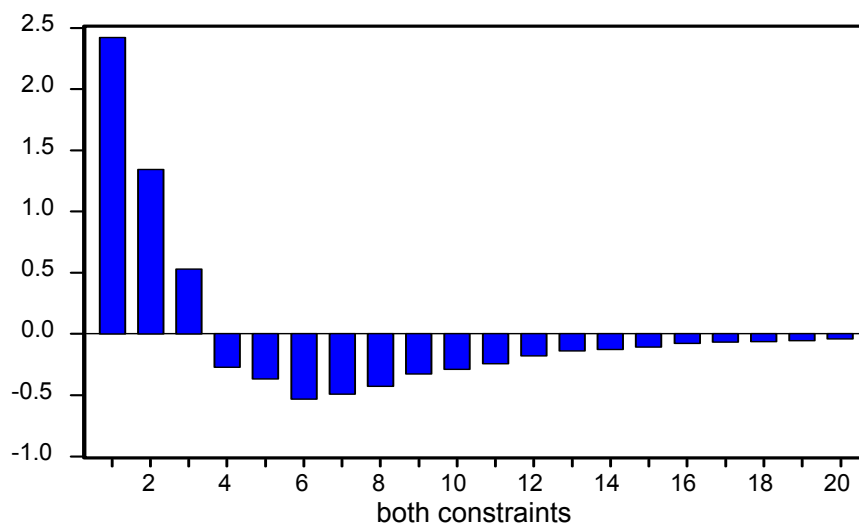
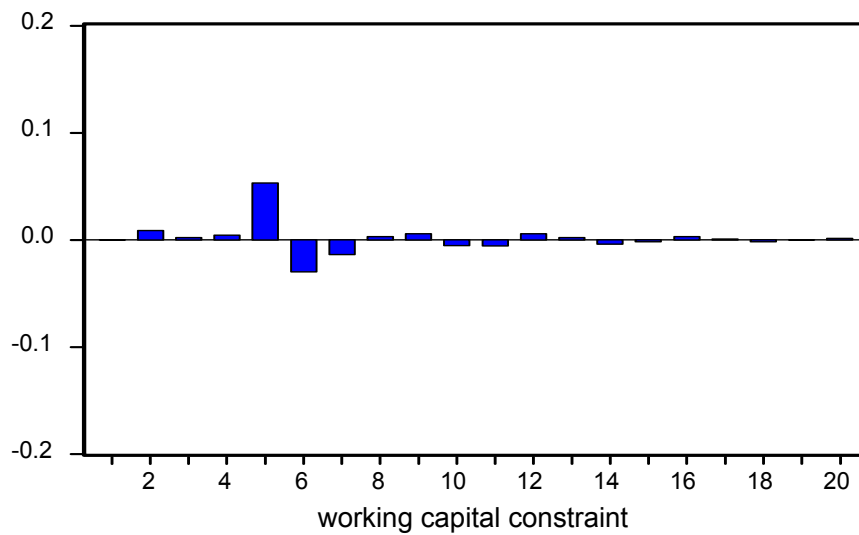
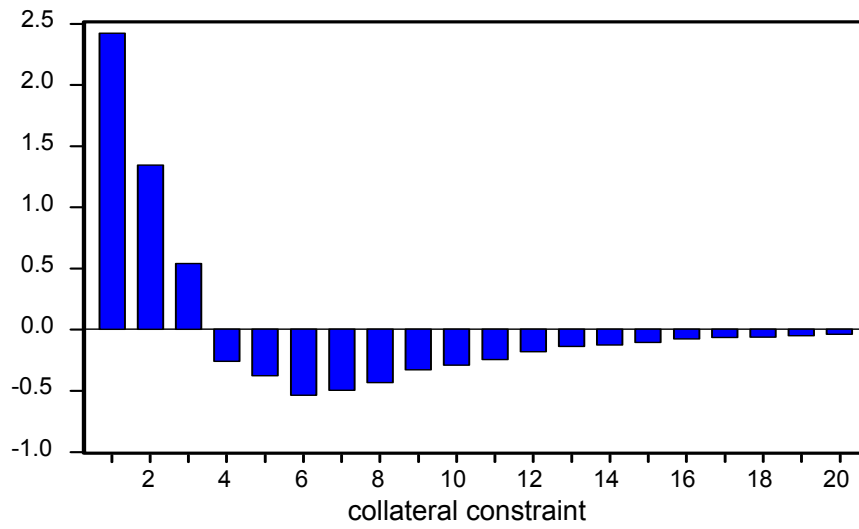
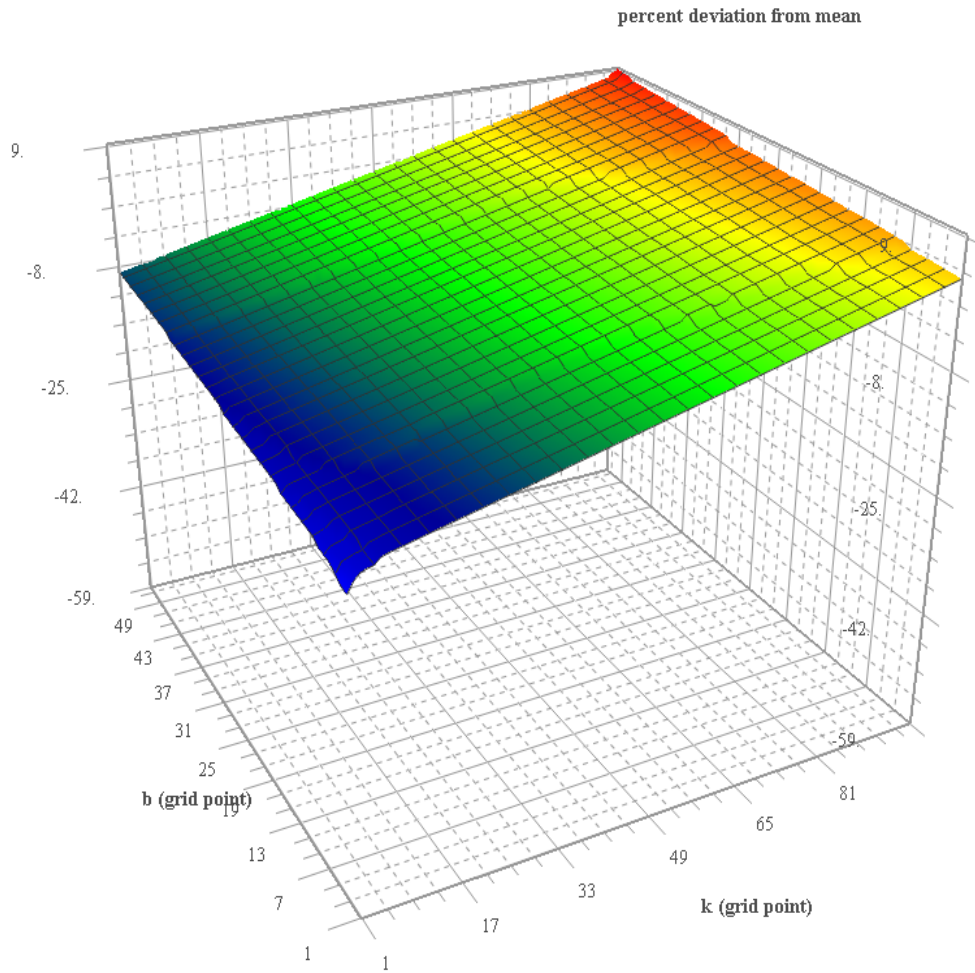
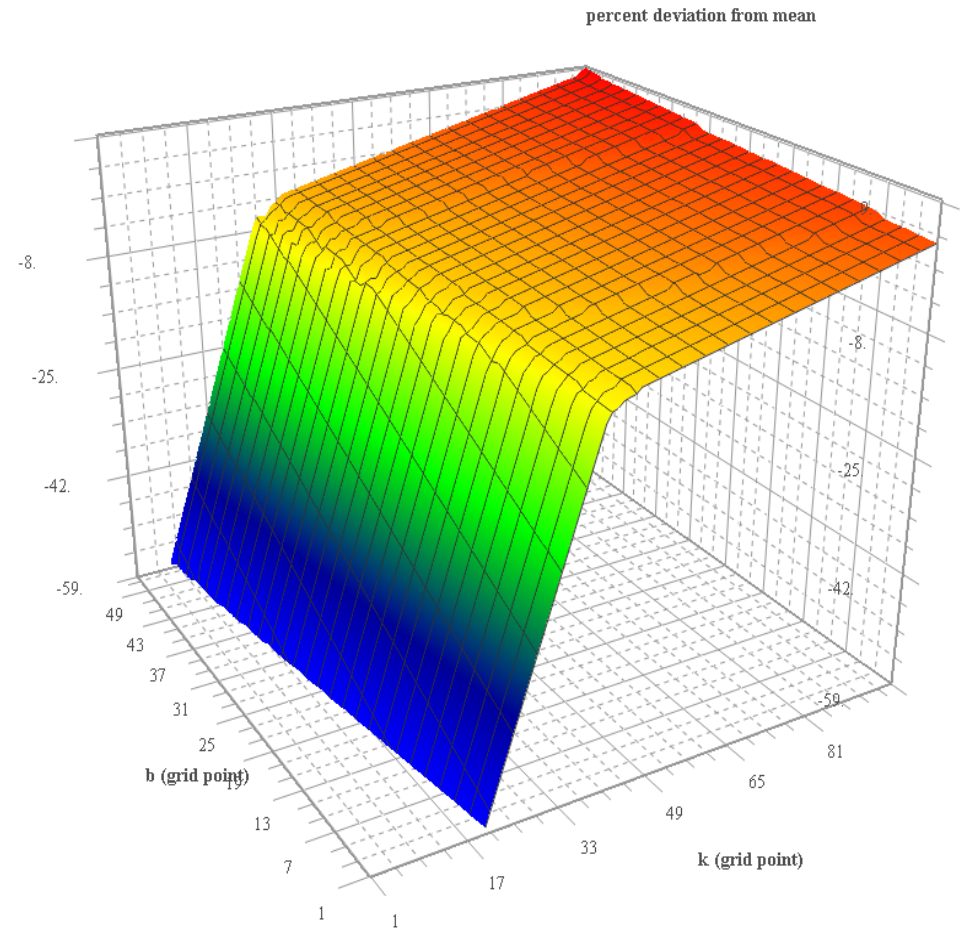


Figure 9. Impact Effects on Consumption with Perfect Credit Markets and with Collateral Constraints
 (as a percent of long-run averages, in response to one-standard-deviation shocks to ε^A , ε^R , and ε^P)

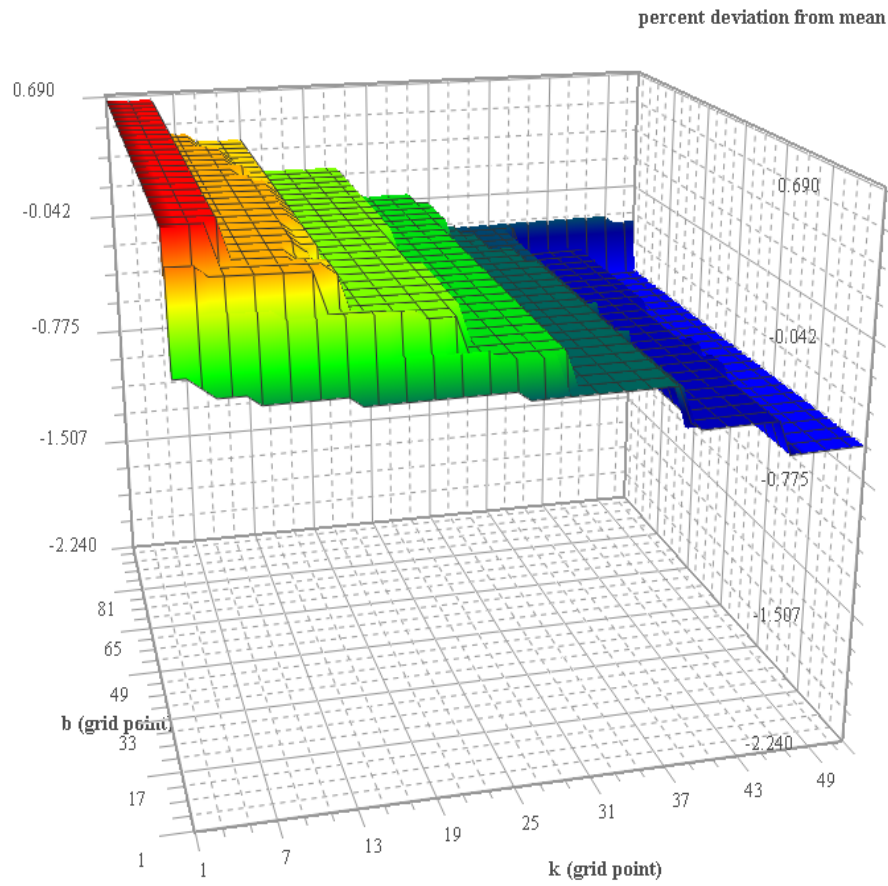


a. economy with perfect credit markets

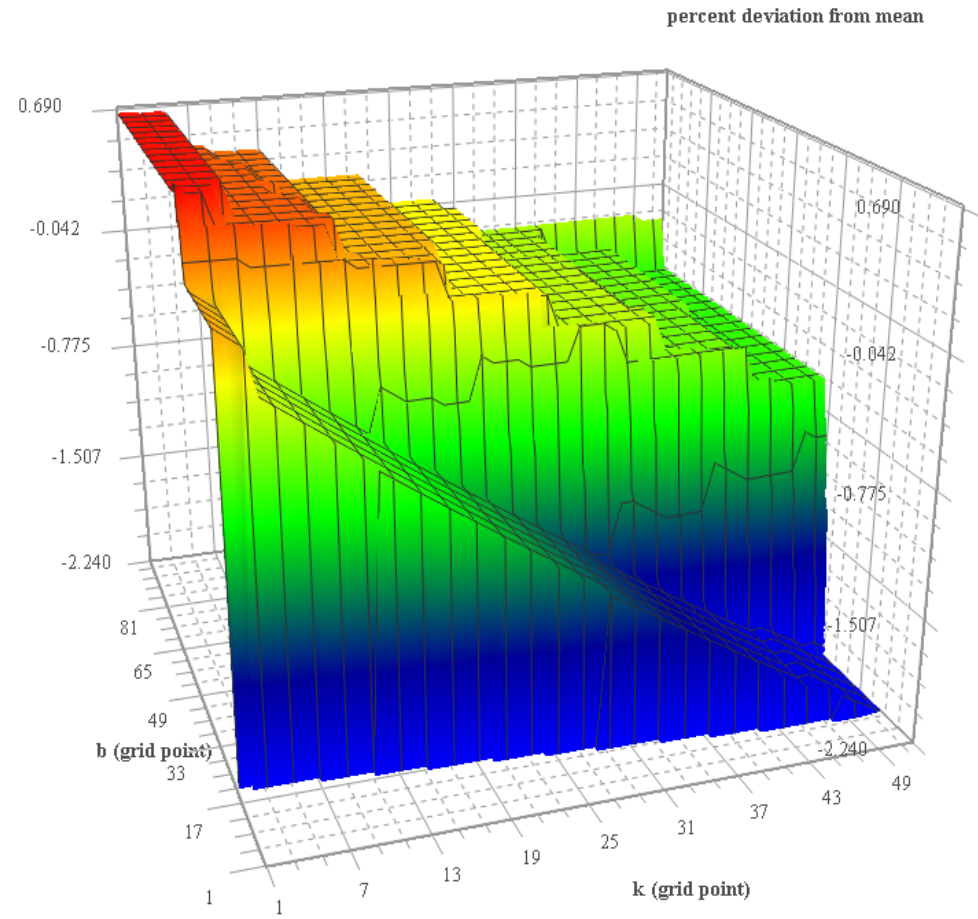


b. economy with collateral constraints

Figure 10. Impact Effects on Tobin's Q with Perfect Credit Markets and with Collateral Constraints
 (as a percent of long-run averages, in response to one-standard-deviation shocks to ε^A , ε^R , and ε^P)

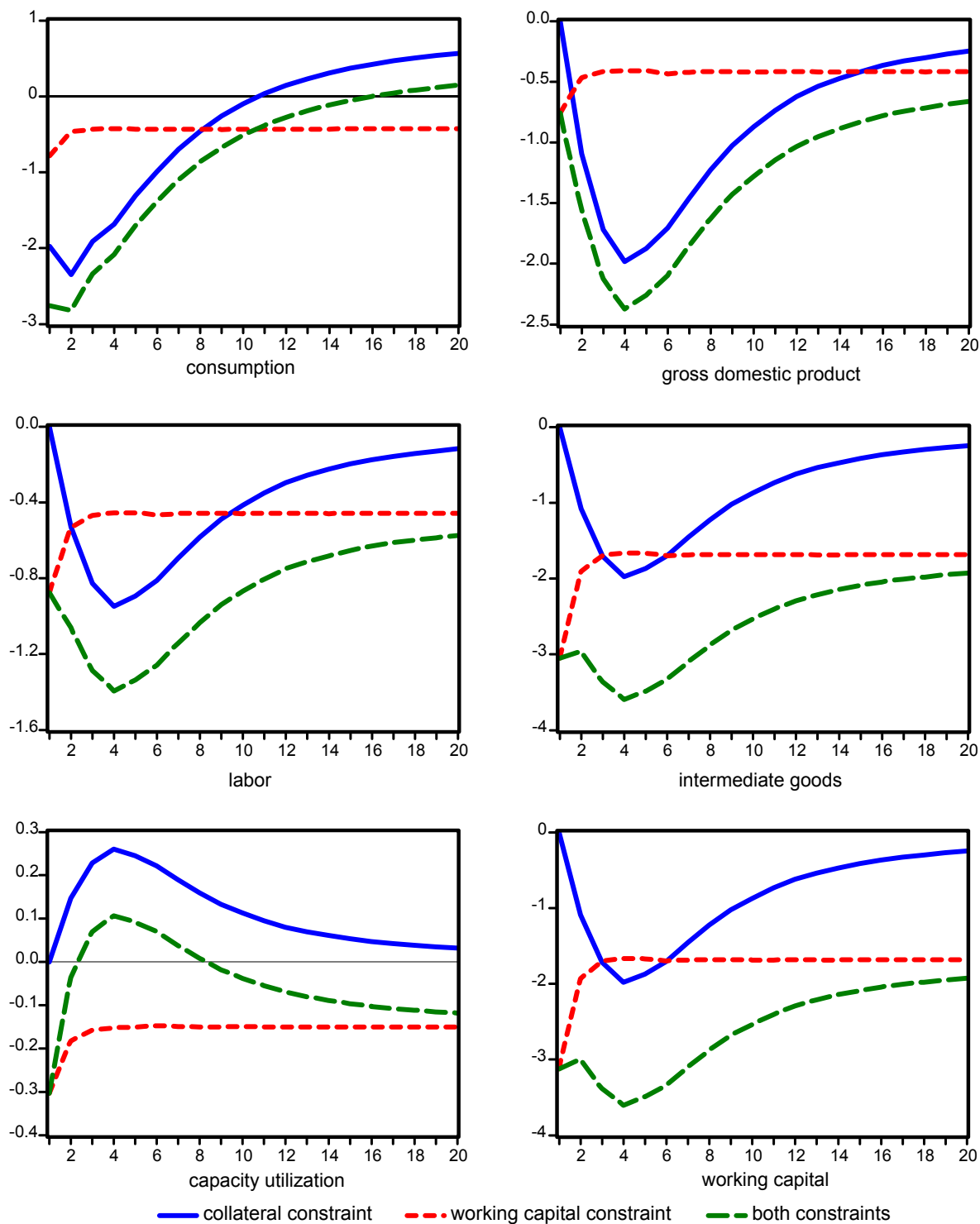


a. economy with perfect credit markets



b. economy with collateral constraints

Figure 11. Difference in Forecasting Functions Relative to Economy with Perfect Credit Markets in Response to Adverse One-Standard-Deviation Shocks to Productivity, Intermediate Goods Prices & World Interest Rate (percent deviations from long-run average in frictionless economy)



Note: Forecasting functions are conditional on the initial conditions $K=45.09$, $B=-18.09$, which imply a debt/GDP ratio of 57.2 percent and a debt-to-value-of-capital ratio of 39.6 percent.

Table 1. Mexico: Business Cycle Statistics and the Sudden Stop of 1995

variable	standard deviation	standard dev. relative to GDP	correlation with GDP	first-order autocorrelation	Sudden Stop (date in brackets)	Sudden Stop relative to standard dev.	sample
Endogenous variables:							
GDP	2.649	1.000	1.000	0.683	-8.313 (1995:2)	3.138	1980:1-2002:4
private consumption	3.260	1.231	0.891	0.711	-8.685 (1995:3)	2.664	1980:1-2002:4
investment	10.264	3.875	0.959	0.813	-29.991 (1995:3)	2.922	1980:1-2002:4
current account-GDP ratio	2.072	0.782	-0.787	0.732	4.840 (1995:1)	2.336	1980:1-2002:4
equity prices	14.199	5.360	0.574	0.558	-27.679 (1995:1)	1.949	1993:1-2002:4
Exogenous shocks:							
real world interest rate	1.595	0.602	-0.001	0.276	1.550 (1994:4)	0.972	1980:1-2002:4
intermediate goods prices	2.978	1.124	-0.558	0.604	5.811 (1995:1)	1.951	1993:1-2002:4
Solow residual for GDP	1.470	0.555	0.174	0.678	-4.447 (1995:2)	3.026	1983:1-2002:4
correlations between shocks	interest rate & prices = 0.161		interest rate & Solow residual = -0.049		prices & Solow residual = -0.250		

Note: The data were logged and detrended using the Hodrick-Prescott filter with the smoothing parameter set at 1600. The real interest rate is the average 3-month U.S. T-bill rate deflated by U.S. average CPI inflation. Equity prices are in units of the GDP deflator. Intermediate goods prices are measured as the ratio of the price index of imported intermediate goods divided by the exports deflator both from the National Accounts. "Sudden Stop" corresponds to the lowest deviation from trend of the corresponding variable (for the current account-GDP ratio it is the largest change in percentage points observed in two consecutive quarters). The GDP correlations are computed for the common sample 1993:1-2002:4. The Solow residual is the fraction of GDP unaccounted for by capital and labor inputs assuming a Cobb Douglas technology with a labor share of 0.7 and using capital and labor measures provided by Rodrigo Garcia at Banco de Mexico.

Table 2. Long-Run Business Cycle Moments in the Simulations of the Baseline Model

variable	mean	standard deviation (in percent)	standard deviation relative to GDP	correlation with GDP	first-order autocorrelation
I. Frictionless Economy					
GDP	33.970	2.641	1.000	1.000	0.660
consumption	22.863	2.240	0.848	0.726	0.854
investment	5.431	10.297	3.899	0.417	-0.023
current account-GDP ratio	0.000	1.420	0.538	0.478	0.288
capital stock	49.265	1.590	0.602	0.581	0.724
foreign assets-GDP ratio	-0.294	15.380	5.823	0.053	0.996
equity prices	1.009	0.192	0.073	0.315	-0.056
debt-value of capital ratio	0.201	10.470	3.964		
intermediate goods	25.521	4.527	1.714	0.912	0.582
capacity utilization	0.841	0.198	0.075	0.793	0.252
working capital	8.012	2.765	1.047	0.194	0.647
working cap.-net sales ratio	0.238	0.179	0.068	0.000	0.202
			Savings-investment correlation	0.584	
			GDP-world interest rate correlation	-0.096	
			GDP-int. goods price correlation	-0.620	
II. Economy with Collateral Constraint set at 40 percent					
GDP	33.967	2.647	1.000	1.000	0.662
consumption	22.881	2.206	0.834	0.747	0.848
investment	5.430	10.321	3.899	0.421	-0.019
current account-GDP ratio	0.000	1.416	0.535	0.466	0.277
capital stock	49.255	1.624	0.614	0.581	0.731
foreign assets-GDP ratio	-0.284	14.326	5.412	0.061	0.995
equity prices	1.009	0.192	0.073	0.317	-0.052
debt-value of capital ratio	0.194	9.760	3.687		
intermediate goods	25.519	4.531	1.712	0.912	0.583
capacity utilization	0.841	0.198	0.075	0.784	0.255
working capital	8.011	2.770	1.047	0.195	0.648
working cap.-net sales ratio	0.238	0.179	0.068	0.000	0.202
			Savings-investment correlation	0.586	
			GDP-world interest rate correlation	-0.095	
			GDP-int. goods price correlation	-0.619	
			prob. of binding collateral constraint	0.017	
III. Economy with Working Capital Constraint set at 23.59 percent					
GDP	33.830	2.711	1.000	1.000	0.656
consumption	22.774	2.350	0.867	0.741	0.810
investment	5.430	10.304	3.801	0.499	-0.023
current account-GDP ratio	0.000	1.425	0.526	0.410	0.288
capital stock	49.255	1.590	0.586	0.583	0.720
foreign assets-GDP ratio	-0.291	15.400	5.681	0.050	0.996
equity prices	1.009	0.192	0.071	0.394	-0.057
debt-value of capital ratio	0.198	10.444	3.852		
intermediate goods	25.092	5.136	1.895	0.889	0.520
capacity utilization	0.840	0.283	0.104	0.700	0.145
working capital	7.877	3.678	1.357	0.229	0.496
working cap.-net sales ratio	0.236	0.000	0.000	0.000	0.203
			Savings-investment correlation	0.585	
			GDP-world interest rate correlation	-0.243	
			GDP-int. goods price correlation	-0.604	
			prob. of binding working capital constraint	0.500	
IV. Economy with Collateral & Working Capital Constraints					
GDP	33.827	2.716	1.000	1.000	0.657
consumption	22.791	2.319	0.854	0.760	0.804
investment	5.429	10.337	3.806	0.502	-0.021
current account-GDP ratio	0.000	1.422	0.523	0.398	0.277
capital stock	49.245	1.624	0.598	0.583	0.730
foreign assets-GDP ratio	-0.282	14.388	5.297	0.058	0.995
equity prices	1.009	0.193	0.071	0.395	-0.053
debt-value of capital ratio	0.192	9.766	3.596		
intermediate goods	25.090	5.140	1.892	0.889	0.520
capacity utilization	0.840	0.283	0.104	0.695	0.147
working capital	7.877	3.682	1.356	0.230	0.498
working cap.-net sales ratio	0.236	0.000	0.000	0.000	0.203
			Savings-investment correlation	0.587	
			GDP-world interest rate correlation	-0.243	
			GDP-int. goods price correlation	-0.604	
			prob. of binding collateral constraint	0.016	
			prob. of binding working capital constraint	0.500	

Note: Standard deviations are in percent of the corresponding mean, except for variables defined originally as ratios.