Macroeconomics and Asset Markets: some Mutual Implications.*

Harald Uhlig
Humboldt University Berlin
Deutsche Bundesbank, CentER and CEPR

PRELMINARY
COMMENTS WELCOME

First draft: August 15, 2004
This revision: September 14, 2004

*I am grateful to John Campbell for a useful exchange of thoughts, to Kjetil Storesletten for a useful conversation and to Wouter den Haan for useful comments. The views expressed herein are those of the author and not necessarily those of the Bundesbank. Address: Prof. Harald Uhlig, Humboldt University, Wirtschaftswissenschaftliche Fakultät, Spandauer Str. 1, 10178 Berlin, GERMANY. e-mail: uhlig@wiwi.hu-berlin.de, fax: +49-30-2093 5934, home page http://www.wiwi.hu-berlin.de/wpol/

Abstract

This paper sheds light on the mutual discipline, which asset market observations and macroeconomic observation impose on each other. Economic choices such as consumption and leisure, which are taken as exogenous in much of the asset pricing literature, and which may suggest certain preference specifications in order to explain asset price observations in turn may have undesirable macroeconomic consequences, once these economic choices are endogenized.

We study a generic representative agent real business cycle economy, and show, how to analyze it in general, and explore the interconnections between asset market observations, macroeconomic observations and theoretical choices of key parameters. We give particular consideration to the nonseparability between consumption and leisure and investigate the scope of this nonseparability to help explain e.g. the equity premium observation.

As an extension, we also study a two-agent economy, following the lead of Guvenen (2003), and found some undesirable implications of that model as well.

We find that the major obstacle to overcome is the endogeneity of labor market movements. We therefore propose an exogenous law of motion for wages and find that simple models can then go remarkably far in jointly explaining the observed facts.

Keywords: consumption-based asset pricing, business cycle, calibration, equity premium, Sharpe ratio, nonseparability between consumption and leisure, two-agent economy

JEL codes: E32, G12, E22, E24

1 Introduction

Economic risks are ubiquitous. Workers face unemployment risk or the risk (and opportunity) of social rise and decline. Firms face risks associated with changing market conditions. Stock market investors face the risk of variable stock returns. Nations jockey for positions in the uncertain international growth race. These risks are important for all economic actors, and they are important for economic policy. Indeed, much of economic policy can be viewed as risk management. While some of it deals with idiosynchratic risks and the associated tradeoffs between incentives and insurance (see e.g. the debate about the rules on unemployment insurance), a substantial effort is directed towards the management of aggregate or macroeconomic risks. These are risks which can not be diversified away: at most they can be mitigated by appropriate policy or the distribution of the risk bearing burden can be optimized.

Thus, to properly conduct macroeconomic risk management policy, it is of paramount importance to understand the nature and the quantitative significance of macroeconomic risks. E.g., how costly - in terms of welfare - are business cycles, see e.g. Lucas (1987), Otrok (1999), Storesletten et al. (2001) or Alvarez and Jermann (2003)?. These risks show their consequences in two important places in particular. First, on asset markets, risks are priced. Second, the allocation in the economy results from risk-averse economic actors taking actions in the face of existing risks and their prices. Thus, asset prices and the allocation of economic resources in the economy as a whole are tightly intertwined. Observations on asset markets impose discipline on economic choices and models of the macroeconomy and vice versa. The quest therefore is on to provide models which can jointly explain the behaviour of asset prices and of the economy as a whole.

This quest has largely been an elusive one. Understanding the behaviour of asset markets, given economic choices such as consumption, has been the focus of a substantial part of the asset pricing literature, see e.g. Cochrane (2001) or Campbell (2004) for excellent surveys. Similarly, the quest for understanding macroeconomic facts, e.g. business cycles, growth and international trade, has generated a huge volume of research. The explanation of asset pricing facts and macroeconomic facts, when both asset prices as well as the allocation of goods is endogenous, is thus a daunting task. Some papers (and this is a very incomplete list!) which have made considerable progress are e.g. Jermann (1998), Lettau and Uhlig (2000), Boldrin, Christiano and Fisher (2001), Hornstein and Uhlig (2001) and recently in particular Guvenen (2003).

This paper aims at contributing to this research agenda by highlighting some important connections between asset pricing facts and macroeconomic facts and the discipline each imposes on the other. The aim here is to provide a bit of simple (or not so simple) algebra in order to provide some guide as to where one may or may not need to go.

I find that the major obstacle to overcome is the endogeneity of labor market movements. These connections and the key role of labor markets have also been emphasized by Lettau and Uhlig (2002), who focus on utility functions with habit formation. The intuition is simple: if agents can endogenously choose their labor input, they can use this an additional insurance device against stock market fluctuations. Indeed, a number of papers in the literature thus either assume labor to be constant, e.g. Jermann (1998) or Guvenen (2003), or assume considerable frictions in adjusting labor input, e.g. Boldrin, Christiano and Fisher (2001). As an alternative, I propose an exogenous law of motion for wages and find that simple models can then go remarkably far in jointly explaining the observed facts, including the movements of employment.

2 Some facts

First, it is useful to list some key facts. They are well known: we will just provide a brief survey, and add some additional details later, in particular on correlations between certain macrovariables and stock returns.

2.1 Asset markets

Campbell (2004) documents, that the average real return on stocks is 8.1% at an annual rate, resulting in a risk premium of 7.2% at an annual rate over 3-month treasury bills. Their volatility at an annual rate is 15.6%, from which one can calculate the Sharpe ratio, i.e. the ratio from excess return to volatility, of 0.46 at an annual basis. It is well known that the equity premium observation is not a U.S phenomenon alone: again, Campbell (2004) provides an excellent summary.

"Excess returns on U.S. stocks ... are highly forecastable. The log price-dividend ratio forecasts 10% of the variance of the excess return at a 1-year horizon, 22% at a 2-year horizon and 38% at a 4-year horizon." (Campbell, 2004). The predictability is mirrored by results obtained by e.g. Lettau and Ludvigson (2004), who document that consumption, asset values and income are cointegrated, and that this cointegrating vector helps to predict returns on assets rather than changes in consumption. Put differently (and in contrast to conventional wisdom), the wealth effect of an increase in asset prices on consumption is weak. Indeed, changes in consumption are hard to forecast as consumption is nearly a random walk (see again Cochrane, 2004).

As for the safe rate, "the annualized standard deviation of the ex post real return on U.S. Treasury bills is 1.7%" (Campbell, 2004), and therefore considerably lower than stocks.

2.2 Macroeconomics

As for macroeconomic facts, tables 1 provide some key facts on volatilities as well as correlations between output, consumption, investment, government spending, hours, productivity and wages, see also Uhlig (2004). Let me add to that, that the share of wage payments is 0.64, whereas the share of capital payments is 0.36, see e.g. Cooley and Prescott (1995).

The typical features of business cycles are easily seen: output and labor are nearly equally volatile, while consumption fluctuates less and investment fluctuates more. These variables as well as labor productivity are procyclical, i.e. positively correlated with output. There is fairly little correlation of government spending or wages with output. It is these facts that any successful business cycle theory must be consistent with.

3 A basic model

To frame the issue further, we shall start from a generic stochastic neoclassical growth model or real business cycle model with a representative agent and a time-separable utility function. This is a good starting point for a number of reasons. First, in order to investigate the connections between macroeconomics and asset pricing and to consider the endogeneity of choices, we need to move beyond the usual asset pricing equation. The neoclassical growth

Table A				
	output	cons.	investm.	gov.spend.
output	1.74			
cons.	0.80	0.82		
investm.	0.83	0.63	6.87	
gov.spend.	0.19	0.08	-0.29	3.72
share of output	(101.7% is sum of:)	58.8%	19.6%	23.3%
share of output2	(100% is sum of:)	75.2%	24.8%	

Table B						
	output	cons.	inv.	hours	labor prod.	wages
output	2.13					
cons.	0.82	1.30				
investm.	0.86	0.66	8.07			
hours	0.86	0.66	0.72	1.79		
labor prod.	0.54	0.53	0.50	0.04	1.08	
real wages	0.14	0.20	0.05	-0.09	0.43	0.89

Table 1: *Some key business cycle facts. The data is HP-filtered and 100 times logs of quarterly postwar US data. Diagonal elements are standard deviations, off-diagonal elements are correlations. Table A. US NIPA data, 1947:1 - 2002:3. Consumption is the sum of nondurables and services, while investment is durable consumption plus gross private domestic investment. Government spending is government consumption and investment. Output is gross domestic product, whereas output2 is the sum of consumption and investment only. Table B. The data is from Francis and Ramey (2001, not the newly revised version), 1947:1-2000:4, focussing on production in the private sector.*

model, as the work horse model of macroeconomics, is the natural choice. Second, the case of the representative agent and time-separable utility functions is the base case, from where further ramifications can be considered. In particular (and largely due to illustrate the findings by Guvenen, 2003), we shall investigate a two-agent economy further below. Third, we shall be fairly general in the formulation of our model, and we shall show that one is nonetheless free to choose only very few parameters, which then govern the dynamics of the model and the asset pricing implications. As stochastic shocks, we only focus on shocks to total factor productivity: it would not be hard to add additional shocks, and it may help to further illuminate some issues. Most key results do not seem depend on that, however. We ignore government spending or distortionary taxation: again, this could be added as a later step, in particular in light of e.g. McGrattan and Prescott (2003). We also abstract from the growth trend: surely, this is a bit of an ommission, as considerations of the growth trend impose additional discipline on the exercise.

A number of papers have stressed non-separabilities across time, in particular habit-formation, see e.g. Campbell and Cochrane (1999), Boldrin, Christiano and Fisher (2001) and Lettau and Uhlig (2002), or the separation of risk aversion and intertemporal substitution, see e.g. Epstein and Zin (1991), Weil (1989), Tallarini (2000) or the relationship to robust control, see Hansen et al. (1999). Furthermore, much work has recently gone into extending the formulation of preferences into "exotic" territory, see e.g. Backus, Routledge and Zin (2004) and the references therein.

Here instead and as a complement to this literature, I shall explore a somewhat underemphasized avenue of asset pricing research: the non-separability between consumption and leisure. We do this to explore some new grounds and provide some new results and insights. Furthermore, the themes that emerge here - in particular, the mutual restrictions between macroeconomic facts and asset pricing facts, disciplined by simple theory and observations - and the techniques employed below for delivering these interconnections can be generalized to more elaborate utility specifications, and, we suspect, with similar implications. Thus, the fairly simple, yet interesting case of a time-separable utility function with nonseparabilities between consumption and leisure also serves as a showcase for a more general approach.

We use capital letters to denote the original variables, and small letters to denote log-deviations from steady state (unless explicitly stated otherwise).

Let the representative agent or the social planner solve

$$\max E\left[\sum_{t=0}^{\infty}\beta^t U(C_t, L_t)\right]$$

$$\begin{aligned} C_t + X_t &= Y_t = Z_t F(K_{t-1}, N_t) \\ K_t &= (1-\delta)K_{t-1} + G\left(\frac{X_t}{K_{t-1}}\right)K_{t-1} \\ 1 &= N_t + L_t \end{aligned}$$

I.e., the social planner maximizes the expected discounted sum of concave, differentiable and strictly increasing period-utilities $U(\cdot,\cdot)$ in consumption C_t and leisure L_t, subject to a feasibility constraint, that consumption and investment X_t add up to output Y_t, which is produced according to the concave, differentiable and strictly increasing production function $f(\cdot,\cdot)$ with predetermined capital K_{t-1} and labor N_t, and subject to the stationary exogenous total factor productivity process Z_t. I assume that the production function has constant returns to scale. Capital in turn can be produced by adding investment, subject to the concave adjustment cost function $G(\cdot)$. As is standard in the literature, I assume that $G(\delta) = \delta$ and $G'(\delta) = 1$, so that the first-order behavior of the capital accumulation is the same as the usual no-adjustment-cost equation. There is one unit of time as endowment, which can be split between labor and leisure.

It is easy to calculate the usual first order necessary conditions, and there is no need to list them here explicitely: below, we shall investigate their log-linearized version instead. We shall use bars on top of all variables to denote the nonstochastic steady state. Introduce (shadow) wages and (shadow) dividends as the marginal product of labor and capital,

$$\begin{aligned} W_t &= Z_t F_N(K_{t-1}, N_t) \\ D_t &= Z_t F_K(K_{t-1}, N_t) \end{aligned}$$

Let R_{t+1} be the gross return in terms of the consumption good of investing in the capital stock. Using the first-order conditions, this is

$$R_{t+1} = G\left(\frac{X_t}{K_{t-1}}\right)\left(D_{t+1} + \frac{1-\delta+G\left(\frac{X_{t+1}}{K_t}\right)}{G'\left(\frac{X_{t+1}}{K_t}\right)} - \frac{X_{t+1}}{K_t}\right)$$

which will simplify considerably in the log-linearized version below. Note already that

$$\bar{R} = \bar{D} + 1 - \delta \tag{1}$$

as usual, where we keep in mind that bars denote the nonstochastic steady state and not the mean of the stochastic economy. Indeed, for asset pricing pricing implications, this difference is key and we shall explore it further below.

To focus on the key parameters, we introduce the following notation. Let

$$\begin{aligned}
\eta_{cc} &= -\frac{U_{CC}(\bar{C},\bar{L})\bar{C}}{U_C(\bar{C},\bar{L})} \\
\eta_{ll} &= -\frac{U_{LL}(\bar{C},\bar{L})\bar{L}}{U_L(\bar{C},\bar{L})} \\
\eta_{cl,c} &= \frac{U_{CL}(\bar{C},\bar{L})\bar{C}}{U_L(\bar{C},\bar{L})} \\
\eta_{cl,l} &= \frac{U_{CL}(\bar{C},\bar{L})\bar{L}}{U_C(\bar{C},\bar{L})}
\end{aligned}$$

which characterize the curvature properties of the utility function. Note that $\eta_{cc} \geq 0$ is the usual risk aversion with respect to consumption, $\eta_{nn} \geq 0$ is risk aversion with respect to leisure, and $\eta_{cn,c}$ as well as $\eta_{cn,n}$ are cross-derivative terms. There are a few additional restrictions on these values, which we shall elaborate upon further below.

Let

$$\begin{aligned}
\theta &= \frac{F_K(\bar{K},\bar{N})\bar{K}}{F(\bar{K},\bar{N})} \\
\phi_{kk} &= -\frac{F_{KK}(\bar{K},\bar{N})\bar{K}}{F_K(\bar{K},\bar{N})} \\
\phi_{nn} &= -\frac{F_{NN}(\bar{K},\bar{N})\bar{N}}{F_N(\bar{K},\bar{N})}
\end{aligned}$$

which characterize the curvature properties of the production function. Note that θ is the capital share, while $\phi_{kk} \geq 0$ and $\phi_{nn} \geq 0$ are the elasticities of dividends with respect to capital and of wages with respect to labor. Due to constant returns to scale, it is easy to see (and probably well known that)

$$\begin{aligned}\phi_{kk} &= \frac{F_{KN}(\bar{K},\bar{N})\bar{N}}{F_K(\bar{K},\bar{N})}\\ \phi_{nn} &= \frac{F_{KN}(\bar{K},\bar{N})\bar{K}}{F_N(\bar{K},\bar{N})}\end{aligned}$$

For a Cobb-Douglas production function, we have $\phi_{kk} = 1 - \theta$ and $\phi_{nn} = \theta$, but in general, this does not have to be the case.

Finally, let

$$\xi = -\frac{1}{G''(\delta)\delta} > 0$$

which is the traditional notation, and coincides with the parameter ξ for the specific cost-of-adjustment functional form

$$G\left(\frac{X_t}{K_{t-1}}\right) = \frac{a_1}{1-1/\xi}\left(\frac{X_t}{K_{t-1}}\right)^{1-1/\xi} + a_2$$

with a_1 and a_2 chosen so that $G(\delta) = \delta, G'(\delta) = 1$, see also Jermann (1998), Hornstein and Uhlig (2000) and Boldrin et al (2001). The benchmark case of no adjustment costs is $\xi = \infty$.

Let Λ_t and $(\Lambda_t + \Psi_t)$ be the Lagrange multipliers on the first and second constraint, i.e. Ψ_t is the difference between the Lagrange multipliers on the second and the first constraint, and is zero, if the adjustment cost function is linear.

Loglinearizing all equations around the steady state (and using small letters to denote the loglinear deviations) leads to

$$y_t = \frac{\bar{X}}{\bar{Y}}x_t + \left(1 - \frac{\bar{X}}{\bar{Y}}\right)c_t \quad (2)$$

$$y_t = \theta k_{t-1} + (1-\theta)n_t \quad (3)$$

$$k_t = (1-\delta)k_{t-1} + \delta x_t \quad (4)$$

$$w_t = z_t + \phi_{nn}(k_{t-1} - n_t) \quad (5)$$

$$d_t = z_t - \phi_{kk}(k_{t-1} - n_t) \quad (6)$$

$$l_t = \frac{1-\bar{L}}{\bar{L}}n_t \quad (7)$$

$$\lambda_t = -\eta_{cc}c_t + \eta_{cl,l}l_t \tag{8}$$

$$\lambda_t + w_t = \eta_{cl,c}c_t - \eta_{ll}l_t \tag{9}$$

$$\psi_t = \frac{1}{\xi}(x_t - k_{t-1}) \tag{10}$$

$$r_t = \frac{\bar{R} - 1 + \delta}{\bar{R}} d_t - \psi_{t-1} + \frac{1}{\bar{R}}\psi_t \tag{11}$$

$$0 = E_t\left[\lambda_{t+1} - \lambda_t + r_{t+1}\right] \tag{12}$$

For convenience, we have collected these equations also as table 18 in the appendix.

A few remarks are in order. First, each of these equations has an obvious economic interpretation and is useful for interpreting the data. (2) is aggregate feasibility. (3) and (4) are the production functions for output and capital. (5) and (6) define wages and dividends as marginal productivity of labor and capital. (7) shows how to split time between leisure and labor, converting the percent units. (8) and (9) are the households first order conditions with respect to consumption and with respect to leisure. Additionally, (8) defines the shadow value λ_t of wealth measured in consumption good units. (10) measures the wedge created by the adjustment costs in capital and is related to Tobin's q. Finally, (11) defines the return on investing in capital, and (12) is the intertemporal Euler equation, this return needs to satisfy.

Second, despite the generality of the model in terms of the utility function, the production function or the adjustment cost function, there are only a few parameters, which determine the dynamics, namely

$$\frac{\bar{X}}{\bar{Y}}, \theta, \delta, \bar{R}, \phi_{nn}, \phi_{kk}, \xi, \eta_{cc}, \bar{L}, \eta_{ll}, \eta_{cl,l}, \eta_{cl,c}$$

Some of these parameters are further tied down by observations and steady state restrictions: we shall discuss this below.

Finally, while it might appear that the model has several endogenous state variables, one can rewrite the equations above in such a way, that only k_{t-1} as endogenous state variable remains[1]. Indeed, one can fairly easily

[1]For this, note that R_{t+1} only shows up in the Euler equation. There, replace r_{t+1} with $\tilde{r}_{t+1} = r_{t+1} + \psi_t$ and add ψ_t separately. Note that now ψ_{t-1} is no longer needed as state variable for $\tilde{r}_t$.

reduce the list of equation above by hand to a two-dimensional first-order difference equation in k_t and λ_t or further to a single second order stochastic difference equation in k_t. Thus, once numerical values for the parameters are given, and once e.g. an AR(1) process is given for the exogenous process z_t, the dynamics can be solved for in closed form by solving a simple quadratic equation, see Uhlig (1999). Adding additional exogenous stochastic processes to the system would not complicate this analysis either. I.e., it is possible in principle to analyze the dynamic properties entirely analytically, although obviously it is more convenient to let a computer perform these calculations.

3.1 Parameter restrictions

The parameters

$$\frac{\bar{X}}{\bar{Y}}, \theta, \delta, \bar{R}, \phi_{nn}, \phi_{kk}, \xi, \bar{L}, \eta_{cc}, \eta_{ll}, \eta_{cl,l}, \eta_{cl,c}$$

cannot be chosen entirely freely: there are some restrictions imposed either by the logic of the model or by observations.

First, equation (1), the capital share θ, and the steady state condition $\bar{X} = \delta \bar{K}$ implies the investment-output ratio

$$\frac{\bar{X}}{\bar{Y}} = \frac{\delta\theta}{\bar{R} - 1 + \delta} \tag{13}$$

One can therefore use observations or calibrations on three of these parameters to tie down the forth. E.g., for quarterly data, $\delta = 0.025, \theta = 0.36$ and $\bar{R} = 1.01$ implies an investment-output ratio of 25.7%, which is consistent with data in table 1.

There are no direct restrictions for $\phi_{nn}, \phi_{kk}, \xi$, but different choices obviously imply different relationships between the volatilities of e.g. wages with respect to fluctuations in labor etc.. In particular $\xi = 0$ effectively turns the economy into an endowment economy in terms of capital, with a highly variable price for capital but zero fluctuations in investment. Exploring these implications is the task of the numerical analysis of the dynamic properties of the model. A typical choice is $\xi = \infty$ (no adjustment cost) or $\xi = 0.23$. For a Cobb-Douglas production function, $\phi_{nn} = \theta, \phi_{kk} = 1 - \theta$, which is the case we shall stick to in the numerical analysis.

Counting hours awake and hours at work, the share $\bar{L}$ of total time spent in the form of leisure is usually calibrated to 2/3. Now, note that

$$\frac{\eta_{cl,c}}{\eta_{cl,l}} = \frac{\bar{C}U_C(\bar{C},\bar{L})}{\bar{L}U_L(\bar{C},\bar{L})} = \frac{\bar{C}}{\bar{L}\bar{W}} = \frac{\bar{C}\frac{\bar{W}\bar{N}}{(1-\theta)\bar{Y}}}{\bar{L}\bar{W}} = \frac{1-\bar{L}}{\bar{L}}\frac{1-\frac{\bar{X}}{\bar{Y}}}{1-\theta}$$

is the ratio of expenditure shares for consumption and leisure. Given the parameter values stated above, we find

$$\kappa = \frac{\eta_{cl,c}}{\eta_{cl,l}} = 0.58$$

where we introduced the symbol κ to refer to this ratio more easily below.

Finally, the utility function must be concave. Aside from $\eta_{cc} > 0$, $\eta_{ll} > 0$, this implies that

$$\eta_{cc}\eta_{ll} - \eta_{cl,l}\eta_{cl,c} > 0$$

For our purposes below, it is more convenient to rewrite this as

$$\eta_{ll} \geq \frac{\kappa\eta_{cl,l}^2}{\eta_{cc}} \tag{14}$$

I.e., the risk aversion with respect to leisure is bound below by an expression, which depends on the risk aversion with respect to leisure, the cross-derivative term $\eta_{cl,l}$ and a parameter κ emerging from macroeconomic observations. The case most often considered in the literature is the case of separability between leisure and consumption, i.e. $\eta_{cl,l} = 0$, in which case one is free to pick η_{ll} to be any positive number. However, when we investigate the asset price implications in the next subsection, it will be interesting to investigate nonseparabilities, which then have implications for leisure risk aversion.

The parameter calibrations and theoretical and numerical restrictions are summarized in table 2.

4 Asset price implications

4.1 General remarks

Equipped with the utility function above, we can study the asset price implications. For convenience, we collect some well-known implications of log-linear asset pricing, see e.g. Lettau and Uhlig (2002) or Campbell (2004).

	Restrictions		
parameter	theoretical	economic	calibration
θ	free	capital share	0.36
δ	free	deprec. rate	0.025
$\bar{R}$	free	gross cap. return	1.01
ϕ_{nn}	free	elast. of wages	θ (Cobb-Douglas)
ϕ_{kk}	free	elast. of div.	$1-\theta$ (Cobb-Douglas)
$\xi \geq 0$	free	adj. cost	0.23 or ∞
$\bar{L}$	free	leisure share	2/3
η_{cc}	free	cons. risk. avers.	$[1, \infty)$
$\eta_{cl,l}$	free	cross derivative	$(-\infty, \infty)$
$\frac{\bar{X}}{\bar{Y}}$	$= \frac{\delta\theta}{\bar{R}-1+\delta}$	investm. share	25.7%
$\kappa = \frac{\eta_{cl,c}}{\eta_{cl,l}}$	$= \frac{(1-\bar{L})}{\bar{L}} \frac{\left(1-\frac{\bar{X}}{\bar{Y}}\right)}{(1-\theta)}$	rel. expend. shares	0.58
η_{ll}	$\geq \frac{\kappa\eta_{cl,l}^2}{\eta_{cc}}$	leisure risk.av.	$[0, \infty)$

Table 2: *The list of parameters of the basic model and their restrictions.*

Generally, for any asset with gross return R_{t+1} (not just investment in physical capital), the Arrow-Lucas-Rubinstein asset pricing equation has to be satisfied,

$$1 = E_t[\beta \frac{\Lambda_{t+1}}{\Lambda_t} R_{t+1}] \tag{15}$$

or

$$0 = \log\beta + \log\left(E_t\left[\exp\left(\Delta\tilde{\lambda}_{t+1} + \tilde{r}_{t+1}\right)\right]\right) \tag{16}$$

where $\tilde{\lambda}_{t+1} = \log\Lambda_{t+1}$, etc., and where Δ denotes the first difference. A "period" here shall be interpreted to be the relevant investment horizon. For example, while trading costs (and, in some countries, Tobin taxes) probably are a major friction for short investment horizons such as a few months, they presumably matter less, if the horizon is several years. Thus, we shall abstract from trading costs, despite the considerable attention they have attracted, see e.g. Luttmer (1999), and instead investigate a variety of investment horizons. A further reason for considering different investment horizons is the return predictability, which has been observed at longer rather than shorter horizons.

Assume that, conditionally on information at date t (and where we assume that Λ_t is part of that information), Λ_{t+1} and R_{t+1} are jointly lognormally distributed. Let $\sigma^2_{\cdot,t}$ denote conditional variances and $\rho_{\cdot,\cdot,t}$ conditional correlations, given information up to and including t. For example (and with some slight further simplification of notation), $\sigma_{\lambda,t} = E_t\left[\left(\tilde{\lambda}_{t+1} - E_t[\tilde{\lambda}_{t+1}]\right)^2\right]$. These variances and correlations may in turn depend on time, but we shall occasionally leave away the additional date subscript to save notation. Using the standard formula for the expectation of lognormally distributed variables, equation (16) can be rewritten as

$$0 = \log\beta + E_t[\Delta\tilde{\lambda}_{t+1}] + E_t[\tilde{r}_{t+1}] + \frac{1}{2}\left(\sigma^2_\lambda + \sigma^2_r + 2\rho_{\lambda,r}\sigma_\lambda\sigma_r\right) \qquad (17)$$

This can be simplified further. First, note that for the risk-free rate r^f_t, i.e. for an asset with $\sigma^2_r = 0$, we have

$$r^f_t = -\log\beta - E_t[\Delta\tilde{\lambda}_{t+1}] - \frac{1}{2}\sigma^2_{\lambda,t} \qquad (18)$$

We see that the risk-free rate varies over time either due to variations in the expected growth rate of the shadow value of wealth, $E_t[\Delta\tilde{\lambda}_{t+1}]$, or its conditional variance, $\sigma^2_{\lambda,t}$. Since the risk-free rate does not fluctuate very much, either these terms do not fluctuate very much, or their fluctuations just offset each other.

Second, for a risky asset, note that

$$\log E_t[R_{t+1}] = E_t[\tilde{r}_{t+1}] + \frac{1}{2}\sigma^2_{r,t}$$

Let $\mathcal{SR}_t$ denote the Sharpe ratio of that asset, calculated as the ratio of the risk premium or equity premium and the standard deviation of the log return,

$$\mathcal{SR}_t = \frac{\log E_t[R_{t+1}] - r^f_t}{\sigma_{r,t}}$$

The Sharpe ratio is the "price for risk", and generally a more useful number than the equity premium itself, see Lettau and Uhlig (2002) for a detailed discussion. We find that

$$\mathcal{SR}_t = -\rho_{\lambda,r,t}\sigma_{\lambda,t} \qquad (19)$$

In particular, we see that the maximally possible Sharpe ratio $\mathcal{SR}^{\max}_t$ for any asset is

$$\mathcal{SR}^{\max}_t = \sigma_{\lambda,t} \qquad (20)$$

which depends on preferences only.

4.2 Consumption and leisure

We now apply this standard logic to the preference specification above. Since the model was formulated such that there is a steady state, the results above stay valid, if we replace the logarithms of the Lagrange multiplier with the log-deviations, etc.., except that for comparison to the data, one ought to keep in mind (and possibly correct the formulas with) the average expected consumption growth rate.

Equation (8) states the log deviation of the Lagrange multiplier to be

$$\lambda_t = -\eta_{cc} c_t + \eta_{cl,l} l_t$$

Consistent but slightly more restrictive than above, we shall assume, that asset returns, consumption and leisure are jointly lognormally distributed, conditional on information at date t. Thus,

$$E_t[\Delta \tilde{\lambda}_{t+1}] = -\eta_{cc} E_t[\Delta \tilde{c}_{t+1}] + \eta_{cl,l} E_t[\Delta \tilde{l}_{t+1}]$$

for the expected change in the shadow value of wealth for the risk free rate equation (18). Further and similar to the derivation of the Sharpe ratio formula above,

$$\mathcal{SR}_t = \eta_{cc} \rho_{c,r,t} \sigma_{c,t} - \eta_{cl,l} \rho_{l,r,t} \sigma_{l,t} \tag{21}$$

as well as

$$\begin{aligned} \mathcal{SR}_t^{\max} &= \sigma_{\lambda,t} \\ &= \sqrt{\eta_{cc}^2 \sigma_{c,t}^2 - 2\eta_{cc}\eta_{cl,l}\rho_{c,l,t}\sigma_{c,t}\sigma_{l,t} + \eta_{cl,l}^2 \sigma_{l,t}^2} \\ &\leq \eta_{cc}\sigma_{c,t} + \mid \eta_{cl,l} \mid \sigma_{l,t} \end{aligned} \tag{22}$$

In principle, thus, it appears as if nonseparability between consumption and leisure can help. A high relative risk aversion η_{cc} is usually required to explain the observed Sharpe ratio. However, with the appropriate value for the cross-derivative term $\eta_{cl,l}$, one can now vary η_{cc} considerably. This comes at a price. A higher absolute value for $\eta_{cl,l}$ requires a higher relative risk aversion in leisure, see equation (14). Furthermore, these choices will have consequences for the endogenous choices in the macroeconomic model above.

4.3 Data

Let us investigate the data on the correlations of log leisure, log consumption and log excess returns. Here, log leisure is taken to be the negative of log labor, calculated from the time series AWHI, and log consumption is calculated from the time series PCENDC96, both available from the St. Louis Federal Reserve Bank. To calculate log excess returns $r_{t+1} - r_t^f$, we used the time series TRSP500, which is the total value of a S&P500 portfolio, with dividends reinvested, took logs and quarterly averages, and subtracted from this series the log of the value of a “safe portfolio of compounded quarterly interest rates, taken from the 1-year treasury bill rate. Of this series, we took k-th differences to vary the length of the asset holding period, and likewise for log leisure and log consumption. The asset market results are in table 3, whereas the standard deviations and correlations with leisure and consumption are in table 4. The time period is 1970:1 to 2003:4. Note that the Sharpe ratio appears to be lower by nearly a factor of two compared to the usual numbers: this is to some degree due to using log returns, which “worsens” negative stock market returns, and “lessens” positive returns, as is necessary for calculating compounded returns (i.e. geometric averages), although that does not appear to explain it entirely.

In principle, one should perhaps also subtract out the part of the excess return which is predictable with e.g. current price-dividend ratios, in order to calculate conditional correlations and standard deviations. The same is true for consumption and leisure. In these calculations, we thus ”pretend”, that these k-th differences are not predictable and calculate their raw, unconditional correlations.

What one can see in tables 3 and 4 is the following. First, there are no surprises as far as the market price for risk is concerned, as one varies the horizon: the annualized Sharpe ratio remains fairly constant at around 0.3. Second, the correlation between leisure and excess returns over a short holding period of one quarter is very low and too low to be of much help in helping with high consumption risk aversion to explain the equity premium observation.

Third, and more interestingly, the picture does change at longer holding horizons. For example, at a holding period of one year or four quarters, the correlation between leisure and excess returns is already -.21, at eight quarters, it is -.39, and generally exceeds the correlation of consumption with excess returns at horizons above two years.

Finally, the correlation between leisure and stock returns is negative, i.e. stocks provide "insurance against fluctuations in leisure. This is intuitively not surprising, since one expects stocks to do well in booms, which are precisely the times when hours and output are high. Since the Sharpe ratio is determined by the cross derivative term $\eta_{cl,l}$ and not the relative risk aversion with respect to leisure, this insurance aspect is not a problem for the preference-based asset pricing framework: we shall examine the precise implications in the following subsection. If relative risk aversion in consumption is not alone to explain the observed Sharpe ratio, then (25) and the negative correlation between leisure and stock returns implies that one needs $\eta_{cl,l} > 0$, i.e. one needs that leisure and consumption are complements.

The asset pricing formulas above in principle allow for time variation in the volatilities. To generate a time-varying volatility series for leisure, I have calculated the GARCH process

$$\sigma_{l,t}^2 = (1 - \phi)\sigma_{l,t-1}^2 + \phi(l_t - l_{t-1} - E[l_t - l_{t-1}])^2$$

initializing the process with the unconditional variance of leisure. I have likewise proceeded for consumption. A plot of the two series is in figure 1.

Equation (25 suggests that changing volatities induce changes in the Sharpe ratio. For example, assuming the correlations to stay constant, we find

$$\Delta \mathcal{SR}_{t+1} = \eta_{cc}\rho_{c,r}\Delta\sigma_{c,t+1} - \eta_{cl,l}\rho_{l,r}\Delta\sigma_{l,t+1} \tag{23}$$

Assuming furthermore, that stock market volatility stays constant as well, a surprise decrease in the Sharpe ratio implies an extra positive surprise in stock returns. Keeping in mind the negative correlation $\rho_{l,r} < 0$ and the positive value for $\eta_{cl,l}$, equation (23) therefore predicts a negative correlation between stock returns and changes in the volatilies of consumption as well as leisure. Table 5 investigates this issue. Indeed, and in particular at longer horizons, we see that the correlation is negative indeed, in particular between the volatility for leisure and stock returns. I.e., decreases in business cycle uncertainty increase stock returns: this makes a lot of intuitive sense. Figure 2 shows that negative correlation for a holding period of $k = 8$ quarters.

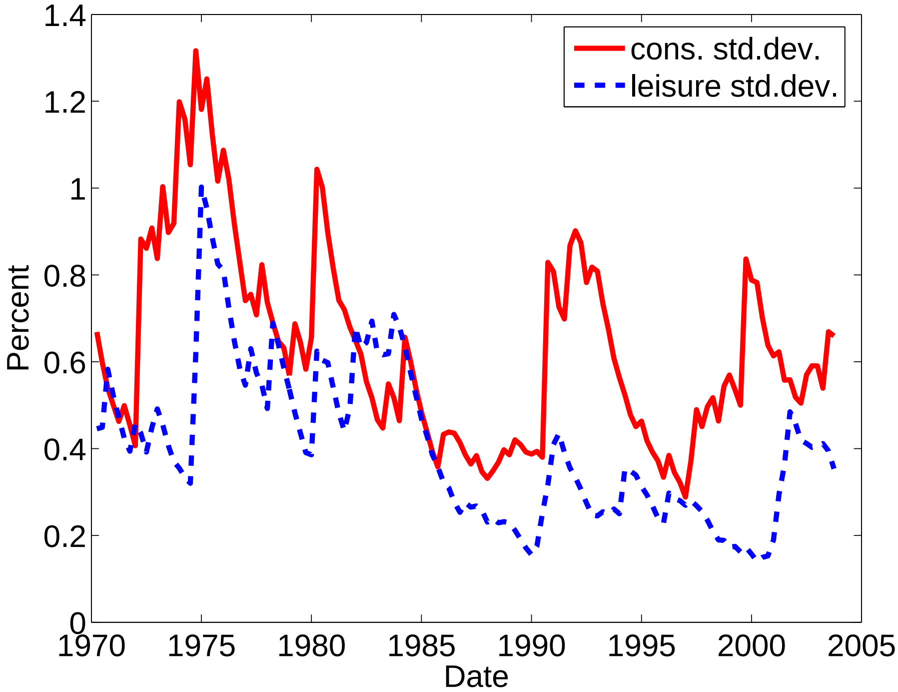

Figure 1: *The time-varying volatilies of leisure and consumption.*

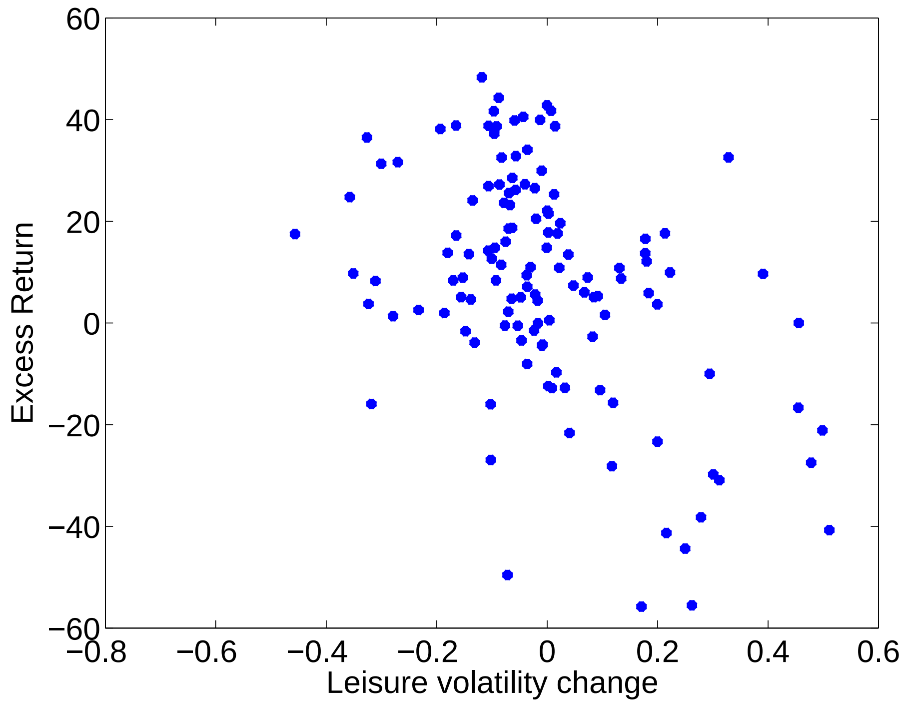

Figure 2: *The correlation between changing leisure volatility and excess stock returns for a holding period of* $k = 8$ *quarters*

Horizon k (Quarters)	std.dev. of r_{t+1}	Sharpe ratio	Annualized Sharpe ratio, $SR\sqrt{4/j}$
1	6.87	0.15	0.30
2	10.37	0.21	0.29
3	13.18	0.24	0.28
4	15.40	0.27	0.27
5	17.51	0.29	0.26
6	19.32	0.31	0.25
7	20.96	0.33	0.25
8	22.21	0.36	0.26
9	23.34	0.39	0.26
10	24.66	0.42	0.26
11	25.81	0.44	0.27
12	26.75	0.47	0.27
13	27.69	0.50	0.28
14	28.42	0.54	0.29
15	29.01	0.58	0.30
16	29.47	0.63	0.31
17	29.99	0.67	0.33
18	30.75	0.71	0.33
19	31.17	0.76	0.35
20	31.41	0.82	0.37

Table 3: *Properties of excess returns, when varying the holding horizon.*

Horizon k (Quarters)	std.dev. of leis., σ_l	std.dev. of cons., σ_c	corr(c,l)	corr(l,r)	corr(c,r)
1	0.45	0.67	-0.33	-0.07	0.27
2	0.80	1.04	-0.42	-0.08	0.34
3	1.11	1.33	-0.51	-0.15	0.37
4	1.36	1.64	-0.55	-0.21	0.39
5	1.58	1.90	-0.58	-0.28	0.39
6	1.78	2.10	-0.61	-0.33	0.40
7	1.95	2.27	-0.62	-0.36	0.41
8	2.10	2.42	-0.62	-0.39	0.42
9	2.23	2.52	-0.61	-0.42	0.40
10	2.32	2.60	-0.62	-0.45	0.37
11	2.40	2.67	-0.63	-0.47	0.36
12	2.46	2.73	-0.62	-0.50	0.34
13	2.50	2.80	-0.62	-0.52	0.35
14	2.51	2.87	-0.60	-0.54	0.36
15	2.51	2.95	-0.59	-0.56	0.37
16	2.49	3.01	-0.57	-0.58	0.39
17	2.47	3.06	-0.55	-0.60	0.41
18	2.45	3.09	-0.53	-0.60	0.41
19	2.42	3.12	-0.51	-0.60	0.41
20	2.39	3.11	-0.48	-0.59	0.41

Table 4: *Variances and correlations of leisure and consumption with excess returns.*

Horizon k (Quarters)	std.dev. of leis.vol.	std.dev. of cons.vol.	$corr(\sigma_c, \sigma_l)$	$corr(\sigma_l, r)$	$corr(\sigma_c, r)$
1	0	0.01	0.18	0.06	0.00
2	0.01	0.02	0.22	-0.01	-0.00
3	0.02	0.02	0.24	-0.13	-0.01
4	0.02	0.03	0.21	-0.23	-0.00
5	0.03	0.04	0.21	-0.28	0.01
6	0.03	0.05	0.18	-0.32	0.02
7	0.03	0.06	0.17	-0.38	0.02
8	0.03	0.07	0.17	-0.46	0.02
9	0.04	0.07	0.18	-0.50	-0.00
10	0.04	0.08	0.18	-0.52	-0.04
11	0.04	0.09	0.20	-0.52	-0.06
12	0.04	0.10	0.24	-0.53	-0.07
13	0.04	0.10	0.28	-0.53	-0.08
14	0.04	0.11	0.31	-0.53	-0.10
15	0.05	0.11	0.35	-0.51	-0.11
16	0.05	0.11	0.38	-0.52	-0.11
17	0.05	0.11	0.41	-0.54	-0.13
18	0.05	0.11	0.44	-0.54	-0.12
19	0.05	0.10	0.45	-0.53	-0.10
20	0.05	0.10	0.43	-0.52	-0.09

Table 5: *Variances and correlations of the volatility of leisure, the volatility of consumption and excess returns.*

4.4 Implications for preferences

We now use these observations to draw out implications for preferences, assuming now that volatilies and correlations stay constant. The standard case, on which practically the entire asset pricing literature has focussed, is the case $\eta_{cl,l} = 0$. In that case, (25) implies

$$\eta_{cc} = \frac{\mathcal{SR}}{\rho_{c,r}\sigma_c} \tag{24}$$

for the level of relative risk aversion in consumption. Using an annual holding period, $k = 4$, and the data of the tables above, one obtains

$$\eta_{cc} = \frac{0.27}{1.64\% * 0.39} = 42$$

Even assuming perfectly positive correlation, one needs $\eta_{cc} = 16.5$. Other authors typically find even much higher values, see Campbell (2004). These values seem high on a priori grounds and incompatible with standard macroeconomic models.

With nonseparabilities between consumption and leisure, however, lower values for η_{cc} are possible, when the value of the cross-derivative is changed simultaneously as well. To that end, rewrite equation (25) as

$$\eta_{cl,l} = \frac{\mathcal{SR} - \eta_{cc}\rho_{c,r}\sigma_c}{-\rho_{l,r}\sigma_l} \tag{25}$$

For the macroeconomic implications, and since leisure is fairly volatile, it is desirable to pick the relative risk aversion with respect to leisure as low as possible. We thus assume that equation (14) holds with equality,

$$\eta_{ll} = \frac{\kappa\eta_{cl,l}^2}{\eta_{cc}}$$

For holding periods of one year, $k = 4$ and two years, $k = 8$, table 6 as well as figures 3 and 4 show the resulting values as a function of the relative risk aversion for consumption, η_{cc}.

We see that explaining the Sharpe ratio remains hard: low values for the relative risk aversion in consumption require dramatically high values for the relative risk aversion in leisure. It is some progress that one can explain

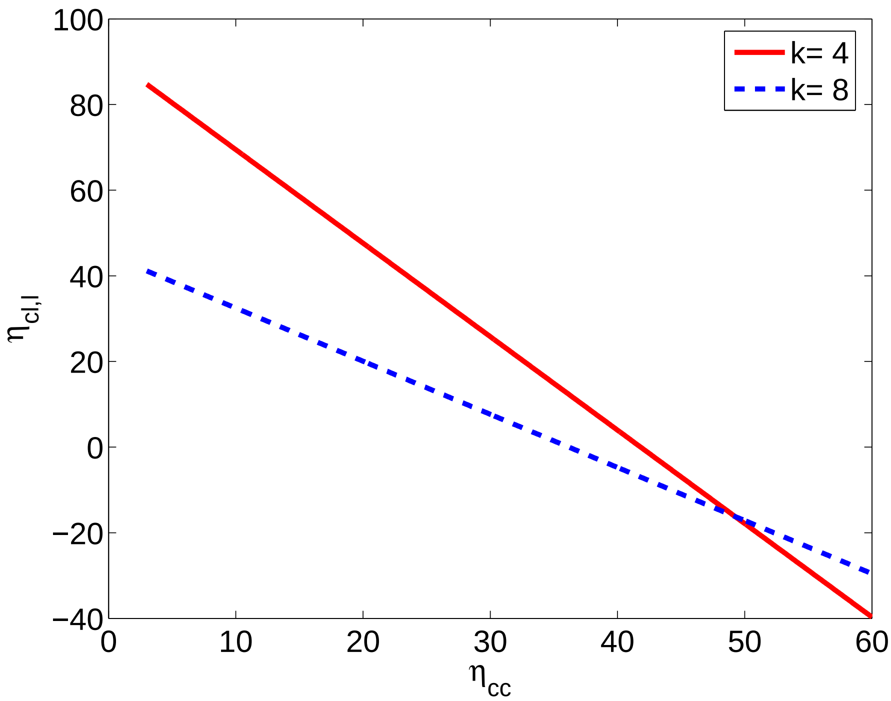

Figure 3: *The implied value for the cross-derivative $\eta_{cl,l}$, when varying the relative risk aversion for consumption between 3 and 60.*

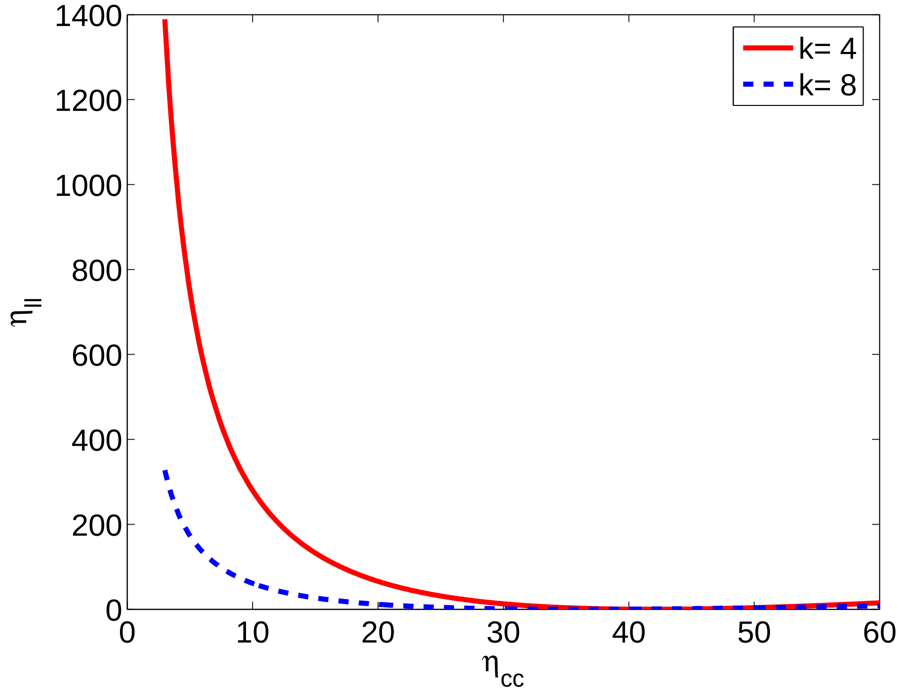

Figure 4: *The implied value for the minimal relative risk aversion in leisure* η_{ll}*, when varying the relative risk aversion for consumption between 3 and 60.*

η_{cc}	$\eta_{cl,l}$		η_{ll}	
	k=4	k=8	k=4	k=8
3.0	84.7	41.1	1389.2	327.5
5.0	80.4	38.7	749.8	173.5
10.0	69.5	32.5	280.0	61.2
15.0	58.5	26.3	132.6	26.7
20.0	47.6	20.1	65.8	11.7
30.0	25.8	7.7	12.9	1.1
40.0	4.0	-4.7	0.2	0.3
50.0	-17.9	-17.1	3.7	3.4

Table 6: *Implied values for the cross-derivative term $\eta_{cl,l}$ and the minimal relative risk aversion in leisure η_{ll}, when varying the relative risk aversion in consumption η_{cc}.*

the observed Sharpe ratio at levels of relative risk aversion below 20, even when taking account the correct correlations, using the calculations based on a holding period of $k = 8$ quarters. Obviously, these are still fairly high numbers.

5 Macroeconomic implications

The asset pricing literature typically takes consumption and leisure choices as given. However, given the calculated preference parameters, these choices need to be regarded as endogenous. The model of section 3 therefore helps to answer the question, how the economy will behave, given these parameters. For the technology process, we have now assumed an AR(1) process,

$$z_t = 0.95z_{t-1} + \epsilon_t$$

where σ_ϵ will be rescaled in such a way, that the HP-filtered standard deviation of output is 2%, as a benchmark number which is roughly consistent with the data. Alternatively, one could have chosen the standard choice for this standard deviation of 0.712 used in the literature. Since much of the information of the model behavior is contained in the volatilities relative to output volatility, it seemed more useful to show the ability (or absence thereof) of the model to generate the observed fluctuations in terms of the necessary scale of σ_ϵ.

The results can be seen in tables 8 and 9, using two different values for the adjustment cost parameter ξ. Impulse responses to a 1% technology shock are shown in table 10. The annualized Sharpe ratio has been obtained directly via equation 26, assuming an asset holding period of 8 quarters. More precisely,

$$\mathcal{SR}_{ann.} = \frac{E[(\lambda_{t+8} - \lambda_t)^2]}{\sqrt{2}} \tag{26}$$

The result here should be compared to the number in the right-most column of table 3, i.e. to 0.3.

Since the relative risk aversions either in consumption or leisure are fairly extreme, we have also chosen preference parameters implied from targeting a quarter of the observed Sharpe ratio, see table 7. The results for the model simulations are now in tables 11 and 12. Impulse responses to a 1% technology shock are shown in table 13.

There is a wealth of results here, on which one can derive solid intuition, using the loglinearized equations of the models as well as the impulse response functions. Let me just point out a few things. First, adjustment costs help in generating sizeable Sharpe ratios, in particular for high levels of relative risk aversion in consumption. However, the fluctuation of the technology shock need to be scaled up by nearly an order of magnitude to make the output fluctuations consistent with the data. Furthermore, labor reacts negatively to a technology shock. Perhaps this is indeed a feature of the data, see the recent literature, e.g. Basu et al. (1999), Shea (1998), Gali (1999), Francis and Ramey (2001,2003), Christiano et al (2003) and Uhlig (2004). However, given the technology-shock driven model here, it makes it impossible to explain the positive comovement between hours, investment, consumption and output. High adjustment costs also make consumption too volatile, and generate too little investment volatility. Interestingly, it does not seem to make much difference in terms of implied Sharpe ratios, as to whether one takes parameter choices implied by targeting the original Sharpe ratio, or the parameter choices from table 7, generated from only targeting a quarter of the observed Sharpe ratio. Clearly then, the discrepancy to the data must then show up in other places for the latter, and it does. Due to the endogeneity of the economic choices, agents smooth those variables considerably stronger,

η_{cc}	$\eta_{cl,l}$		η_{ll}	
	k=4	k=8	k=4	k=8
1.0	20.6	10.0	247.2	57.8
3.0	16.3	7.5	51.2	10.9
5.0	11.9	5.0	16.5	2.9
7.0	7.5	2.5	4.7	0.5

Table 7: *Reducing the Sharpe Ratio by a factor of 4: implied values for the cross-derivative term $\eta_{cl,l}$ and the minimal relative risk aversion in leisure η_{ll}, when varying the relative risk aversion in consumption η_{cc}.*

where they dislike fluctuations a lot. Thus, e.g. a higher risk aversion in consumption results ceteris paribus in lower consumption fluctuations, and thus possibly no change in the Sharpe ratio. This is a lesson, which has also been emphasized by Lettau and Uhlig (2000), investigating the implications of habit formation.

Finally, note that consumption and labor always move in opposite directions in these simulations. There are two reasons for this. First, the agent can use labor movements as insurance against consumption fluctuations. I.e., if productivity is unusually low, the agent can compensate with high labor in order to keep consumption from dropping too much, and vice versa in times of high productivity. The second reason is the large positive value for $\eta_{cl,l}$: this turns consumption and leisure into complements. I.e., if consumption is high, the agent also wishes labor to be low or vice versa.

The lesson here is that implications from asset prices for preferences in turn have implications for the endogenous choices of consumption and leisure, which need to be compared to the data. This additional discipline on the choice of parameters or preferences is worth emphasizing more, and this paper provides a machinery for doing so.

6 Exogenous wage movements

The key difficulty of the simple model to jointly explain asset pricing facts and macroeconomic facts lies in the labor market. The intuition is simple. We observe that hours worked fluctuate nearly as much as output over the cycle. In the standard model, agents equate the marginal utility of leisure

Parameters		Labor		Cons.		Inv.	
η_{cc}	σ_ϵ	$\sigma_{n,HP}$	$\rho(n,y)$	$\sigma_{c,HP}$	$\rho(c,y)$	$\sigma_{x,HP}$	$\rho(x,y)$
$\xi = 0.23$:							
5	1.85	0.64	-1	2.56	1	0.31	1
10	2.27	1.48	-1	2.52	1	0.43	1
15	2.84	2.64	-1	2.47	1	0.60	1
20	3.67	4.33	-1	2.37	1	0.88	1
$\xi = \infty$:							
5	1.07	1.04	0.66	4.02	-0.65	18.32	0.87
10	1.11	1.06	0.74	1.71	-0.72	12.20	0.96
15	1.16	0.99	0.73	0.85	-0.69	9.94	0.98
20	1.23	0.91	0.69	0.44	-0.61	8.87	0.99

Table 8: *Results for the basic model, when using preferences targeted at matching the Sharpe ratio observation for a holding period of* $k = 8$ *periods. The volatility of the technology shock has been rescaled so that the HP-filtered standard deviation of output equals 2%: compare it to the standard value of 0.7 in the literature. The table shows results for the HP-filtered model output.*

η_{cc}	σ_c	σ_l	$\mathcal{SR}_{ann.}$	σ_{rf}	σ_r
$\xi = 0.23$:					
5	5.25	0.65	0.01	0.06	1.08
10	5.00	1.47	0.02	0.14	1.46
15	4.72	2.53	0.03	0.26	2.05
20	4.79	4.37	0.06	0.48	3.00
$\xi = \infty$:					
5	7.37	0.95	0.00	0.06	0.08
10	3.36	1.03	0.01	0.08	0.10
15	1.91	1.08	0.01	0.11	0.12
20	1.13	1.10	0.02	0.13	0.14

Table 9: *Further results, choices as in the previous table, original Sharpe ratio target. Listed are the volatilities of the* $k = 8$*-period differenced consumption and leisure series, the annualized Sharpe ratio, the volatility of the risk-free rate and of the return to capital.*

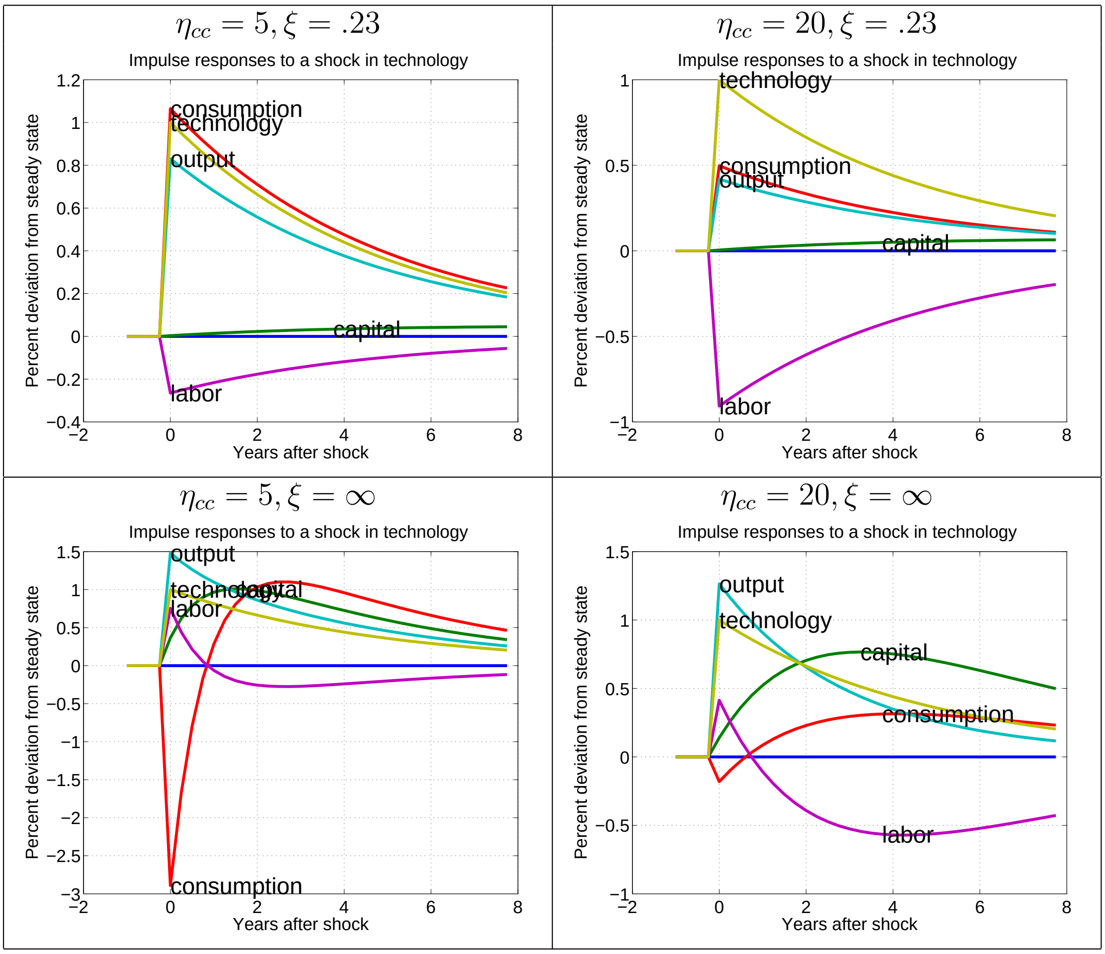

Table 10: *Impulse responses for four of the eight model variations, original Sharpe ratio target*

Parameters		Labor		Cons.		Inv.	
η_{cc}	σ_ϵ	$\sigma_{n,HP}$	$\rho(n,y)$	$\sigma_{c,HP}$	$\rho(c,y)$	$\sigma_{x,HP}$	$\rho(x,y)$
$\xi = 0.23$:							
1	1.76	0.44	-1	2.57	1	0.29	1
3	2.34	1.62	-1	2.51	1	0.47	1
5	3.23	3.43	-1	2.40	1	0.79	1
7	4.79	6.59	-1	2.21	1	1.38	1
$\xi = \infty$:							
1	1.05	1.02	0.65	5.04	-0.61	20.99	0.82
3	1.09	1.08	0.78	1.29	-0.70	11.05	0.97
5	1.16	0.97	0.78	0.44	-0.50	8.74	0.99
7	1.23	0.87	0.72	0.19	0.26	7.87	1

Table 11: *Results for the basic model, when using preferences targeted at matching the Sharpe ratio observation divided by the factor of 4, for a holding period of $k = 8$ periods. The volatility of the technology shock has been rescaled so that the HP-filtered standard deviation of output equals 2%: compare it to the standard value of 0.7 in the literature. The table shows results for the HP-filtered model output.*

η_{cc}	σ_c	σ_l	$SR_{ann.}$	σ_{rf}	σ_r
$\xi = 0.23$:					
1	5.16	0.44	0.01	0.04	1
3	5.12	1.65	0.02	0.18	1.63
5	5.05	3.60	0.05	0.41	2.71
7	4.49	6.71	0.10	0.85	4.75
$\xi = \infty$:					
1	8.80	0.88	0.00	0.05	0.08
3	2.64	1.05	0.01	0.09	0.10
5	1.18	1.07	0.02	0.12	0.14
7	0.59	1.04	0.02	0.15	0.16

Table 12: *Further results, choices as in the previous table, Sharpe ratio target divided by 4. Listed are the volatilities of the $k = 8$-period differenced consumption and leisure series, the annualized Sharpe ratio, the volatility of the risk-free rate and of the return to capital.*

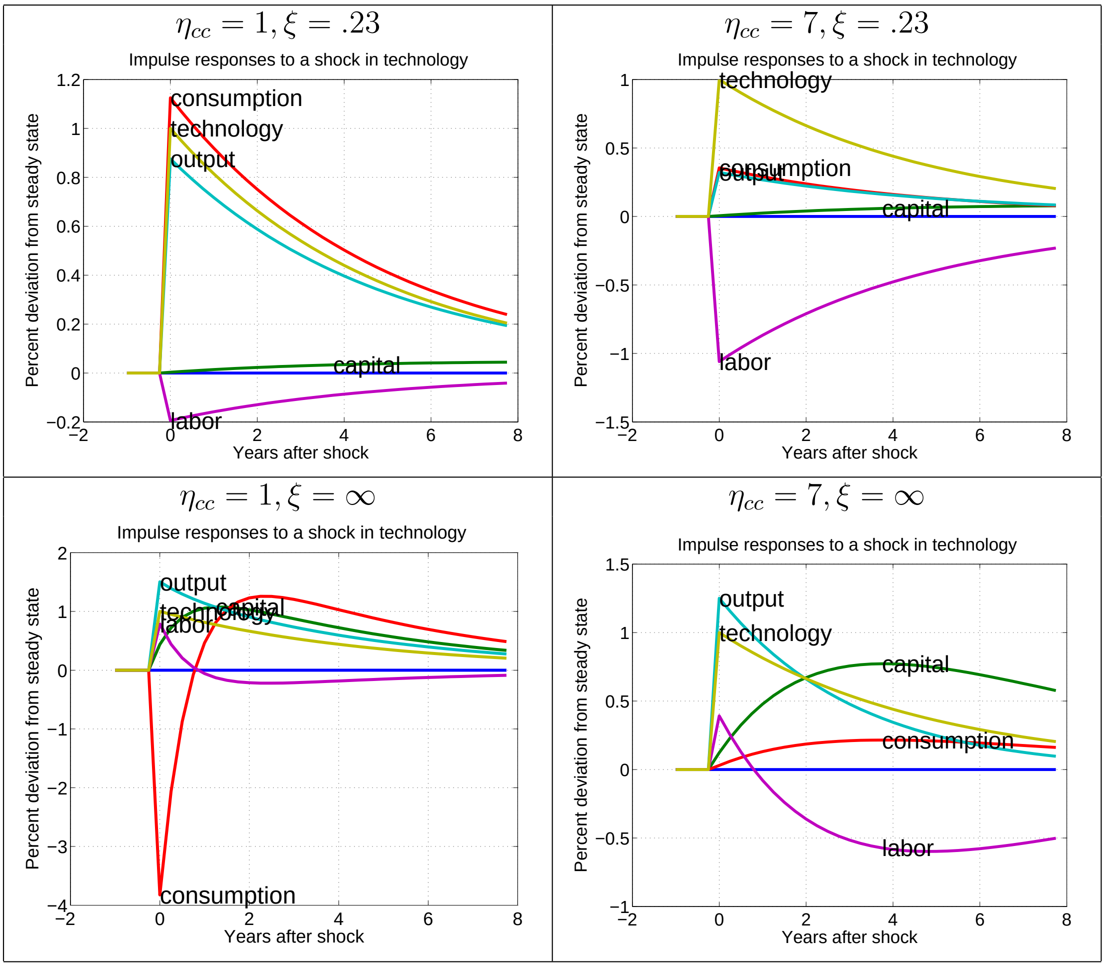

Table 13: *Impulse responses for four of the eight model variations, Sharpe ratio target divided by 4*

to the opportunity costs of working, i.e. the real wage. They can thus use the labor-leisure choice as an insurance device. E.g. with high risk aversion in consumption, they can work hard, should consumption otherwise be low, and work less, should consumption otherwise be high.

These connections and the key role of labor markets have also been emphasized by Lettau and Uhlig (2002), who focus on utility functions with habit formation. Indeed, a number of papers in the literature thus either assume labor to be constant, e.g. Jermann (1998), or assume considerable frictions in adjusting labor input, e.g. Boldrin, Christiano and Fisher (2001).

Understanding labor markets obviously is a major subject on its own, and an entire branch of the economics literature is devoted to studying it. That literature has investigated and emphasized a number of frictions on labor markets. Based on that research, one may want to question the possibility to use endogenous leisure-labor choices to smooth out stock market fluctuations.

6.1 The evolution of wages

As an alternative to the equation (9), equating wages to marginal utility of leisure, I propose that wages adjust sluggishly to labor market conditions, and that workers are always lined up to take a job, if one is available. I.e., I assume that the wage is always below the labor-market clearing price, and assume instead that the log real wage evolves according to

$$w_t = \gamma w_{t-1} + \alpha n_{t-1} \tag{27}$$

where γ is close to unity and α is positive. The idea is that real wages move sluggishly, adjusting upwards when labor markets get tighter and downwards, as unemployment rises. I do not claim that I have good microfoundations for this equation. Rather, I regard it as a heuristically plausible starting point. It turns out that it works remarkably well in moving the theoretical predictions closer to the data, and thus opens a fruitful avenue in jointly explaining macroeconomic facts and asset pricing facts by paying greater attention to labor market frictions. I can imagine that a microfoundation for equation (27) could e.g. be found, following Hall (2003) or the labor market search literature, see e.g. Petrongolo and Pissarides (2001) and the references therein.

To find the parameters for (27), the natural thing to do is to simply run a regression[2]. However, this generates misleading results. Hours worked are trending in the data because of population growth and long-run shifts in e.g. the labor supply by women, and wages are growing due to trend productivity growth, while the model is formulated in terms of stationary variables, i.e. (27) should be understood to refer to log-deviations from a steady state growth path. But even removing a quadratic trend from the logs of both of these variables generates little or even negative correlations at short lags, see table 14 or figure 6. Therefore, rather than estimating $[w_t, n_t]' = B[w_{t-1}, n_{t-1}]$ directly, I instead run a first-order VAR of quarterly quadratically detrended real wages and hours on its 12th lag, and take the resulting quadratic matrix to the power of 1/12: one can view this as an IV-estimate of the first-order autocorrelation matrix B. I obtain

$$\begin{bmatrix} w_t \\ n_t \end{bmatrix} = \begin{bmatrix} 0.29 & 0.29 \\ -0.55 & -0.34 \end{bmatrix} \begin{bmatrix} w_{t-12} \\ n_{t-12} \end{bmatrix}$$

which thus implies

$$\begin{bmatrix} w_t \\ n_t \end{bmatrix} = \begin{bmatrix} 1.04 & 0.15 \\ -0.28 & 0.72 \end{bmatrix} \begin{bmatrix} w_{t-1} \\ n_{t-1} \end{bmatrix}$$

This is economically reasonable. The first line of coefficients in this matrix shows, that wages indeed move sluggishly and react positively to a tightening of the labor market, as expected. The second line states, that lower wages imply higher employment. The roots of the implied first-order matrix are complex, and have 0.89 as their absolute value. The implied dynamics is actually quite interesting and shown in figure 5 in response to a one-time surprise increase of wages by 1%. This depresses the labor market, and eventually, this decline in hours forces wages down, overshooting slightly to the other side after about six years. Whether the complex roots in this system might be a contributor or even a key source of business cycle fluctuation could merit further investigation.

Based on these estimates, I fix $\gamma = 1.04$ and $\alpha = 0.15$ for the calculations to follow.

[2] For the empirical analysis, I use the data series AWHI and COMPRNFB, available from the St. Louis Federal Reserve Bank web site. The data is from 1964 to 2004.

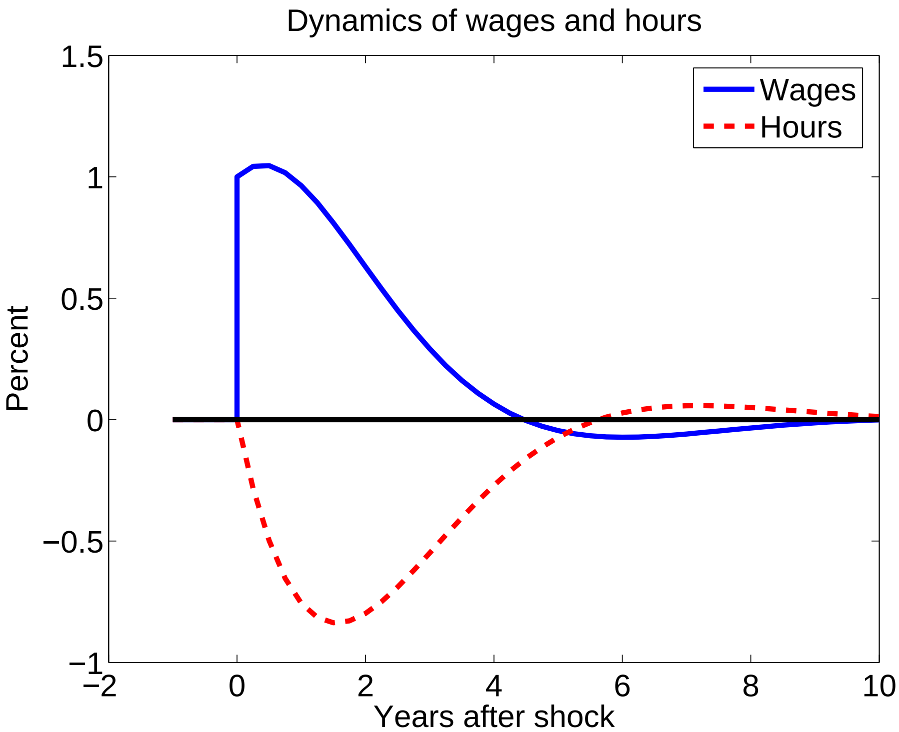

Figure 5: *The value of the criterion function χ, as ξ and η_{cc} are varied in the representative agent model with exogenous wages.*

j	$\rho_{w(t),n(t-j)}$
0	-0.06
1	-0.03
2	0.00
3	0.04
4	0.07
5	0.11
6	0.14
7	0.18
8	0.23
9	0.27
10	0.31
11	0.33
12	0.36
13	0.37
14	0.37
15	0.36
16	0.34
17	0.33
18	0.31
19	0.28
20	0.25

Table 14: *Correlation of log real wages and log hours worked, after removal of a quadratic trend. The data is from the St. Louis Federal Reserve.*

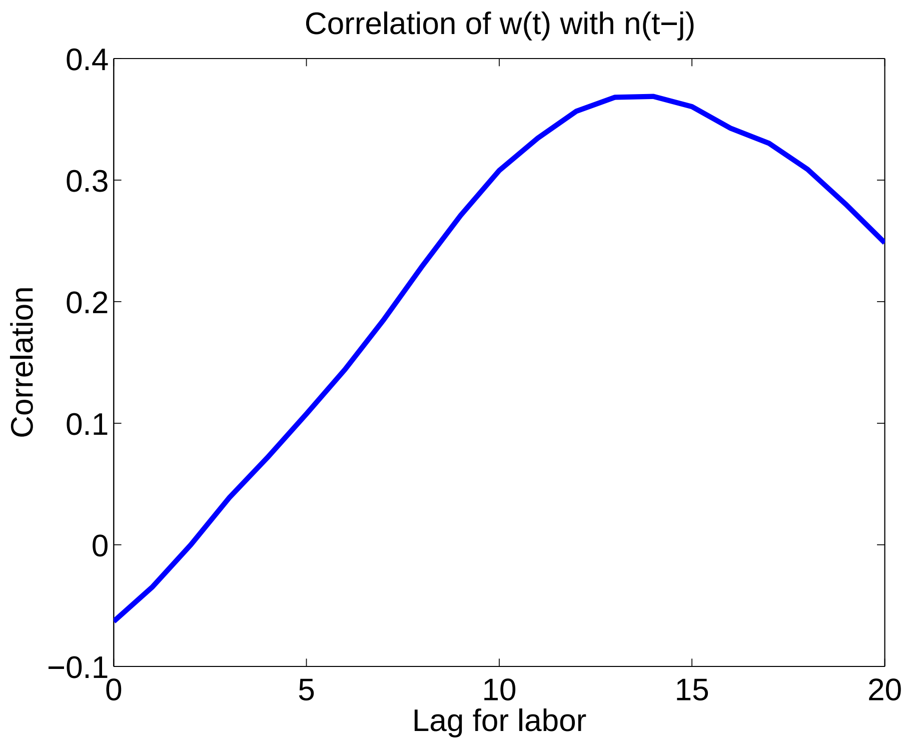

Figure 6: *The value of the criterion function* χ, *as* ξ *and* η_{cc} *are varied in the representative agent model with exogenous wages.*

6.2 Model implications

To investigate the scope of the alternative model, where (9) has been replaced with (27), $\gamma = 1.04$, $\alpha = 0.15$, I allow η_{cc} as well as ξ to vary over some reasonable range. I compare the models, using the criterion function

$$\begin{aligned}\chi \quad = \quad & 1000 * (\mathcal{SR}\text{ann.} - 0.27)^2 + (\sigma_{rf} - 1.7)^2 \\ & +(\sigma_{n,HP} - 1.79)^2 \\ & +(\max(\sigma_{c,HP} - 1.30/2.13, 0) + \min(\sigma_{c,HP} - 0.82/1.74, 0))^2 \\ & +(\max(\sigma_{x,HP} - 6.87/1.74, 0) + \min(\sigma_{x,HP} - 8.07/2.13, 0))^2 \\ & +(\rho_{c,y} - 0.80)^2 + (\rho_{n,y} - 0.86)^2 + (\rho_{x,y} - 0.83)^2\end{aligned}$$

One model is better than another model, if it generates a lower value for χ. This could be viewed as a rough GMM procedure. Alternatively, it can be viewed as reflecting the tastes of this author: I want the model to be close in particular on the Sharpe ratio prediction, and I also want it to be close on a number of other macroeconomic business cycle features. Certainly, one can move to more sophisticated estimation and calibration techniques: again, this would make sense for “fine tuning” the results here, alongside a more refined version of the labor market modelling. The point here is simply to show the path to a potentially very fruitful field of further research.

I allow for $\eta_{cc} \in [1, 40]$ and $\xi \in [0.05, 1.95]$, solving for $\eta_{cl,l}, \eta_{cl,c}$ and η_{ll}, as described in the previous section, when targeting the original Sharpe ratio. I also tried out wider ranges. The results for the criterion function can be seen in figure 7. The criterion “desires a large value for η_{cc}, and ties down ξ fairly sharply at about 0.55, a value nearly twice as high than the traditional value of 0.23 in the literature. Obviously, since this is not an estimation and the weights in the function χ are a bit arbitrary, it is more important to understand how this comes about, i.e. what the tradeoffs are, as the parameters are varied.

The Sharpe ratio increases with larger values for η_{cc} and lower values for ξ, see figure 8. On the other hand, low values for ξ generate high values for the interest rate volatility, see figure 9. Note also, how the output-consumption correlation and the consumption volatility is quite sensitive to these paramters, see figures 10 as well as figure 11, keeping in mind that the point of view has changed. Importantly, due to the change in my assumptions regarding the labor market, the labor-output correlation now stays positive

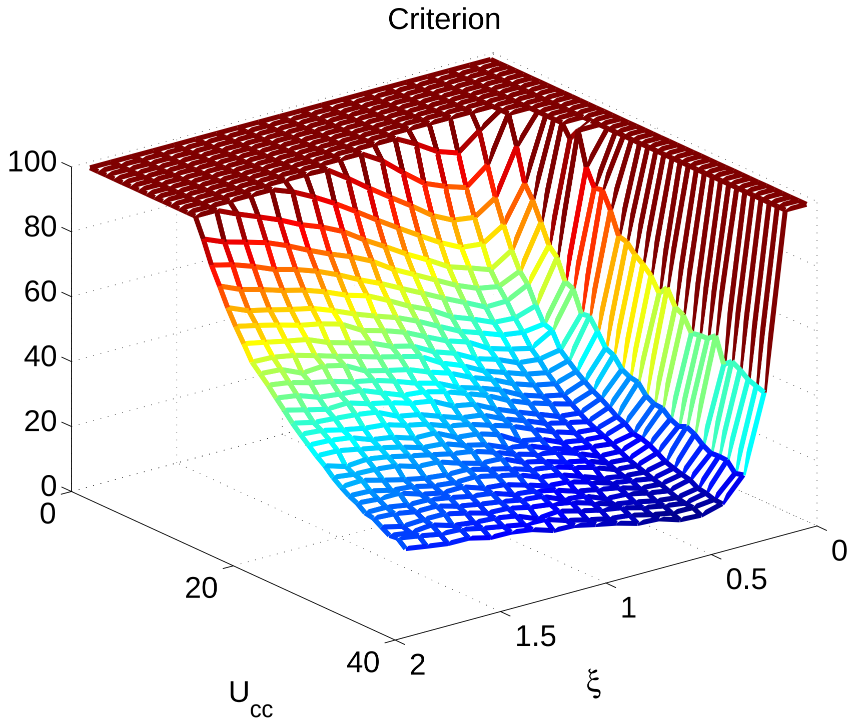

Figure 7: *The value of the criterion function χ, as ξ and η_{cc} are varied in the representative agent model with exogenous wages.*

in the range of parameters considered, see figure 12.

For $\eta_{cc} = 40$ and $\xi = 0.55$, I obtain the results in table 15. I have rescaled σ_ϵ in the process for technology such that the HP-filtered variance of output equals 2, which seems more or less the value in the data. As one can see, not much of a change is required: I need $\sigma_\epsilon = 0.65$. Overall, the quantitative features of this model compare remarkably well to the data. Note that the risk-free rate is not particularly volatile, but that the returns to capital are, and that this model is therefore quite capable of producing a sizeable equity premium, even if stocks are viewed as an unlevered claim to capital. The impulse response functions for a technology shock are shown in figure 13.

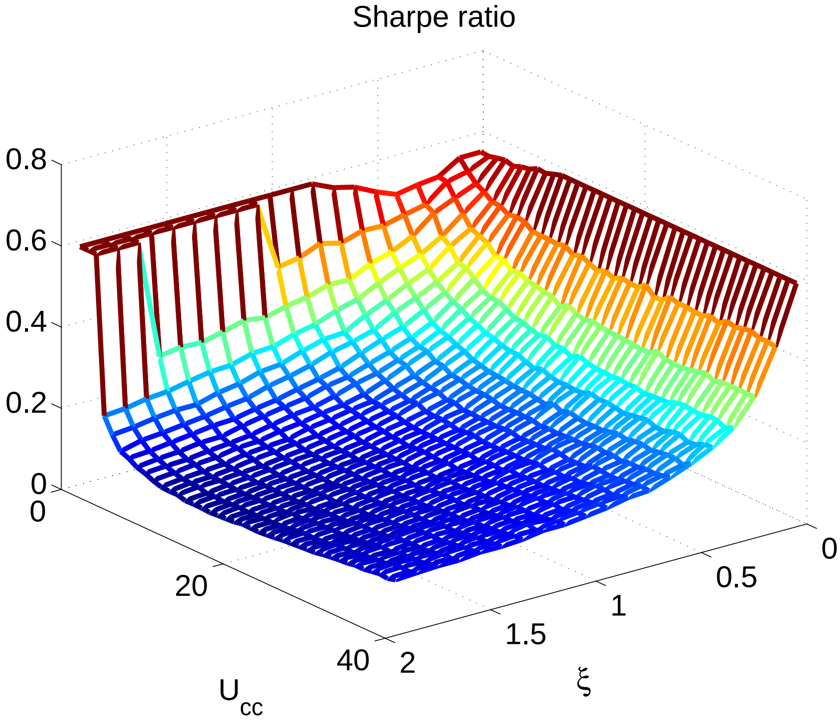

Figure 8: *The value for the Sharpe ratio, as ξ and η_{cc} are varied in the representative agent model with exogenous wages.*

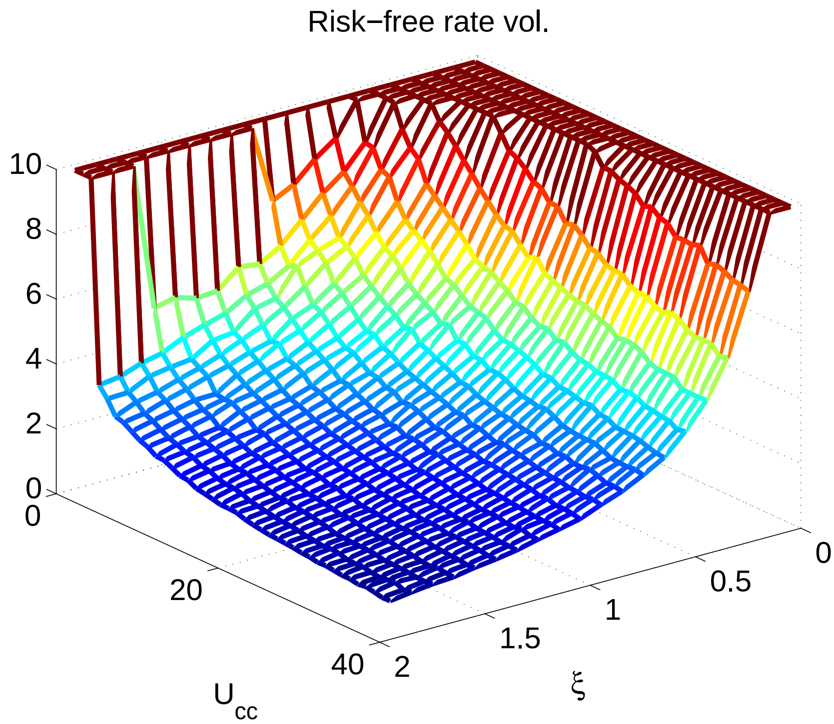

Figure 9: *The volatility of the risk-free rate, as* ξ *and* η_{cc} *are varied in the representative agent model with exogenous wages.*

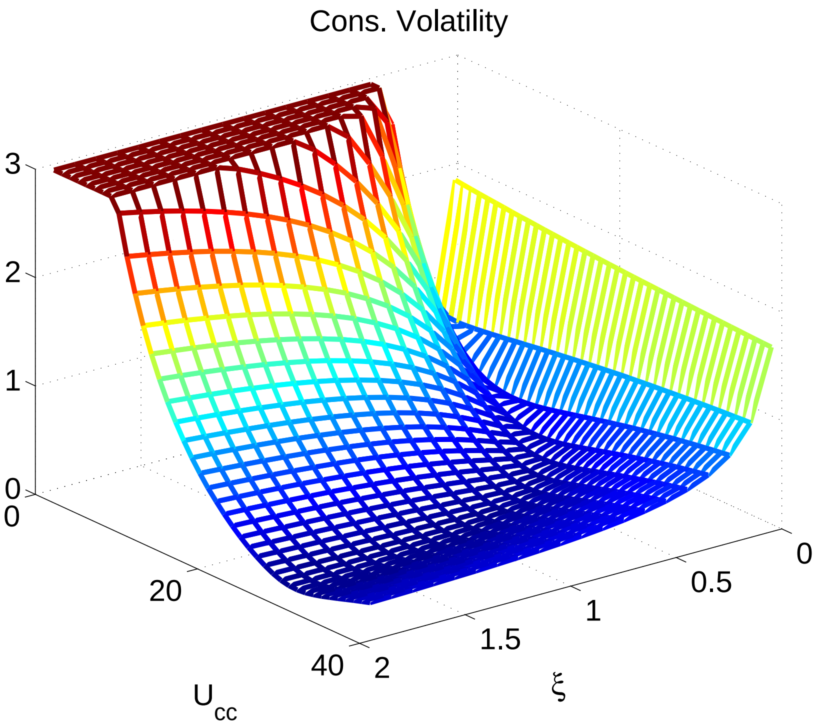

Figure 10: *The volatility of consumption, as* ξ *and* η_{cc} *are varied in the representative agent model with exogenous wages.*

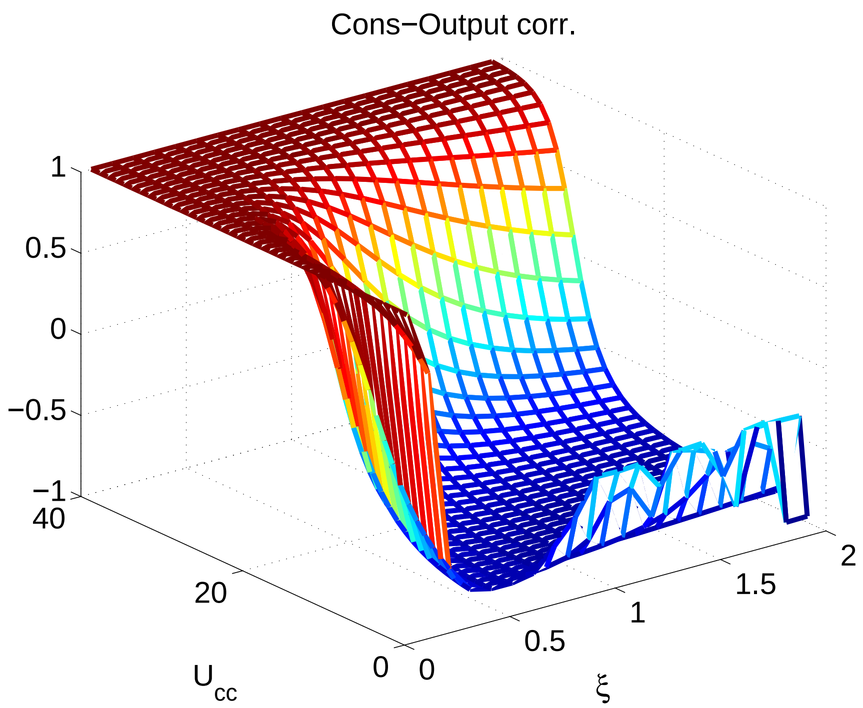

Figure 11: *Output-consumption correlation, as* ξ *and* η_{cc} *are varied in the representative agent model with exogenous wages. Note the change in view point.*

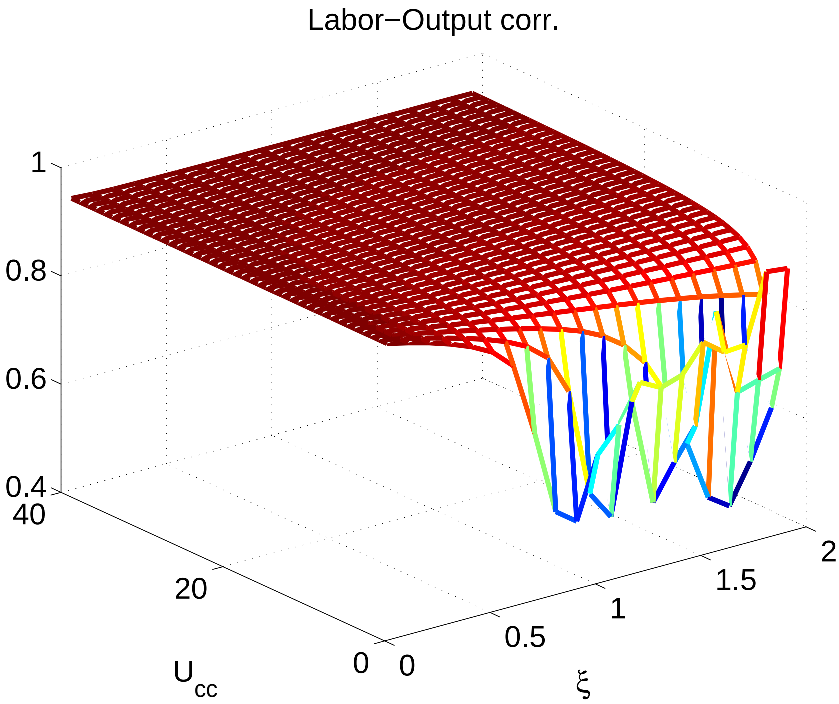

Figure 12: *Output-labor correlation, as ξ and η_{cc} are varied in the representative agent model with exogenous wages. Note the change in view point.*

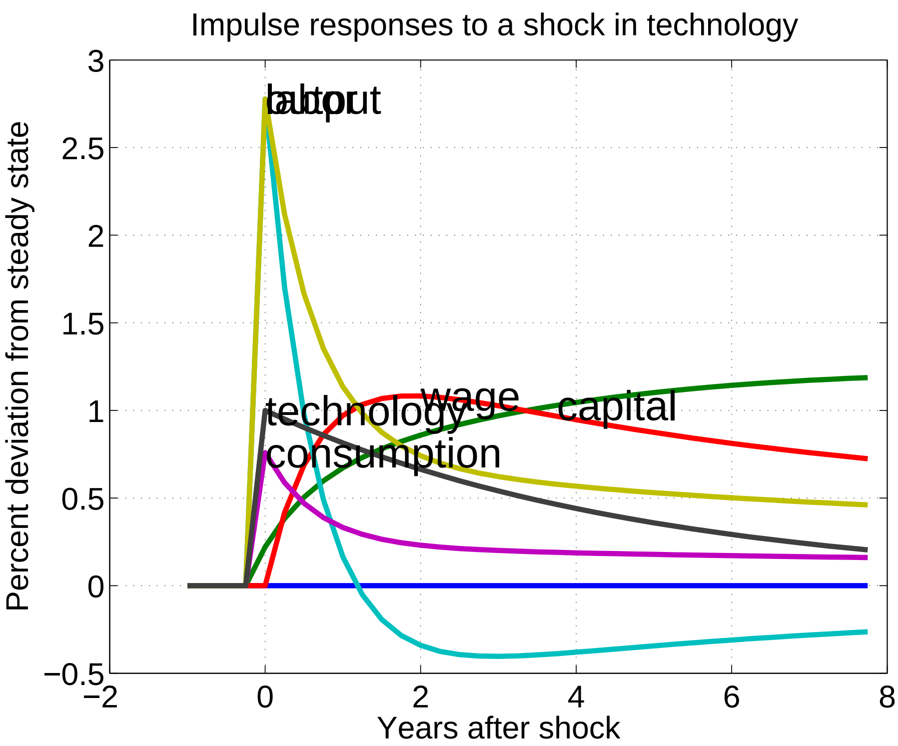

Figure 13: *Impulse response to a technology shock for the 1-agent economy with an exogenous law for wages, when fixing* $\eta_{cc} = 9$ *and* $\xi = 0.1$.

HP-filtered moments	
$\sigma_{y,HP}$	2
σ_{ϵ}	0.65 (cmp. to 0.7)
$\sigma_{n,HP}$	2.09
$\rho_{n,y}$	0.93 .
$\sigma_{c,HP}$	0.55
$\rho_{c,y}$	1.00
$\sigma_{x,HP}$	6.36
$\rho_{x,y}$	1.00
unfiltered moments	
$\sigma_{\Delta_8 c}$	0.96
$\sigma_{\Delta_8 l}$	1.77
$\mathcal{SR}_{ann}$	0.22
σ_{rf}	3.95
$\sigma_r$capital	11.12

Table 15: *Results for the 1-agent economy with an exogenous law for wages, when fixing* $\eta_{cc} = 40$ *and* $\xi = 0.55$.

7 A two-agent economy

Campbell and Cochrane (1999) have emphasized that the absence of arbitrage implies the existence of a stochastic discount factor, which can explain observed asset prices, and have postulated a specific preference formulation for a representative agent, which gives rise to many of the key asset pricing observations, if one assumes that log consumption is an exogenously given random walk. Campbell and Cochrane assume that the agent is subject to an external habit, for which they specify a nonlinear law of motion.

Again, the question arises, which consumption choices an agent would endogenously make, if provided with these preferences. Ljungqvist and Uhlig (2003) consider this problem in an endowment economy, where the endowment is constant, and show that the agent can vastly improve welfare by going through repeated bouts of destroying parts of his endowment. This unusual feature is due to a local as well as a global nonconcavity of the preferences: indeed, solving the dynamic programming problem even in this simple endowment economy is a daunting task.

Guvenen (2003) has recently proposed to instead view these preferences as the preferences of some "rich" stockholder, reinterpreting the habit level of Campbell-Cochrane as the consumption of the "poor" worker. Real business cycles with "capitalists" and "workers" have been used before, see e.g. Danthine et al. (1992), Danthine and Donaldson (1994) or Hornstein and Uhlig (2000).

In particular, Guvenen (2003) assumes that the two types of agents can only trade the riskless bond with each other. He assumes that "workers" are prevented from investing in capital and are highly risk averse, and thus desire a smooth consumption paths. The "capitalists", who own the capital stock, are less risk averse, but since they provide business cycle insurance to the "workers" via the riskless bond, and since they also finance investments, their own consumption ends up sufficiently volatile to generate the observed Sharpe ratio. In terms of the asset pricing equation 24, applied to the consumption of the capitalists, and assuming perfect correlation of capitalist consumption with stock returns, a relative risk aversion of, say, 2 implies that the capitalist consumption volatility of 13.5% at an annual level (and using our moderate-to-low values for the Sharpe ratio). This arguably is a high number. Guvenen (2003) argues, that this is consistent with some recent observations on the consumption of luxury goods, see Aït-Sahalia et al (2002). Further investigation of this issue is certainly warranted.

Guvenen (2003) emphasizes nonlinearities and higher-order moments to generate a rich array of asset pricing implications, but one can already develop much of the intuition and the features of his approach by using a simple extension of our basic model. Like Guvenen (2003), assume that there are two types of agents, who only trade the riskless bond with each other. Assume that the "workers" provide labor, but cannot invest in capital, whereas the "capitalists" only invest, consume and trade in the riskless bond. In other words, the "capitalist" chooses investment X_t, bond holdings B_t and consumption $C_t^{(C)}$ to solve

$$\max E\left[\sum_{t=0}^{\infty} \beta^t U^{(C)}(C_t^{(C)})\right]$$

$$\begin{aligned} C_t^{(C)} + B_t + X_t &= D_t K_{t-1} + R_{t-1}^f B_{t-1} \\ K_t &= (1-\delta) K_{t-1} + G\left(\frac{X_t}{K_{t-1}}\right) K_{t-1} \end{aligned}$$

whereas the "worker" chooses leisure L_t and labor N_t, consumption $C_t^{(W)}$ and debt $-B_t$ to solve

$$\begin{aligned} \max E\left[\sum_{t=0}^{\infty}\beta^t U^{(W)}(C_t^{(W)}, L_t)\right] & \\ C_t^{(W)} - B_t &= W_t N_t - R_{t-1}^f B_{t-1} \\ 1 &= N_t + L_t \end{aligned}$$

where we use superindices (W) and (C) to distinguish between the worker and the capitalist, whenever necessary. Production is given by

$$Y_t = Z_t F(K_{t-1}, N_t)$$

as before, and likewise are dividends D_t and wages W_t (which one derive in the usual manner by formulating the problem of a competitive firm, maximizing profits etc.). In equilibrium, markets clear and agents and firms maximize.

Loglinearizing the model results in some small changes, compared to the basic model of section 3. Equation (2) needs to be replaced by

$$y_t = \frac{\bar{X}}{\bar{Y}} x_t + \frac{\bar{C}^{(C)}}{\bar{Y}} c_t^{(C)} + \frac{\bar{C}^{(W)}}{\bar{Y}} c_t^{(W)} \tag{28}$$

With the first-order conditions, one now needs to be careful in choosing the Lagrange multipliers for the appropriate agent. Equation (8) and (9) refer to the working decision and thus need to be replaced by

$$\lambda_t^{(W)} = -\eta_{cc}^{(W)} c_t^{(W)} + \eta_{cl,l}^{(W)} l_t \tag{29}$$

$$\lambda_t^{(W)} + w_t = \eta_{cl,c}^{(W)} c_t - \eta_{ll}^{(W)} l_t \tag{30}$$

There is an analogue to equation (29) for the capitalist, but it is simpler, since the capitalist does not work,

$$\lambda_t^{(C)} = -\eta_{cc}^{(C)} c_t^{(C)} \tag{31}$$

This equation needs to be added. We also need to add the loglinearized budget constraint of e.g. the worker in order to determine the evolution of debt (it is not needed for anything else). Since one might want to assume that the steady state level of debt is zero, we express the deviation of debt from its steady state values in percent of steady state output rather than steady-state debt, $B_t = \bar{B} + b_t \bar{Y}$. The log-linearized budget constraint then reads

$$\frac{\bar{C}^{(W)}}{\bar{Y}} c_t^{(W)} - b_t = (1-\theta)(w_t + n_t) - \bar{R}\frac{\bar{B}}{\bar{Y}} r_{t-1}^f - \bar{R} b_{t-1} \qquad (32)$$

The asset pricing equation (12) is to be replaced with the three equations

$$0 = E_t \left[\lambda_{t+1}^{(C)} - \lambda_t^{(C)} + r_{t+1} \right] \qquad (33)$$

$$0 = E_t \left[\lambda_{t+1}^{(C)} - \lambda_t^{(C)} + r_t^f \right] \qquad (34)$$

$$0 = E_t \left[\lambda_{t+1}^{(W)} - \lambda_t^{(W)} + r_t^f \right] \qquad (35)$$

Note that the latter two equations result from the trade in the riskless bond, whereas the first of these three is the equation resulting from the intertemporal investment decision problem of the "capitalist" agent.

All other equations remain as before. The model now no longer has a one-dimensional state variable, but standard techniques are available for solving it, see e.g. Uhlig (1999). We use the calibration given in Guvenen (2003), see table 16. The table is structured similarly to table 2. Given the worker steady-state debt-to-GDP ratio $\bar{B}/\bar{Y}$ of the worker, note that the budget constraints imply that

$$\frac{\bar{C}^{(W)}}{\bar{Y}} = 1 - \theta - (\bar{R} - 1)\frac{\bar{B}}{\bar{Y}}$$

$$\frac{\bar{C}^{(C)}}{\bar{Y}} = 1 - \frac{\bar{X}}{\bar{Y}} - \frac{\bar{C}^{(C)}}{\bar{Y}}$$

We are free to choose the debt-to-GDP ratio, as long as $\frac{\bar{C}^{(W)}}{\bar{Y}} > 0, \frac{\bar{C}^{(C)}}{\bar{Y}} > 0$: in the interest of space, we shall drop that condition in table 16.

For the exogenous technology process, we assume

$$z_t = 0.95 z_{t-1} + \epsilon_t$$

as before. Guvenen assume $\sigma_\epsilon = 2$ as do we except for the exogenous law of wages case: this exceeds the usual value by a factor of three.

We shall actually consider two values for $\eta_l l$, namely 0 and ∞. In the second case $\eta_{ll} = \infty$, labor input is essentially fixed: this seems to be the case considered by Guvenen (2003): his model therefore punts on explaining

parameter	Restrictions: theoretical	economic	calibration
θ	free	capital share	0.4
δ	free	deprec. rate	0.02
$\bar{R}$	free	gross cap. return	1.01
ϕ_{nn}	free	elast. of wages	θ
ϕ_{kk}	free	elast. of div.	$1-\theta$
$\xi \geq 0$	free	adj. cost	0.23
$\bar{L}$	free	leisure share	2/3
$\eta_{cc}^{(C)}$	free	cons. risk. avers. cap.	2
$\eta_{cc}^{(W)}$	free	cons. risk. avers. worker	10
$\eta_{cl,l}^{(W)}$	free	cross derivative	0
$\frac{\bar{B}}{\bar{Y}}$	free	debt-to-GDP ratio	0
$\frac{\bar{X}}{\bar{Y}}$	$= \frac{\delta\theta}{\bar{R}-1+\delta}$	investm. share	25.7%
$\frac{\bar{C}^{(W)}}{\bar{Y}}$	$1-\theta-(\bar{R}-1)\frac{\bar{B}}{\bar{Y}}$	cons. share of worker	60%
$\frac{\bar{C}^{(C)}}{\bar{Y}}$	$1-\frac{\bar{X}}{\bar{Y}}-\frac{\bar{C}^{(C)}}{\bar{Y}}$	cons. share of cap.	14.3%
$\kappa = \frac{\eta_{cl,c}^{(W)}}{\eta_{cl,l}^{(W)}}$	$= \frac{(1-\bar{L})}{\bar{L}} \frac{\frac{\bar{C}^{(W)}}{\bar{Y}}}{(1-\theta)}$	rel. expend. shares	0.5
$\eta_{ll}^{(W)}$	$\geq \frac{\kappa\left(\eta_{cl,l}^{(W)}\right)^2}{\eta_{cc}}$	leisure risk.av.	$0, \infty$

Table 16: *The list of parameters of the basic model and their restrictions.*

	$\eta_{ll} = 0$	$\eta_{ll} = \infty$	exog. wages
HP-filtered moments			
σ_ϵ	2	2	0.7
$\sigma_{y,HP}$	0.62	2.59	1.96
$\sigma_{n,HP}$	3.30	0.00	2.04
$\rho_{n,y}$	-1.00	n.a.	0.93
$\sigma_{c^{(W)},HP}$	0.39	1.82	1.16
$\rho_{c^{(W)},y}$	1.00	1.00	1.00
$\sigma_{c^{(C)},HP}$	1.46	5.90	4.95
$\rho_{c^{(C)},y}$	1.00	1.00	1.00
$\sigma_{x,HP}$	0.70	2.66	2.26
$\rho_{x,y}$	1.00	1.00	1.00
unfiltered moments			
$\sigma_{\Delta_8 c^{(W)}}$	0.81	3.75	2.01
$\sigma_{\Delta_8 c^{(C)}}$	3.04	12.08	8.48
$\sigma_{\Delta_8 l}$	3.41	0.00	1.70
$\mathcal{SR}_{ann}$	0.04	0.17	0.12
σ_{rf}	0.47	1.73	3.31
$\sigma_r$capital	2.39	9.02	9.45

Table 17: *Results for the 2-agent economy. The third column holds labor input fixed, and can be compared to the results in Guvenen (2003). The second column assumes that the worker has linear utility in leisure. The forth column assumes an exogenous law of motion for wages.*

fluctuations in employment, a key feature of business cycles. Thus, in order to investigate the implications of choosing hours endogenously, we consider also the other extreme $\eta_{ll} = 0$. In that case, labor is very elastic and can be used by the worker to insure against business fluctuations (in terms of wages) by offsetting movements in labor such as to smooth consumption.

Finally, I also consider a version of this economy with an exogenous lawo of motion for wages, i.e. with (27), $\gamma = 1.04$, $\alpha = 0.15$ replacing the standard first order condition of the worker with respect to the choice of leisure. Note that the value for η_{ll} is now irrelevant for the model results: obviously, it would matter a lot for welfare calculations.

The results can be seen in table 17 as well as in figures 14, 15 and 17.

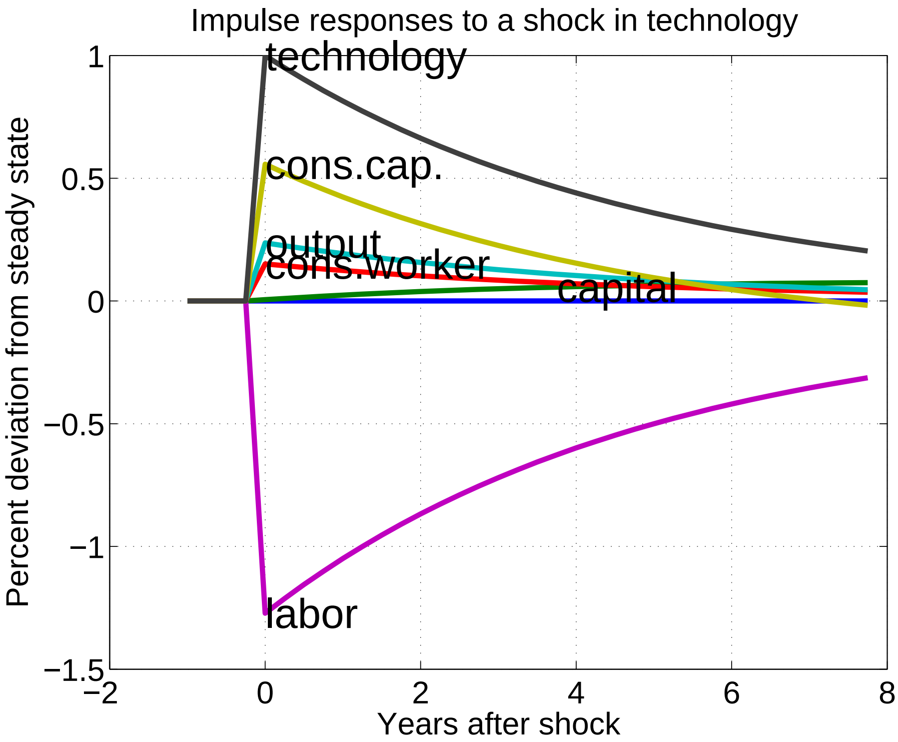

Figure 14: *Impulse response in the two-agent economy, assuming linear utility in leisure, i.e.* $\eta_{ll} = 0$

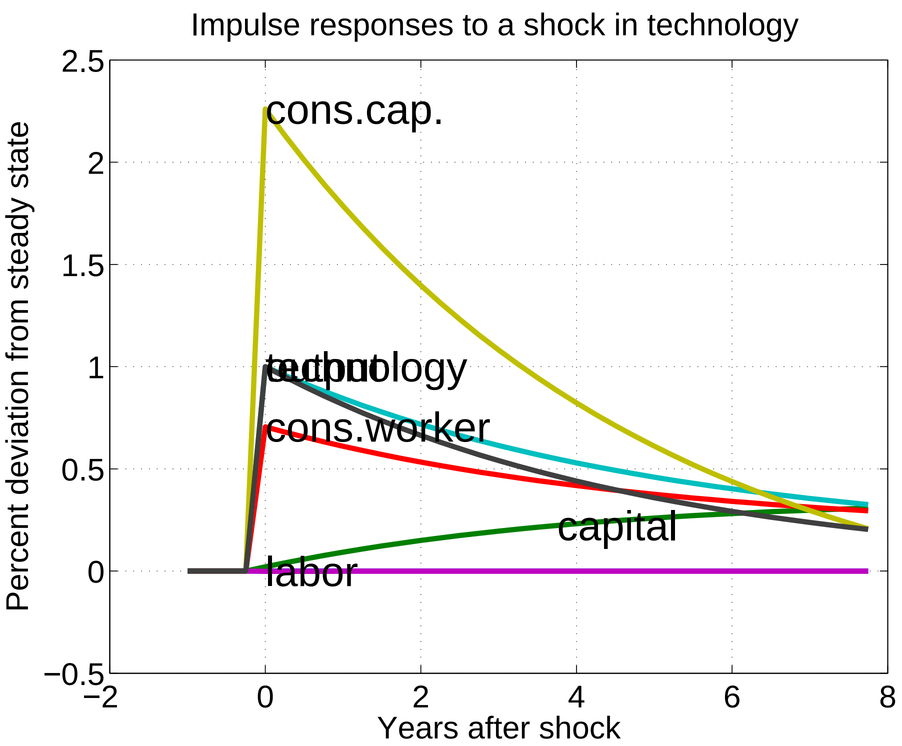

Figure 15: *Impulse response in the two-agent economy, assuming a fixed endowment of labor, i.e.* $\eta_{ll} = \infty$

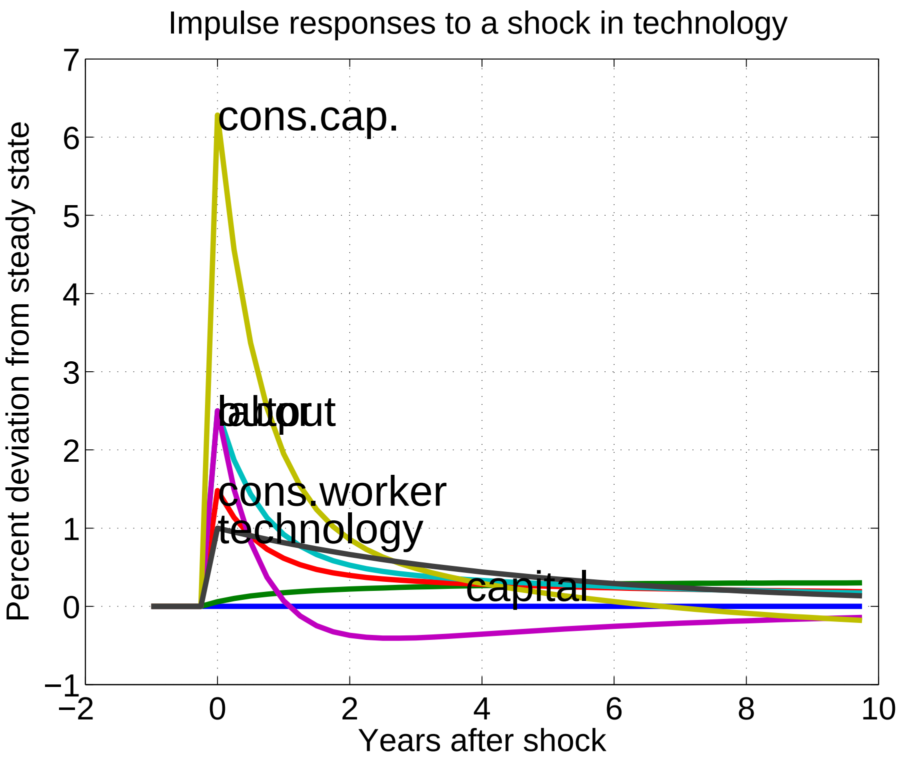

Figure 16: *Impulse response in the two-agent economy, assuming an exogenous law of motion for wages*

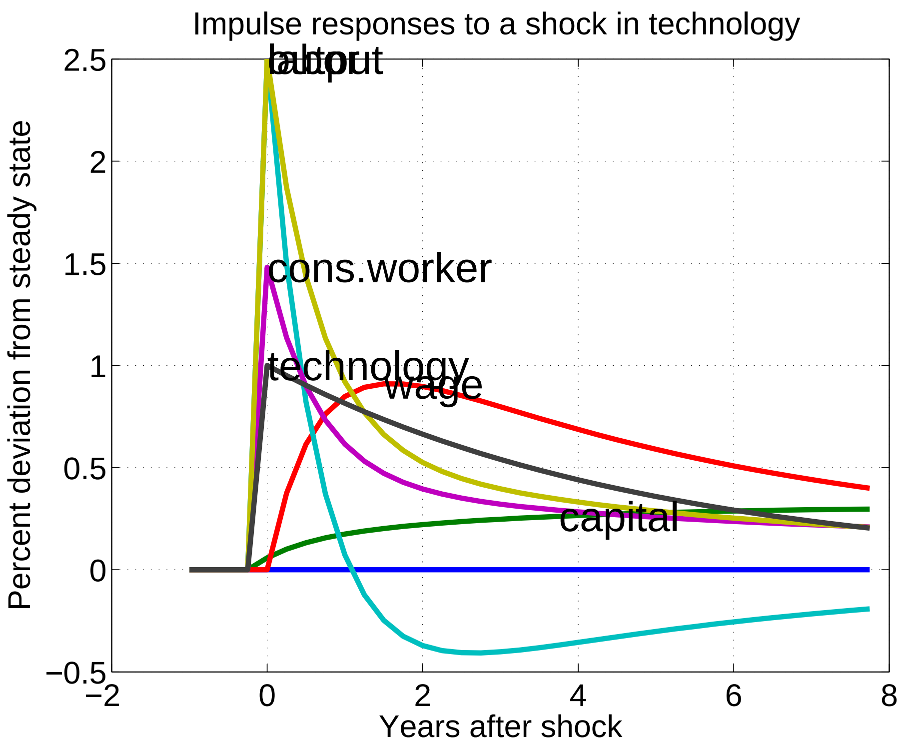

Figure 17: *Impulse response in the two-agent economy, assuming an exogenous law of motion for wages: selecting a different set of variables for impulse responses. Compare this figure to the "optimal" version of the one-agent model.*

The message of this exercise is three-fold. First, the same techniques that have been used above for the basic model can be brought to bear on more elaborate model like this two-agent economy to draw out the interconnections between asset markets and the macroeconomy with little effort. Second, the proposed "solution" of moving to a two-agent economy is no panacea. For either one punts on explaining a key feature of business cycles, namely the movements in hours worked. Or one permits movements in hours, but in that case, the risk-averse worker can use hour fluctuations to smooth consumption (and thus output!), rather than rely on the bond market. It may be interesting to explore what happens, if nonseparabilities between consumption and leisure are allowed for, i.e. to consider the case $\eta_{cl,l}^{(W)} \neq 0$, but we have not pursued this here (yet), and it seems doubtful that this can resolve the difficulties of this particular model in explaining business cycles.

Third, introducing an exogenous law of motion for wages, as done for the simple representative-agent model above, fixes this problem, as it did before. This altered version of the Guvenen model now has reasonable properties, including labor market behaviour, even though the Sharpe ratio has decreased a bit more still. So, again, understanding the relationship between labor markets and asset markets is key.

8 Conclusions

This paper has shed light on the mutual discipline, which asset market observations and macroeconomic observation impose on each other. Economic choices such as consumption and leisure, which are taken as exogenous in much of the asset pricing literature, and which may suggest certain preference specifications in order to explain asset price observations in turn may have undesirable macroeconomic consequences, once these economic choices are endogenized.

We have studied a generic representative agent real business cycle economy, and shown, how to analyze it in general. We have explored the interconnections between asset market observations, macroeconomic observations and theoretical choices of key parameters. We have considered nonseparabilities between consumption and leisure in particular and have investigated the scope of this nonseparability to help explain e.g. the equity premium observation.

feasibility:	y_t	$=$	$\frac{\bar{X}}{\bar{Y}}x_t + \left(1 - \frac{\bar{X}}{\bar{Y}}\right)c_t$
goods production:	y_t	$=$	$\theta k_{t-1} + (1-\theta)n_t$
cap. production:	k_t	$=$	$(1-\delta)k_{t-1} + \delta x_t$
wages:	w_t	$=$	$z_t + \phi_{nn}(k_{t-1} - n_t)$
dividends:	d_t	$=$	$z_t - \phi_{kk}(k_{t-1} - n_t)$
time endowment:	l_t	$=$	$-\frac{1-\bar{L}}{\bar{L}}n_t$
shadow value of wealth:	λ_t	$=$	$-\eta_{cc}c_t + \eta_{cl,l}l_t$
shadow value of time:	$\lambda_t + w_t$	$=$	$\eta_{cl,c}c_t - \eta_{ll}l_t$
adj. cost friction / Tobin's q:	ψ_t	$=$	$\frac{1}{\xi}(x_t - k_{t-1})$
return on capital:	r_t	$=$	$\frac{\bar{R}-1+\delta}{\bar{R}}d_t - \psi_{t-1} + \frac{1}{\bar{R}}\psi_t$
Lucas asset pricing:	0	$=$	$E_t\left[\lambda_{t+1} - \lambda_t + r_{t+1}\right]$

Table 18: *List of the log-linearized equations of the basic model.*

As an extension, we have also studied a two-agent economy, following the lead of Guvenen (2003), and found some undesirable implications of that model as well.

I found that the major obstacle to overcome is the endogeneity of labor market movements. The intuition is simple: if agents can endogenously choose their labor input, they can use this an additional insurance device against stock market fluctuations. Indeed, a number of papers in the literature thus either assume labor to be constant, e.g. Jermann (1998) or Guvenen (2003), or assume considerable frictions in adjusting labor input, e.g. Boldrin, Christiano and Fisher (2001). As an alternative, I have proposed an exogenous law of motion for wages and found that simple models can then go remarkably far in jointly explaining the observed facts, including the movements of employment. The same device can repair and improve upon the Guvenen (2003) model. Thus, the key to understanding macroeconomic facts and asset pricing facts jointly may be in understanding labor markets rather than agent heterogeneity.

A The loglinear equations of the basic model

References

[1] Aït-Sahalia, Yacine, Jonathan A: Parker and Motohiro Yogo (2002), "Luxury Goods and the Equity Premium," draft, Princeton University.

[2] Alvarez, Fernando and Urban J. Jermann (2003), "Using asset prices to measure the cost of business cycles," draft, The Wharton School of the University of Pennsylvania.

[3] Backus, David, Bryan Routledge and Stanley Zin (2004), "Exotic Preferences for Macroeconomists," draft, Stern School of Business, New York University.

[4] Basu, Susanto, Miles Kimball and John Fernald, "Are Technology Improvements Contractionary?" Board of Governors of the Federal Reserve System, International Finance Discussion Paper no. 625, 1999.

[5] Boldrin, Michele, Lawrence J. Christiano and Jones D.M. Fisher, "Habit Persistence, Asset Returns, and the Business Cycle", American-Economic-Review. March 2001; 91(1): 149-66.

[6] Campbell, John Y., "Understanding Risk and Return," Journal of Political Economy, 104(2), April, 298-345.

[7] Campbell, John Y. and John H. Cochrane (1999), "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," Journal of Political Economy. April 1999; 107(2): 205-51.

[8] Campbell, John Y.,"Consumption-Based Asset Pricing," Chapter 13 in Constantinides et al., eds., *Handbook of the Economics of Finance, vol. 1B, Financial Markets and Asset Pricing*, Elsevier, North Holland (2004).

[9] Christiano, Lawrence J., Martin Eichenbaum and Robert Vigfusson (2003). "What Happens After a Technology Shock?" Mimeo, Northwestern University.

[10] Cochrane, John H., *Asset Pricing*, Princeton University Press, Princeton, NJ (2001).

[11] Cooley, Thomas F. and Edward C. Prescott, eds. (1995). *Frontiers in Business Cycle Research.* Princeton, NJ: Princeton University Press.

[12] Danthine, Jean-Pierre, John B. Donaldson, Rajnish Mehra, “The equity premium and the allocation of income risk,” Institute for Empirical Macroeconomics, Discussion Paper 60, Federal Reserve Bank of Minneapolis (March 1992).

[13] Danthine, Jean-Pierre and John B. Donaldson, “Asset pricing implications of real market frictions,” Discussion Paper 9504, Universite de Lausanne (December 1994).

[14] Epstein, L., and S. Zin (1989), ”Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework,” Econometrica 57: 937-968.

[15] Francis, Neville and Valerie Ramey (2001). “Is the Technology-Driven Real Business Cycle Hypothesis Dead? Shocks and Aggregate Fluctuations Revisited.” Mimeo. University of California, San Diego, Department of Economics.

[16] Francis, Neville and Valerie Ramey (2003). ”The Source of Historical Economic Fluctuations: An Analysis using Long-Run Restrictions.” Mimeo. University of California, San Diego, Department of Economics.

[17] Gali, Jordi (1999). ”Technology, Employment and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?” *American Economic Review* 89(1), pp. 249-271.

[18] Guvenen, Fatih (2003), “A Parsimonious Macroeconomic Model for Asset Pricing: Habit Formation or Cross-Sectional Heterogeneity?”, draft, Economics Department, University of Rochester.

[19] Hall, Robert (2003). ”Wage Determination and Employment Fluctuations.” Mimeo, Stanford University.

[20] Hansen, L.P., T.J. Sargent and T.D. Tallarini (1999), “Robust permanent income and pricing,” Review of Economic Studies 66: 873-907.

[21] Hornstein, Andreas and Harald Uhlig, "What Is the Real Story for Interest Rate Volatility?", German Economic Review. February 2000; 1(1): 43-67.

[22] Jermann, U.J. (1998), "Asset pricing in production economies," Journal of Monetary Economics 41: 257-275.

[23] Lettau, Martin and Harald Uhlig, "Can Habit Formation Be Reconciled with Business Cycle Facts?", Review of Economic Dynamics. January 2000; 3(1): 79-99

[24] Lettau, Martin and Harald Uhlig, "The Sharpe Ratio and Preferences: A Parametric Approach," Macroeconomic Dynamics. April 2002; 6(2): 242-65.

[25] Lettau, Martin and Sidney C. Ludvigson, "Understanding Trend and Cycle in Asset Values: Reevaluating the Wealth Effect on Consumption," American Economic Review (2004), vol. 94, no. 1, 276-299.

[26] Ljungqvist, Lars and Harald Uhlig (2003), "Optimal Endowment Destruction under Campbell-Cochrane Habit Formation," draft, Humboldt University Berlin.

[27] Lucas, Robert E. Jr. (1987), *Models of Business Cycles,* Basil Blackwell, New York.

[28] Luttmer, Erzo G.J. , "What Level of Fixed Costs Can Reconcile Consumption and Stock Returns?," Journal of Political Economy. October 1999; 107(5): 969-97

[29] McGrattan, Ellen and Edward C. Prescott, "Average Debt and Equity Returns: Puzzling?", American Economic Review (2003), vol. 93, no.2: 392-397.

[30] Otrok, Christopher (1999), "On Measuring the Welfare Cost of Business Cycles, draft, University of Virginia

[31] Petrongolo, Barbara and Christopher A. Pissarides, "Looking into the Black Box: A Survey of the Matching Function, " Journal of Economic Literature 38 (June 2001) 390-431.

[32] Shea, John, (1998). "What do Technology Shocks Do?" *NBER Macroeconomics Annual 1998*, pp. 275-310.

[33] Storesletten, Kjetil, Chris I. Telmer, Amir Yaron, "The Welfare Cost of Business Cycles Revisited: Finite Lives and Cyclical Variation in Idiosyncratic Risk," European Economic Review. June 2001; 45(7): 1311-39.

[34] Tallarini, Thomas D. Jr., "Risk-Sensitive Real Business Cycles," Journal of Monetary Economics. June 2000; 45(3): 507-32.

[35] Uhlig, Harald, "A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily," in Ramon Marimon and Andrew Scott, eds., *Computational methods for the study of dynamic economies*, Oxford and New York: Oxford University Press, 1999; 30-61.

[36] Uhlig, Harald, "Do Technology Shocks Lead to a Fall in Total Hours Worked?", Journal of the European Economic Association, April-May 2004, vol. 2, 2-3: 361-371.

[37] Weil, P. (1989), "The equity premium puzzle and the risk-free rate puzzle," Journal of Monetary Economics 24: 401-421.