Can A Representative Agent Model Represent A Heterogeneous Agent Economy?*

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Abstract

Accounting for observed fluctuations in aggregate employment, consumption, and real wage using optimality conditions of a representative household often requires preferences that are incompatible with economic priors (e.g., Mankiw, Rotemberg, and Summers, 1985). This discrepancy between the equilibrium model and the aggregate data is often viewed as evidence of the failure of labor-market clearing. We argue that such a conclusion is premature. We construct a model economy where all prices are flexible and all markets clear at all times but household decisions are not readily aggregated because of incomplete capital markets and the indivisible nature of labor supply. We demonstrate that if we were to explain the model-generated aggregate time series using decisions of a "fictitious" stand-in household, such a household is likely to have a non-concave or unstable utility. Our analysis suggests that the representative agent model often fails to represent an equilibrium outcome of a heterogeneous agent economy.

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1 Introduction

Modern business cycle theories posit that observed aggregate fluctuations of U.S. economy correspond to optimal decisions of a stand-in household (e.g., Kydland and Prescott, 1982; King, Plosser, and Rebelo, 1988). In these models, the cyclical variation of aggregate consumption and employment is a result of the continuous optimum of a household that trades current and future goods and leisure in response to stochastic movements in prices. However, studies that use aggregate time-series data to test the hypothesis of intertemporal substitution often reach negative conclusions. For example, Mankiw, Rotemberg, and Summers (1985) (denoted MRS hereafter) found that the over-identifying restrictions implied by the theory are almost always rejected, the estimated parameters of preferences are highly unstable, and the utility function is often non-concave, leading to elasticities of wrong signs. This incompatibility between the representative agent model and aggregate data is often viewed as a failure of labor-market clearing (e.g., Galí, Gertler, and Lopéz-Salido, 2007).[1]

In this paper we argue that such conclusion is premature. We demonstrate that an attempt to account for the aggregate behavior of an heterogeneous agent economy by a "fictitious" representative household often fails. We construct a model economy where all prices are flexible and all markets clear at all times. In our model, individual households possess identical preferences but face a limit on the amount they can borrow and cannot perfectly insure against idiosyncratic productivity shocks (Aiyagari, 1994). Moreover, households supply their labor in an indivisible manner (Rogerson, 1988). Under this environment, the optimality condition for the choice of hours worked holds as inequality at the individual

[1] Alternatively, time-varying factors in the marginal rate of substitution between commodity consumption and leisure (e.g., stochastic shifts in home production technology or changes in labor-income tax rates) are suggested to fill the gap between the optimality condition and the aggregate data (e.g., Hall, 1997; Chari, Kehoe, and McGrattan, 2007).

level. Both incompleteness of capital markets and indivisibility of labor supply prevent a nice aggregation of individual optimality conditions.

The equilibrium path of our model economy under the exogenous aggregate productivity shocks reproduces the volatility and correlation structure of key aggregate variables (consumption, hours, and wages) from the U.S. economy. We then ask whether outcomes of our heterogenous-agent model economy are readily characterized as realizations of an optimizing representative agent. We estimate three optimality conditions that a representative agent would face when choosing hours worked and commodity consumption. If we were to explain the model-generated aggregate time series using decisions of a stand-in household, such a household must have a highly unstable or non-concave utility: the estimated representative household often works longer hours and consumes more commodities when the real wage is low. Similarly to the finding by MRS from the actual U.S. aggregate data, the generalized method of moments (GMM) estimates of preference parameters of a representative household are highly unstable or often wrong signs.

In order to investigate the marginal contributions of each friction (capital market incompleteness and indivisibility of labor), we also consider additional model economies: the incomplete capital market with divisible labor economy (referred as "Incomplete-market" model) and the complete capital market with indivisible labor economy (referred as "Indivisible-labor" model). According to the GMM estimation of model-generated aggregate time series, we find these economies can be well represented by optimal choices of a representative agent. In "Incomplete-market" model (with divisible labor), the GMM estimates based on model-generated aggregate times series accurately reveal the households' preference parameters. When households can choose the length of working hours, hours worked are highly correlated across households in response to aggregate (productivity) shocks, allowing for a fairly precise aggregation despite incomplete capital markets and confirming the result of Krusell and Smith (1998). In the "Indivisible-labor" model (with complete capital markets), the allocation of a heterogeneous agent economy can be well described by an efficient allocation of a social planner who maximizes the aggregated utilities of households. However, it is important to note that the aggregate preferences is not necessarily identical to those of individual households. With heterogeneous productivity in the population, the efficient allocation requires an assignment of workers (work vs. leisure) based on their comparative advantages. In other words, the GMM estimates based on model-generated aggregate time series reveals the social planner's objective function, the (equally) weighted average of household utility functions.

Confronted with the inability of an equilibrium model to account for the joint behavior of aggregate hours worked, wages, and consumption, MRS proposed mainly three hypotheses: (1) aggregation error (2) economy-wide time-varying preferences and (3) failure of market clearing.[2] While it is highly plausible that all of these have contributed to the discrepancy between the representative model and aggregate data, our analysis suggests that the incompatibility between the representative household's optimization and the aggregate data may reflect a poor aggregation rather than a failure of the market or exogenous time-varying preferences. Nevertheless, our analysis also shows that when the model economy consists of heterogeneous agents and the individual decision rules are hard to aggregate, an attempt to

[2]It is also well-known that low-wage and less-skilled workers enter the labor market during expansions and exit during recessions, making aggregate hours more volatile than the effective unit of hours (Hansen, 1993), and making the aggregate wages less volatile than individual wages (Bils, 1985; Solon, Barsky, and Parker, 1994). However, this so-called compositional bias has an impact mostly on the volatilities, not on the correlations. Both in the model and data, the poor GMM estimates of preference parameters stems mostly from the lack of correlation between employment and productivity (wage), which is 0.03 (0.2).

account for the aggregate time series by an optimizing behavior of the representative household fails. The relative risk aversion of consumption is significantly underestimated when the aggregate consumption Euler equation is used. The parameter that governs the behavior of the labor supply is estimated with great uncertainty, just like those from the actual aggregate data.

The paper is organized as follows. Section 2 briefly discusses the GMM estimate of three optimality conditions based on the aggregate U.S. time series. In Section 3 we compute the equilibrium fluctuations of the heterogeneous agent economy with incomplete capital markets and indivisible labor using the method developed by Krusell and Smith (1998). In Section 4, based on the aggregate time series generated from the heterogeneous agent model economy, we estimate three optimality conditions that a "fictitious" representative agent would satisfy. Section 5 is the conclusion.

2 GMM estimates based on aggregate data from the U.S. economy

Consider a representative household whose preferences are given by:

$$\max \; E_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{C_{t+s}^{1-\sigma} - 1}{1-\sigma} - \psi \frac{H_{t+s}^{1+\gamma}}{1+\gamma} \right\}$$

where C_t is consumption and H_t is hours worked in period t.[3] The preference parameters are β, the discount factor, σ, the inverse of the intertemporal substitution elasticity of consumption, γ, the inverse of the intertemporal substitution elasticity of hours, and a constant ψ. When the representative household follows the optimal path, three first-order conditions

[3]We assume a utility function separable between consumption and hours worked, which is popular in both business cycle analysis and the empirical labor supply literature. However, non-separability does not change the main result of the article.

must hold:

$$\psi \frac{H_t^{\gamma}}{C_t^{-\sigma}} \frac{P_t}{W_t} - 1 = 0. \tag{S}$$

$$E_t \left[\beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t(1+R_t)}{P_{t+1}} - 1 \right] = 0. \tag{EC}$$

$$E_t \left[\beta \frac{H_{t+1}^{\gamma}}{H_t^{\gamma}} \frac{W_t(1+R_t)}{W_{t+1}} - 1 \right] = 0. \tag{EL}$$

Here P_t is the nominal price of a unit of C_t, W_t is the wage rate, and R_t is the nominal return from holding a security between t and $t+1$. The static first-order condition (S) holds regardless of the household's decisions in the capital market. The Euler equation for consumption (EC) will hold even if labor supply cannot be freely chosen, and trading is not possible in many assets, as long as some asset exists that is either held in positive amounts or for which borrowing is possible. The Euler equation for leisure (EL) asserts that along an optimal path the representative household cannot improve its welfare by working one hour more at t and using its earnings W_t to purchase a security whose proceeds will be used to buy back $W_t(1+R_t)/W_{t+1}$ of leisure at $t+1$ in all states of nature.

If the static first-order condition (S) held exactly, one of (EC) and (EL) would be redundant. However, since (S) is unlikely to hold exactly in the data, we use the information in all three of these first-order conditions to estimate the parameters of utility function. Following MRS, σ, γ, β and ψ are estimated by the GMM using the quarterly U.S. aggregate time series for the period 1964:I-2003:IV. Aggregate real per capita consumption is the sum of consumption expenditures on nondurable goods and services. The aggregate price is the price deflator that corresponds to our measure of consumption. Aggregate hours worked represent the total hours employed in the non-agricultural business sector. The nominal wage is the nominal hourly earnings of production and non-supervisory workers in the non-agricultural sector. The nominal interest rate is the 3-month Treasury bill rate. All quantities are divided by the working-age (ages between 16 and 65) population.

We use two sets of instruments in the GMM estimation. Instrument I consists of the following variables for periods $t-1$ and $t-2$: growth rates of consumption, real interest rates, hours worked, and real wages. Instrument II consists of the same variables as Instrument I but for periods t and $t-1$. Hence, we can check through Instrument II if the estimates are severely affected when current variables are used as instrument variables. While it is common to include period t variables as instruments in the asset pricing literature (see Hansen, 2007; Cochrane, 2001, Chapter 10, for a detailed explanation of the GMM procedure), existence of predetermined prices (such as sticky wages) may warrant the exclusion period t-variable from the list of instruments (see MRS for this argument). We report the estimates using both instruments and they are not very different from each other. The standard two-stage approach as in Hansen and Singleton (1982) is used in performing the GMM estimation. At the first stage, the identity weighting matrix is applied to get preliminary estimates of coefficients. The inverse of Newey and West's (1987) heteroskedasticity and autocorrelation consistent (HAC) covarince matrix is used as the second-stage weighting matrix to derive asymptotically efficient estimates.[4]

Estimates in Table 1 basically replicate those in MRS. They also share the common shortcomings of preference parameter estimates in aggregate time series such as Dunn and Singleton (1986), Hansen and Singleton (1982, 1984), and Ghysels and Hall (1990). According to these estimates, preferences are often found to be unreasonable. In the static first-order condition (S), the households are not risk averse enough. The estimate of σ is 0.210 (with standard er-

[4]The HAC covariance matrix is calcuated with a Bartlett kernel and Newey and West's (1987) fixed bandwidth selection criterion.

ror of 0.062) and 0.188 (0.067) with Instruments I and II, respectively. The marginal disutility from working is not increasing in hours worked as the estimate of γ is negative: it is -0.569 (0.198) and -0.473 (0.210), with Instruments I and II, respectively. According to these estimates, households would often work longer hours when the real wage is low (i.e., consume less leisure despite the low real price of leisure).

In the Euler equation for consumption (EC), the intertemporal substitution elasticity of consumption turns to a negative value (-0.210 and -0.129, respectively, with Instruments I and II), although it is not statistically significant. In the Euler equation for leisure (EL), the estimate for γ is 0.179 and 0.089 respectively with Instruments I and II, implying a fairly elastic labor supply. One of the stylized facts in aggregate labor-market fluctuation is that hours worked vary greatly without a corresponding movement of wages. To account for these, the representative household must have had a very elastic labor-supply schedule. According to these point estimates, the implied value for the intertemporal substitution elasticity of hours worked ($1/\gamma$) is 5.6 and 11.2. These are clearly beyond the admissible values based on empirical micro studies such as MaCurdy (1981) and Altonji (1986). When all three optimality conditions are estimated together as a system of equations in the last column of the table, σ is -0.046 (with standard error of 0.027) and γ is 0.023 (0.044), according to which the representative household exhibits a non-concave utility in consumption and is willing to shift its work schedule even for a tiny movement in anticipated wage changes. Each optimality condition is rejected according to Hansen's (1982) J-test of over-identifying restrictions at the significance level of 5%. When the three optimality conditions are tested together, the intertemporal substitution hypothesis is not rejected at the significance level of 10%. When expenditures on nondurable goods (excluding services) are used for aggregate consumption, the estimation result moves slightly toward our economic priors. The estimate of σ in Table 2 is now between 0.136 (0.333) and 0.843 (0.049), depending on the optimality condition and instrument. However, the estimate of γ is still highly unstable (either negative or a small value) as it ranges between -0.450 (0.115) and 0.413 (0.138).

In sum, two features in the aggregate labor-market data led to the wrong sign or a small value of γ. A lack of systematic correlation between the cyclical components of hours worked and wages (which is 0.39 in the aggregate data after Hodrick-Prescott (HP) filtering) results in either non-concave or unstable utility. Accounting for the volatility of hours worked relative to wages (more precisely, relative to the real wage evaluated by the marginal utility of consumption) requires an elastic labor supply schedule. (At business cycle frequencies, the ratio of the standard deviation of hours to that of wages is 1.52). The discrepancy between the optimality conditions and aggregate data is often interpreted as an evidence for the failure of labor market clearing due to, say, sticky wages. In the next section, we show that a competitive equilibrium obtained from a reasonably calibrated heterogeneous agent model can lead to estimates similar to those we see in the U.S data, which in turn implies that nonsensible estimates of preference parameters in the aggregate data do not necessarily reflect a failure of market clearing or stochastic components of preferences. Rather, they can reflect imperfect aggregation of individual optimality conditions.

3 The Benchmark Model

The model economy is based on Chang and Kim (2007) which extends Krusell and Smith's (1998) heterogeneous-agent model with incomplete capital markets (Aiyagari, 1994) to indivisible labor supply (Rogerson, 1988). Both frictions break the tight link between individual and aggregate labor supply schedules. The indivisibility of labor implies that the optimality condition for hours worked holds as inequality at the individual level. The incompleteness of capital markets implies an imperfect aggregation of individual optimality conditions.

There is a continuum (measure one) of workers who have identical preferences but different productivity. Individual productivity varies exogenously according to a stochastic process with a transition probability distribution function $\pi_x(x'|x) = \Pr(x_{t+1} \leq x'|x_t = x)$. A worker maximizes his utility by choosing consumption c_t and hours worked h_t:

$$\max \ E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma} - 1}{1-\sigma} - \psi \frac{h_t^{1+\gamma}}{1+\gamma} \right\}$$

subject to

$$a_{t+1} = w_t x_t h_t + (1 + r_t) a_t - c_t.$$

Workers trade claims for physical capital, a_t, which yields the rate of return r_t and depreciates at rate δ each period. They face a borrowing constraint: $a_t \geq \bar{a}$ for all t. Workers supply their labor in an indivisible manner; i.e., h_t takes either zero or $\bar{h}(< 1)$. If he works, a worker supplies $\bar{h}$ units of labor and earns $w_t x_t \bar{h}$, where w_t is the wage rate per effective unit of labor. The representative firm produces output according to a constant-returns-to-scale Cobb-Douglas technology in capital, K_t, and efficiency units of labor, L_t.

$$Y_t = F(L_t, K_t, \lambda_t) = \lambda_t L_t^{\alpha} K_t^{1-\alpha},$$

where λ_t is the aggregate productivity shock with a transition probability distribution function $\pi_\lambda(\lambda'|\lambda) = \Pr(\lambda_{t+1} \leq \lambda'|\lambda_t = \lambda)$. In this model economy, a technology shock is the only aggregate shock. This does not necessarily reflect our view on the source of business cycles. Since we would like to estimate preferences using aggregate time series, we intentionally rule out stochastic shocks that may shift the labor-supply schedule itself and cause identification problem in estimating preferences (e.g., exogenous shifts in aggregate preferences, government spending, or income tax rate).[5]

The value function for an employed worker, denoted by V^E, is:

$$V^E(a,x;\lambda,\mu) = \max_{a'\in\mathcal{A}} \left\{ \frac{c^{1-\sigma}-1}{1-\sigma} - \psi\frac{\bar{h}^{1+\gamma}}{1+\gamma} + \beta E\Big[\max\left\{ V^E(a',x';\lambda',\mu'), V^N(a',x';\lambda',\mu') \right\} \big| x,\lambda \Big] \right\}$$

subject to

$$c = wx\bar{h} + (1+r)a - a',$$

$$a' \geq \bar{a},$$

$$\mu' = \mathbf{T}(\lambda,\mu),$$

where $\mathbf{T}$ denotes a transition operator that defines the law of motion for the distribution of workers $\mu(a,x)$.[6] The value function for a non-employed worker, denoted by $V^N(a,x;\lambda,\mu)$, is defined similarly with $h=0$. Then, the labor-supply decision is characterized by:

$$V(a,x;\lambda,\mu) = \max_{h\in\{0,\bar{h}\}} \left\{ V^E(a,x;\lambda,\mu), V^N(a,x;\lambda,\mu) \right\}.$$

The competitive equilibrium consists of a set of value functions, $\{V^E(a,x;\lambda,\mu)$, $V^N(a,x;\lambda,\mu)$, $V(a,x;\lambda,\mu)\}$, a set of decision rules for consumption, asset holdings, and labor supply, $\{c(a,x;\lambda,\mu)$, $a'(a,x;\lambda,\mu)$, $h(a,x;\lambda,\mu)\}$, aggregate capital and labor inputs, $\{K(\lambda,\mu)$, $L(\lambda,\mu)\}$, factor prices, $\{w(\lambda,\mu)$, $r(\lambda,\mu)\}$, and a law of motion for the distribution $\mu' = \mathbf{T}(\lambda,\mu)$ such that:

1. Individuals optimize:

[5] According to Chari, Kehoe, and McGrattan (2007), such shocks will affect the measurement of marginal rate of substitution between consumption and leisure, and as a result show up in the so-called labor-market wedge.

[6] Let $\mathcal{A}$ and $\mathcal{X}$ denote sets of all possible realizations of a and x, respectively. The measure $\mu(a,x)$ is defined over a σ-algebra of $\mathcal{A}\times\mathcal{X}$.

Given $w(\lambda,\mu)$ and $r(\lambda,\mu)$, the individual decision rules $c(a,x;\lambda,\mu)$, $a'(a,x;\lambda,\mu)$, and $h(a,x;\lambda,\mu)$ solve $V^E(a,x;\lambda,\mu)$, $V^N(a,x;\lambda,\mu)$, and $V(a,x;\lambda,\mu)$.

2. The representative firm maximizes profits:

$$w(\lambda,\mu) = F_1\big(L(\lambda,\mu), K(\lambda,\mu), \lambda\big)$$

$$r(\lambda,\mu) = F_2\big(L(\lambda,\mu), K(\lambda,\mu), \lambda\big) - \delta$$

for all (λ,μ).

3. The goods market clears:

$$\int \{a'(a,x;\lambda,\mu) + c(a,x;\lambda,\mu)\} d\mu = F\big(L(\lambda,\mu), K(\lambda,\mu), \lambda\big) + (1-\delta)K$$

for all (λ,μ).

4. Factor markets clear:

$$L(\lambda,\mu) = \int xh(a,x;\lambda,\mu) d\mu$$

$$K(\lambda,\mu) = \int a d\mu$$

for all (λ,μ).

5. Individual and aggregate behaviors are consistent:

$$\mu'(A^0, X^0) = \int_{A^0,X^0} \left\{ \int_{\mathcal{A},\mathcal{X}} \mathbf{1}_{a'=a'(a,x;\lambda,\mu)} \, d\pi_x(x'|x) d\mu \right\} da' dx'$$

for all $A^0 \subset \mathcal{A}$ and $X^0 \subset \mathcal{X}$.

3.1 Calibration

We briefly explain the choice of the model parameters. A detailed discussion of the calibration can be found in Chang and Kim (2006, 2007). The unit of time is a business quarter.

We assume that the individual productivity shock (a source of the cross-sectional heterogeneity in our model economy) x_t follows an AR(1) process: $\ln x' = \rho_x \ln x + \varepsilon_x$, where $\varepsilon_x \sim N(0, \sigma_x^2)$. The values of $\rho_x = 0.939$ and $\sigma_x = 0.287$ reflect the persistence and standard deviation of innovation to individual wages in the Panel Study of Income Dynamics (PSID).[7] A working individual spends one-third of his discretionary time ($\bar{h} = 1/3$). The intertemporal substitution elasticity of consumption is one ($\sigma = 1$). The intertemporal substitution elasticity of hours worked is 0.4 ($\gamma = 2.5$). We set ψ such that the steady state employment rate is 60%. The discount factor β is chosen so that the quarterly rate of return to capital is 1% in the steady state. An aggregate productivity shock, λ_t, follows an AR(1) process: $\ln \lambda' = \rho_\lambda \ln \lambda + \varepsilon_\lambda$, where $\varepsilon_\lambda \sim N(0, \sigma_\lambda^2)$. We set $\rho_\lambda = 0.95$ and $\sigma_\lambda = 0.007$. Table 3 summarizes the parameter values of the benchmark economy.

3.2 Cross-sectional distribution and aggregate fluctuations of the model

As we investigate the aggregation issue, it is desirable for the model economy to possess a reasonable amount of heterogeneity and volatility of key aggregate variables similar to that in the U.S. data. We compare cross-sectional earnings and wealth—two important observable dimensions of heterogeneity in the labor market—found in the model and in the data. Figure 1 shows the Lorenz curves of family wealth and earnings distributions from both the PSID and the model. Family wealth in the PSID (1984 survey) reflects the net worth of houses, other real estate, vehicles, farms and businesses owned, stocks, bonds, cash accounts, and other assets. Looking at the left panel of the figure, the wealth distribution is found to be more skewed in the data; the Gini coefficient of wealth distribution in the PSID is 0.76,

[7]These are maximum-likelihood estimates of Heckman (1979) correcting for a sample selection bias. See Chang and Kim (2006) for the details.

whereas that in our model is 0.62. The right panel of the figure shows the Lorenz curves of earnings. Family earnings in the PSID are the sum of earnings of the household head and spouse. The earnings distribution appears more skewed in our model than in the data. This is because on average 40% of agents are not working in our model (recall that the steady state employment rate is calibrated to 60%) whereas according to the 1984 PSID, only 18% of households reported zero earnings. However, when we restrict the observations to the working households only the earnings distributions in the model and PSID are very close to each other as we estimate the stochastic process of individual productivity shocks from the hourly wages in the PSID. Overall, the model economy appears to possess a reasonable degree of heterogeneity, thus making it possible to study the effects of aggregation in the labor market.

To obtain the aggregate fluctuations, we solve the equilibrium of the model using the "bounded rationality" method developed by Krusell and Smith (1998)—agents make use of a finite set of moments of μ in forecasting aggregate prices. As in Krusell and Smith (1998) we achieve a fairly precise forecast when we use the first moment of μ only (i.e., aggregate capital, K). The detailed description of our computation procedure is given in Chang and Kim (2007). Table 4 compares the cyclical property of key aggregate variables of the model economy to that in the U.S. aggregate data for 1964:I - 2003:IV. All variables are logged and de-trended by the HP filter. Our model with an aggregate productivity shock generates about 63% of business cycle volatility in the data: the standard deviation of output in the U.S. data is 2.04% and in our model it is 1.28%. The relative (to output) volatilities of consumption, hours of work, and real wages are, however, pretty close to those in the data. They are, respectively, 0.43, 0.85, and 0.56, in the data, whereas they are 0.39, 0.76, and 0.50 in our model. The correlations with output are 0.83, 0.87, and 0.60, respectively, for consumption, hours, and real wages in the data. They are, respectively, 0.84, 0.87, and 0.68 in the model. One of the distinguishing features of our model is that hours worked is fairly volatile—in fact, more volatile than wages, yet not highly correlated with wages in the face of aggregate productivity shifts. The correlation between hours and wages is 0.39 in the data and 0.23 in our model. As demonstrated in Chang and Kim (2006), the indivisibility of labor and capital market incompleteness break a tight link between individual and aggregate labor supplies. In a similar economic environment to this model, they show that the aggregate labor supply elasticity has little to do with individual intertemporal substitution elasticity of leisure. Rather, it depends on the reservation wage distribution.[8]

4 Estimation based on the model-generated aggregate data

4.1 Representative-agent model

In order to confirm that the GMM procedure recovers the true underlying preference parameters, we first estimate optimality conditions using the time series generated from the representative agent model with productivity shocks (i.e., the standard real business cycle model). We assume that the preference parameters of the stand-in household are the same as those in the benchmark economy: $\sigma = 1$ and $\gamma = 2.5$. All parameters except for ψ are also identical to those in the benchmark model. We choose ψ so that the steady state hours worked is 1/3. We simulate 100,000 observations and discard the first 500 observations. We do not estimate the static first-order condition (S) because it holds exactly. The top panel of Table 5 reports the estimates using the 2484 sets of estimation, each of which has a sample

[8]In fact, as the GMM estimate reveals, the aggregate elasticity is around 1 in the model economy despite the fact that the intertemporal substitution elasticity assumed at the individual level is 0.4.

size of 160 observations, three-fourths of which overlap with the next set. We report the estimates based on Instrument I only because they are not greatly affected by the choice of instrument. According to the Euler equation for consumption (EC), the point estimate of σ is 0.670 (with standard error 0.197). According to the Euler equation for leisure (EL), the estimate of γ is 3.227 (0.252). When both equations are estimated jointly (System), σ is 0.754 (0.198) and γ is 3.019 (0.239). The estimate for β is always 0.99 with high precision. The estimates of σ are slightly smaller than the true value 1 and those of γ are slightly bigger than its true value of 2.5. These discrepancies between the estimate and true value disappear when we use a large sample size. The large-sample-size estimates consist of 618 sets of estimation, each of which has a sample size of 640 observations, three-fourths of which overlap with the next set. In the bottom panel of Table 5 according to the estimates based on the large sample size, σ is 0.929 (with standard error of 0.093) and 1.008 (0.070), respectively, in (EC) and (System). The estimate of γ is 2.800 (0.197) and 2.651 (0.137) in (EL) and (System), respectively.

Figure 2 exhibits the distribution (kernel density) of estimates for σ, γ, β, and J-statistic from both small-sample-size (solid line) and large-sample-size (dashed line) data sets generated from the model.[9] With a large sample size, both σ and γ are now highly concentrated around their true values. In sum, the GMM estimation accurately recovers true parameters with a large enough sample size.

[9]In estimating the kernel density, Gaussian kernel with Silverman's (1986) automatic bandwidth selection criterion is used.

4.2 Heterogeneous-agent model

We now apply the same GMM procedure to the aggregated time series of consumption, employment, wages, and the rate of return to capital generated from our benchmark heterogeneous agent model with aggregate productivity shocks. According to Table 6 the estimate for σ based on the small sample size (160 observations in each estimation) is 1.116 (0.079) and 1.107 (0.064) in (S) and (System), respectively. The estimation based on the large sample size (640 observations in each estimation) delivers similar values. While the estimate for the intertemporal substitution elasticity of consumption is close to the assumed value for individual households, when (EC) is estimated alone, the estimate of σ is below that of households. It is only 0.422 (0.220) and 0.639 (0.127), respectively for small and large sample sizes, which resemble the low value of risk aversion often reported in the literature based on the aggregate consumption Euler equation (e.g., Hansen and Singleton, 1984; Ghysels and Hall, 1990).

The estimation result of the intertemporal substitution elasticity of hours worked is striking. As we found from the actual U.S. data (e.g., Table 1, Table 2, or MRS), γ is estimated to be either negative or close to zero (although statistically insignificant). According to the small-sample-size estimation, the estimate of γ is -0.065 (0.160), -0.158 (0.143) and 0.002 (0.101), respectively, in (S), (EL), and (System). This pattern persists in the large-sample-size estimation. They are all negative values and occasionally statistically significant: -0.139 (0.075), -0.235 (0.064) and -0.013 (0.051), respectively, in (S), (EL) and (System). We noted earlier that U.S. aggregate data led to the wrong sign or small value of γ for two reasons: (i) a lack of systematic correlation between hours worked and wages results in either non-concave or unstable utility, (ii) accounting for the volatility of hours worked relative to wages requires an elastic labor supply schedule. Our heterogeneous agent model also shows similar patterns of relative volatility and correlation in aggregate employment and wages and has led to similar GMM estimates of γ.

Figure 3 exhibits the kernel density of estimates based on both small-sample-size (solid line) and large-sample-size (dashed line) data sets. The estimates for σ are clustered between 1 and 1.5 in both (S) and (System) among small-sample-size estimates and a similar pattern persists in the estimates based on the large sample size, while the estimates are more clustered as the sample size increases. Interestingly, estimates based on (EC) exhibit a somewhat bimodal distribution at 0 and 0.5 among small-sample estimates. With a large sample size the estimates are clustered around 0.64. Estimates of γ exhibit either a wrong sign or a small positive number regardless of the sample size. They are distributed between -0.5 and 0.5, with more concentration among the large-sample-size estimates. In terms of the J-statistic (bottom row), the hypothesis that a "representative" household optimally chooses hours worked and consumption is often rejected, similar to the pattern we observe from the GMM estimation based on actual U.S. data. In particular, with a large sample size, the intertemporal substitution hypothesis is rejected at the frequency of 98 out of 100.

When the model economy consists of heterogeneous agents and the individual decision rules are hard to aggregate, an attempt to account for the aggregate time series by an optimizing behavior of the representative household fails. The relative risk aversion of consumption is significantly underestimated when the aggregate consumption Euler equation is used. The parameter that governs the behavior of the labor supply is estimated with great uncertainty regardless of the equation and instrument, just like those from the actual aggregate data.

4.3 Auxiliary model economies

In our benchmark model economy with heterogenous agents the difficulty of aggregation stems from two frictions: incomplete capital markets and indivisible labor. In order to distinguish the contribution of each, we consider two additional model economies which feature each friction only: the "Incomplete-market" with divisible labor and the "Indivisible-labor" with complete capital markets.

"Incomplete-market" model Households can choose any length of working hours but still face the borrowing constraint and (uninsurable) idiosyncratic productivity shocks. This is essentially the same specification as in Krusell and Smith (1998) with endogenous choice of leisure. The equilibrium of this economy can be defined similarly to that of the benchmark model with the worker's value function with divisible labor, $V^D(a, x; \lambda, \mu)$:

$$V^D(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}, h \in (0,1)} \left\{ \ln c - B \frac{h^{1+1/\gamma}}{1+1/\gamma} + \beta E \Big[V^D(a', x'; \lambda', \mu') \big| x, \lambda \Big] \right\}$$

subject to

$$c = w(\lambda, \mu)xh + (1 + r(\lambda, \mu))a - a',$$

$$a' \geq \bar{a},$$

$$\mu' = \mathbf{T}(\lambda, \mu).$$

"Indivisible-labor" model The next model economy we consider allows for complete capital markets but maintains indivisible labor and heterogeneity through idiosyncratic productivity shocks. Thanks to perfect risk sharing, agents enjoy the same level of consumption regardless of their employment status, productivity, or asset holdings.[10] The equilibrium of this economy is identical to the allocation made by a social planner who maximizes the equally weighted utility of the population. The planner chooses the sequence of consumption $\{C_t\}_{t=0}^{\infty}$ and the cut-off productivity $\{x_t^*\}_{t=0}^{\infty}$ for labor-market participation. To ensure an efficient allocation, the planner assigns workers who have a comparative advantage in the market (more productive workers) to work. If a worker's productivity is above x_t^*, he supplies $\bar{h}$ hours of labor. The planner's value function in the complete market, denoted by $V^C(K,\lambda)$, and the decision rules for consumption, $C(K,\lambda)$, and cut-off productivity, $x^*(K,\lambda)$, satisfy the following Bellman equation:

$$V^C(K,\lambda) = \max_{C,x^*}\left\{ \ln C - B\frac{\bar{h}^{1+1/\gamma}}{1+1/\gamma}\int_{x_t^*}^{\infty}\phi(x)dx + \beta E\left[V^C(K',\lambda')\middle|\lambda\right]\right\}$$

subject to

$$K' = F(K,L,\lambda) + (1-\delta)K - C,$$

where $L = \bar{h}\int_{x^*}^{\infty} x\phi(x)dx$ is the aggregate effective unit of labor, and $\phi(x)$ is the productivity distribution of workers. The planner chooses the cut-off productivity x^* so that:

$$\frac{1}{C}F_L(K,L,\lambda)\bar{h}x^*\phi(x^*) = B\frac{\bar{h}^{1+1/\gamma}}{1+1/\gamma}\phi(x^*). \tag{1}$$

The left-hand side is the (society's) utility gain from assigning the marginal worker to production. There are $\phi(x^*)$ number of workers with productivity x^* in the economy. Each of them supplies $\bar{h}x^*$ units of effective labor, and the marginal product of labor is F_L. The right-hand side represents the disutility incurred by these workers. The key point here is that, under complete markets, the first-order condition for the choice between hours and con-

[10] The distribution of workers is no longer a state variable in the individual optimization problem. Moreover, because of the ergodicity of the stochastic process for idiosyncratic productivity, the cross-sectional distribution of workers is always stationary.

sumption is *exactly* defined in terms of effective units of labor and wages at the aggregate level.

GMM estimates from the model-generated aggregate data Except for β and ψ, the same parameter values are used across all models. For "Indivisible-labor" model, β is set to 0.99 and ψ is chosen to be consistent with 60% employment along with $\bar{h} = 1/3$. For "Incomplete-market" model, β and ψ are jointly searched to be consistent with average hours of 0.2 ($= 60\% \times 1/3$) and an interest rate of 1% in a steady state. These economies are simulated by the same aggregate productivity shocks.

Table 7 shows the parameter estimates from the aggregate time series of "Incomplete-market" (with divisible labor) model. Despite incomplete capital markets, the aggregate data fairly accurately reveals the individual preference parameters with a high statistical precision. With a large sample size, σ is 1.058 (0.024), 0.828 (0.072), and 0.855 (0.279), according to (S), (EC) and (System), respectively. The labor supply parameter also reveals the value assumed at the individual household level. With a large sample, γ is 2.588 (0.089), 2.828 (0.258), and 2.626 (0.213), according to (S), (EL) and (System), respectively. Figure 4 shows that the estimates are also highly concentrated around their means. The capital-market incompleteness alone does not generate a large aggregation error because, with divisible labor supply, in response to aggregate productivity shocks, hours are highly correlated across households, allowing for a fairly precise aggregation.

According to Table 8, the aggregate consumption from the "Indivisible-labor" (with complete capital markets) economy reveals the relative risk aversion of individual households. The estimate of σ is 0.963 (0.101) and 1.011 (0.064) according to (EC) and (System) with a large sample size.[11] The labor supply elasticity at the aggregate level is, however, very different from that of households. The estimates of γ is 0.840 (0.096)and 0.793 (0.065) respectively for (EL) and (System) with a large sample size, implying the labor supply elasticity of 1.19 and 1.26, higher than the individual elasticity of 0.4. While the aggregate preferences is not necessarily identical to individual preferences, the GMM estimates based on model-generated aggregate time series reveals the social planner's objective function, the (equally) weighted average of household utility functions as there is a well defined optimality condition for a social planner under the complete capital market. Figure 4 confirms that the distributions of parameter estimates are concentrated around their means.

In sum, when the model economy consists of heterogeneous agents and the individual decision rules are hard to aggregate (due to incomplete capital markets *and* indivisible labor), an attempt to account for the aggregate time series by an optimizing behavior of the representative household often ends up with nonsensible estimates for preferences. MRS interpreted the nonsensible preference parameters estimated from the aggregate time series as evidence of the failure of market clearing. Our analysis suggests that the incompatibility between the equilibrium outcome of a representative household's optimization and the aggregate data may actually reflect poor aggregation rather than the failure of the market. However, our analysis also shows that equilibrium outcomes of a heterogeneous agent economy cannot be easily represented by a stand-in household.

[11]Note that we do not estimate the static first order condition (S) for this model economy because it holds exactly in (1). Potentially, the estimation of (S) is still possible because aggregate hours and wages are subject to the so-called compositional bias. However, the composition bias does not a have enough time-varying component as the economy moves near the deterministic steady state.

5 Summary

The cyclical behavior of aggregate hours worked, wages, and consumption is hard to reconcile with the equilibrium outcome of the representative agent model with standard preferences. Attempts to estimate preferences based on optimality conditions of a stand-in household often fail to deliver economically meaningful estimates. Either a commodity or leisure has to be an inferior good for the observed allocation to be an optimum. Unreasonable estimates of preference parameters are interpreted as evidence that the economy operates outside the labor-supply schedule in the short run due to, say, sticky wages. We demonstrate that this incompatibility between the equilibrium of a representative agent model and the aggregate data can reflect a failure of aggregation rather than that of the market. Nevertheless, our analysis suggests that outcomes of a heterogeneous agent economy are not readily represented by an optimum of a representative agent with stable preferences.

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Table 1: Parameter estimates based on U.S. data (Nondurables & Services)

Equations	(S)	(EC)	(EL)	System
Instrument I				
σ	0.210	-0.210		-0.046
	(0.062)	(0.330)		(0.027)
γ	-0.569		0.179	0.023
	(0.198)		(0.145)	(0.044)
β		0.994	0.997	0.996
		(0.002)	(0.0008)	(0.0004)
ψ	0.156			0.113
	(0.017)			(0.011)
J-statistic	14.368	14.985	14.400	14.493
p-value	0.026	0.036	0.045	0.186
Instrument II				
σ	0.188	-0.129		-0.040
	(0.067)	(0.308)		(0.024)
γ	-0.473		0.089	0.0006
	(0.210)		(0.122)	(0.035)
β		0.995	0.997	0.996
		(0.002)	(0.0008)	(0.0004)
ψ	0.164			0.112
	(0.017)			(0.011)
J-statistic	15.710	14.793	14.987	15.083
p-value	0.015	0.039	0.036	0.208

Table 2: Parameter estimates based on U.S. data (Nondurables)

Equations	(S)	(EC)	(EL)	System
Instrument I				
σ	0.834	0.243		0.589
	(0.045)	(0.381)		(0.034)
γ	−0.444		0.379	0.074
	(0.108)		(0.152)	(0.058)
β		0.996	0.997	0.996
		(0.002)	(0.0008)	(0.0006)
ψ	9.790			5.110
	(1.336)			(0.716)
J-statistic	6.048	11.664	12.229	12.308
p-value	0.418	0.112	0.093	0.422
Instrument II				
σ	0.843	0.136		0.624
	(0.049)	(0.333)		(0.029)
γ	−0.450		0.413	0.018
	(0.115)		(0.138)	(0.050)
β		0.996	0.996	0.996
		(0.001)	(0.0008)	(0.0005)
ψ	10.189			5.765
	(1.470)			(0.710)
J-statistic	5.203	12.957	13.570	13.657
p-value	0.518	0.073	0.059	0.418

Table 3: Parameters of the benchmark model economy

Parameter	Description
$\alpha = 0.64$	Labor share in production function
$\beta = 0.9785504$	Discount factor
$\sigma = 1$	Inverse of intertemporal substitution elasticity of consumption
$\gamma = 2.5$	Inverse of intertemporal substitution elasticity of leisure
$\psi = 151.28$	Utility parameter
$\overline{h} = 1/3$	Labor supply if working
$\bar{a} = -2.0$	Borrowing constraint
$\rho_x = 0.939$	Persistence of idiosyncratic productivity shock
$\sigma_x = 0.287$	Standard deviation of innovation to idiosyncratic productivity
$\rho_\lambda = 0.95$	Persistence of aggregate productivity shock
$\sigma_\lambda = 0.007$	Standard deviation of innovation to aggregate productivity

Table 4: Cyclical property of aggregate variables: benchmark model

Variable	U.S. Data	Model
σ_Y	2.04%	1.28%
σ_C/σ_Y	0.43	0.39
σ_H/σ_Y	0.85	0.76
σ_W/σ_Y	0.56	0.50
$cor(Y, C)$	0.83	0.84
$cor(Y, H)$	0.87	0.87
$cor(Y, W)$	0.60	0.68
$cor(H, W)$	0.39	0.23

Note: All variables are logged and de-trended by the HP filter. The volatility of output is measured by its standard deviation and that of all other variables is measured by the standard deviations relative to output.

Table 5: Parameter estimates : representative agent model

Equations	(EC)	(EL)	System
Small Sample Size:			
σ	0.670		0.754
	(0.197)		(0.198)
γ		3.227	3.019
		(0.252)	(0.239)
β	0.990	0.990	0.990
	(0.0002)	(0.0002)	(0.0001)
Size	0.170	0.084	
Large Sample Size:			
σ	0.929		1.008
	(0.093)		(0.070)
γ		2.800	2.651
		(0.197)	(0.137)
β	0.990	0.990	0.990
	(0.0001)	(0.0001)	(0.0001)
Size	0.129	0.049	

Note: For upper (lower) panel, means and standard errors are calculated from 2484 (618) estimations. Each estimation has a sample size of 160 (640) observations, three-fourths of which overlap with the next set. "Size" represents the empirical size (fraction of estimates rejected) of J-test with nominal size of 5%.

Table 6: Parameter estimates: heterogeneous agent model

Equations	(S)	(EC)	(EL)	System
Small Sample Size:				
σ	1.116	0.422		1.107
	(0.079)	(0.220)		(0.064)
γ	−0.065		−0.158	0.002
	(0.160)		(0.143)	(0.101)
β		0.990	0.990	0.990
		(0.0002)	(0.0003)	(0.0002)
Large Sample Size:				
σ	1.116	0.639		1.095
	(0.034)	(0.127)		(0.023)
γ	−0.139		−0.235	−0.013
	(0.075)		(0.064)	(0.051)
β		0.990	0.990	0.990
		(0.0001)	(0.0001)	(0.0001)

Note: For upper (lower) panel, means and standard errors are calculated from 2484 (618) estimations. Each estimation has a sample size of 160 (640) observations, three-fourths of which overlap with the next set.

Table 7: Parameter estimates: "Incomplete-market" model

Equations	(S)	(EC)	(EL)	System
Small Sample Size:				
σ	1.061	0.578		0.881
	(0.123)	(0.181)		(2.206)
γ	2.559		3.310	2.655
	(0.798)		(0.370)	(0.368)
β		0.990	0.990	0.990
		(0.0001)	(0.0002)	(0.0003)
Large Sample Size:				
σ	1.058	0.828		0.855
	(0.024)	(0.072)		(0.933)
γ	2.588		2.828	2.626
	(0.095)		(0.258)	(0.150)
β		0.990	0.990	0.990
		(0.0001)	(0.0001)	(0.0001)

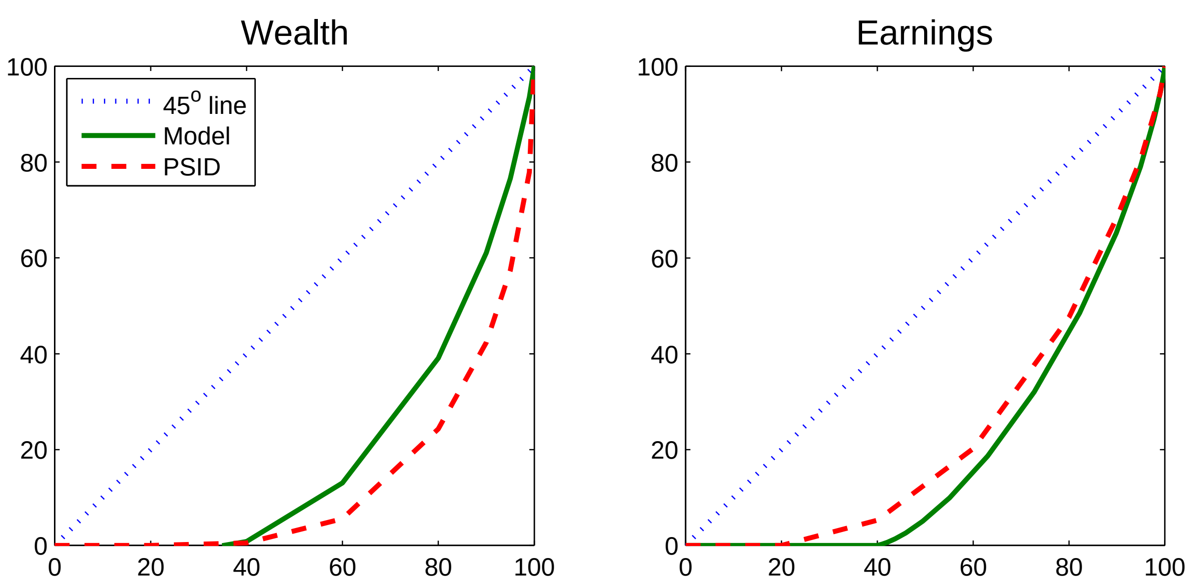
Note: For upper (lower) panel, means and standard errors are calculated from 2484 (618) estimations. Each estimation has a sample size of 160 (640) observations, three-fourths of which overlap with the next set.

Table 8: PARAMETER ESTIMATES: "INDIVISIBLE-LABOR" MODEL

Equations	(EC)	(EL)	System
Small Sample Size:			
σ	0.672		0.742
	(0.209)		(0.201)
γ		1.057	0.964
		(0.121)	(0.114)
β	0.990	0.990	0.990
	(0.0002)	(0.0002)	(0.0002)
Large Sample Size:			
σ	0.963		1.011
	(0.101)		(0.064)
γ		0.840	0.793
		(0.096)	(0.065)
β	0.990	0.990	0.990
	(0.0001)	(0.0001)	(0.0001)

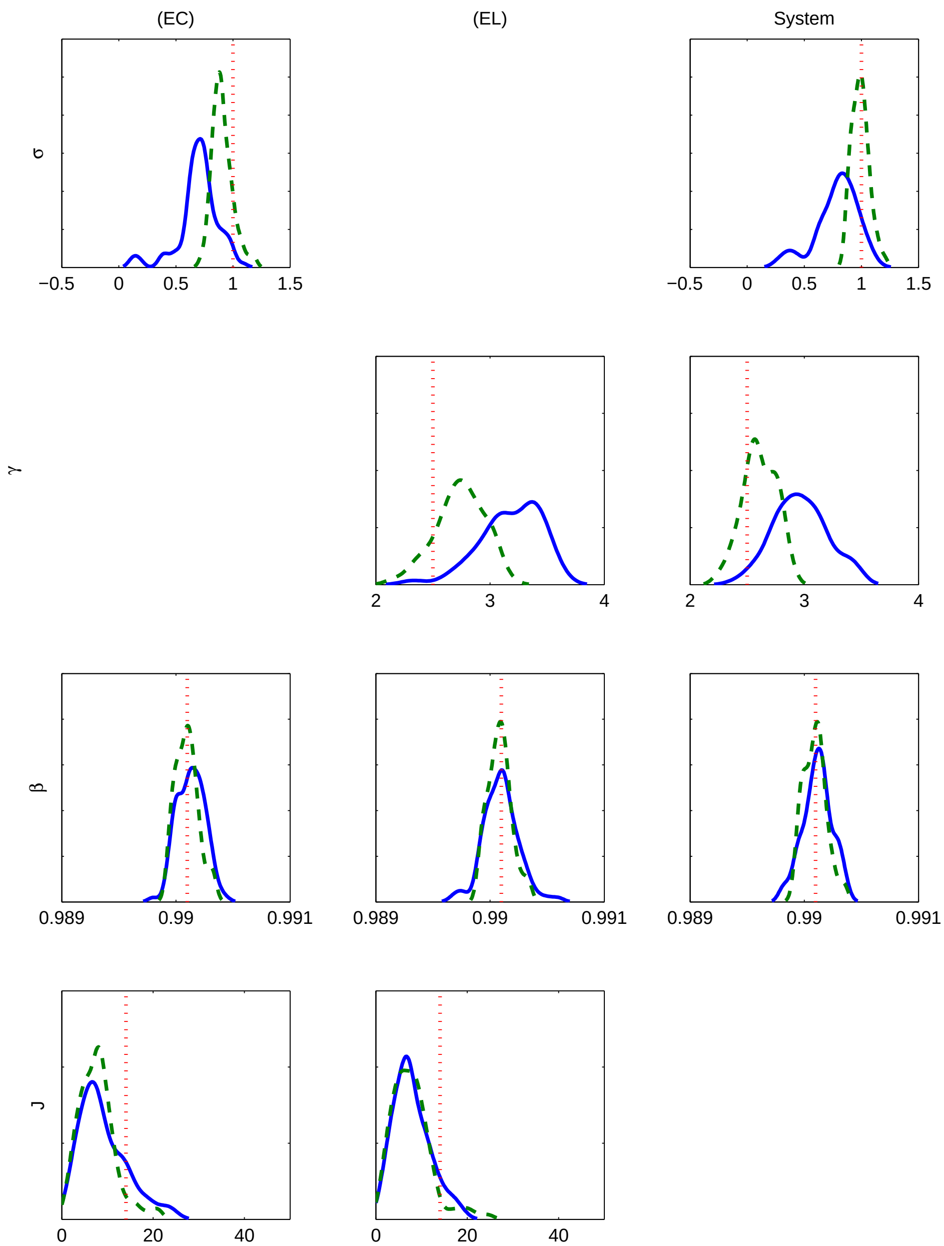
Note: For upper (lower) panel, means and standard errors are calculated from 2484 (618) estimations. Each estimation has a sample size of 160 (640) observations, three-fourths of which overlap with the next set.

Figure 1: LORENZ CURVES OF WEALTH AND EARNINGS



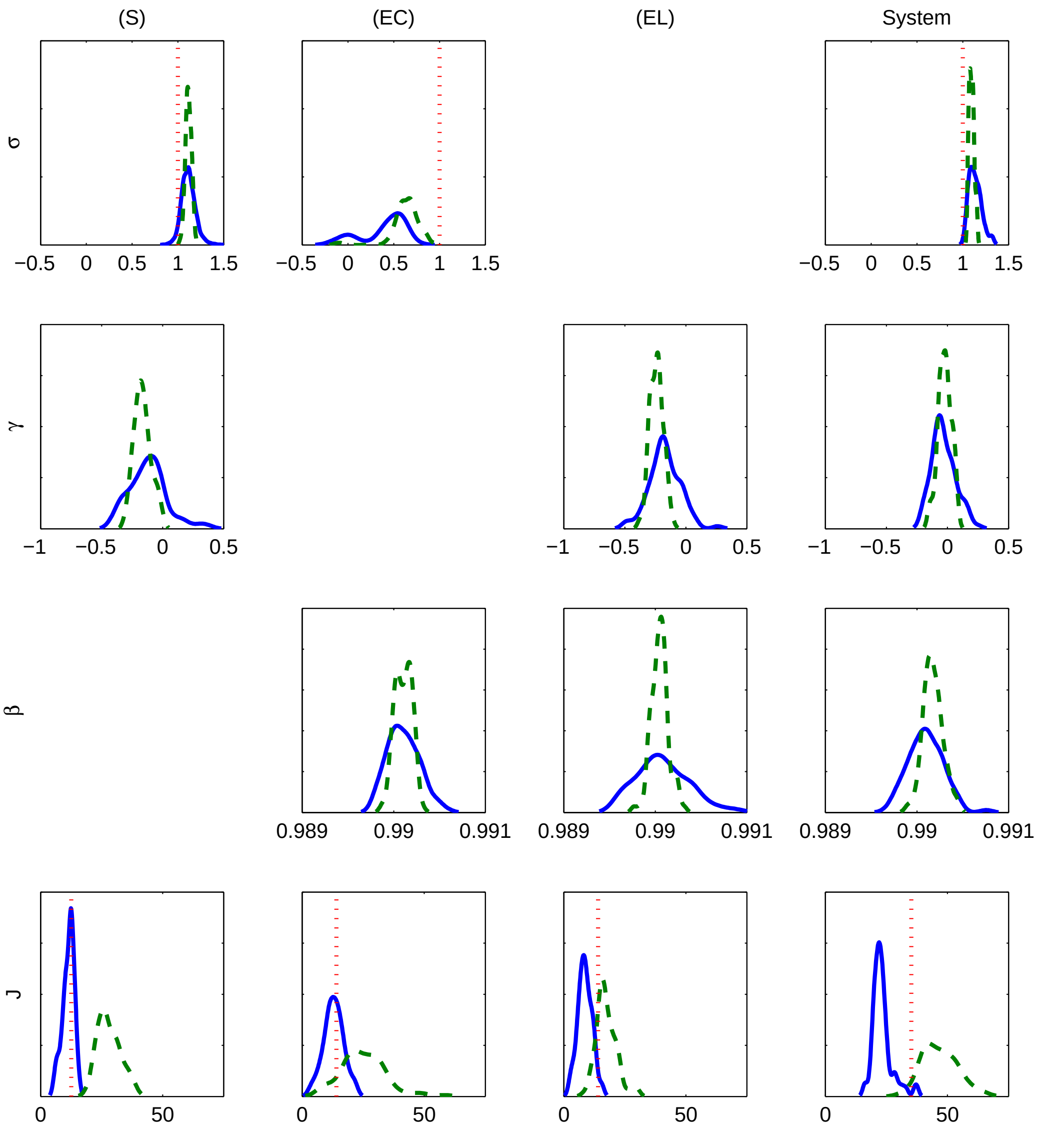
Note: The PSID statistics reflect family wealth and earnings in the 1984 survey.

Figure 2: Kernel density of parameter estimates: representative agent model



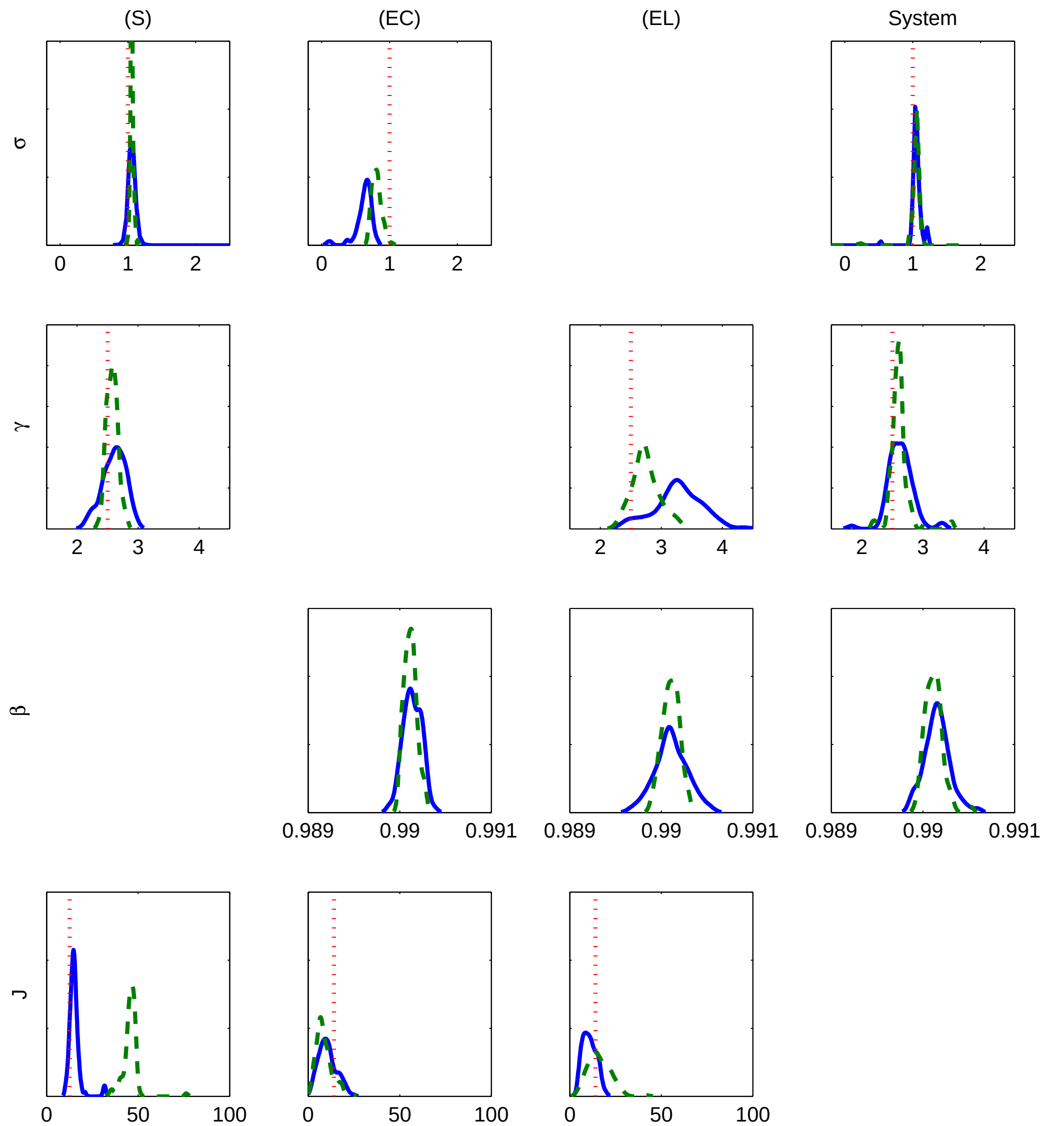
Note: The density of parameter estimates are calculated by the kernel method (Gaussian kernel with automatic bandwidth). The solid line represents the small-sample-size estimates (160 observations for each estimation), while the dashed line describes the large-sample-size estimates (640 observations for each estimation). The vertical dotted lines in the bottom panels represent 5% critical values of the *J*-statistic.

Figure 3: Kernel density of parameter estimates: heterogeneous agent model



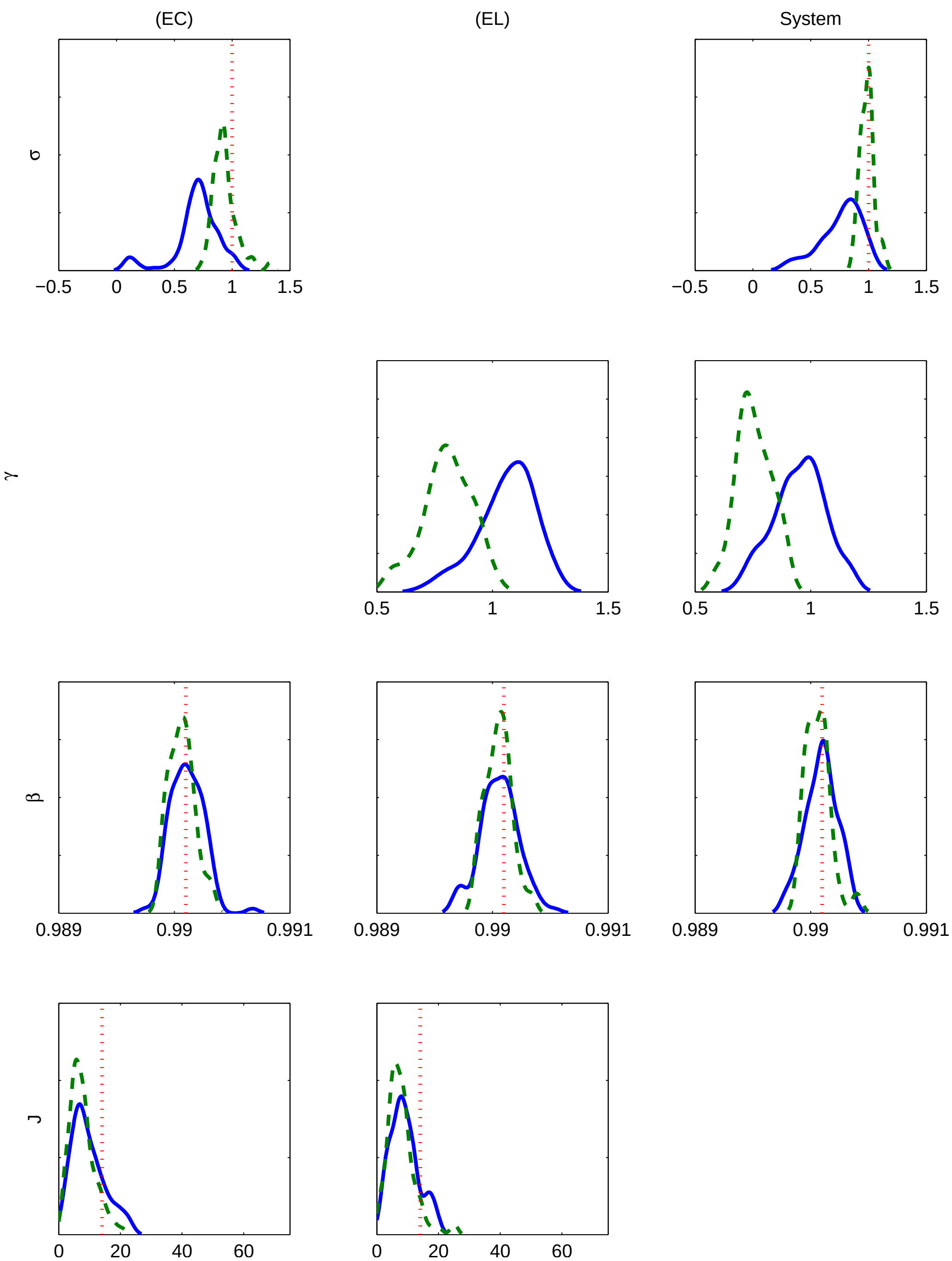
Note: The density of parameter estimates are calculated by the kernel method (Gaussian kernel with automatic bandwidth). The solid line represents the small-sample-size estimates (160 observations for each estimation), while the dashed line describes the large-sample-size estimates (640 observations for each estimation). The vertical dotted lines in the bottom panels represent 5% critical values of the *J*-statistic.

Figure 4: Kernel density of parameter estimates: "Incomplete-market" model



Note: The density of parameter estimates are calculated by the kernel method (Gaussian kernel with automatic bandwidth). The solid line represents the small-sample-size estimates (160 observations for each estimation), while the dashed line describes the large-sample-size estimates (640 observations for each estimation). The vertical dotted lines in the bottom panels represent 5% critical values of the *J*-statistic.

Figure 5: Kernel density of parameter estimates: "Indivisible-labor" model



Note: The density of parameter estimates are calculated by the kernel method (Gaussian kernel with automatic bandwidth). The solid line represents the small-sample-size estimates (160 observations for each estimation), while the dashed line describes the large-sample-size estimates (640 observations for each estimation). The vertical dotted lines in the bottom panels represent 5% critical values of the J-statistic.