Privately Efficient Bargaining between Workers and Large Firms

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Abstract

I propose a model of bargaining between workers and large firms that resembles the Stole and Zwiebel (1996*a*,*b*) model, but has the desirable feature that workers and firms behave in a privately efficient manner. I assume that on hiring, firms and workers can contract to behave efficiently in the future, while maintaining the assumption that the marginal surplus associated with the match is split in a constant ratio. I show that the equilibrium of the model is constrained efficient under my bargaining assumption if the Hosios (1990) condition holds; in Stole and Zwiebel's bargaining model, the incentive of firms to extract rent from their existing employees by over-hiring means that the equilibrium is never constrained efficient in the absence of taxes. I show that the two models cannot be distinguished from data only on the worker side of the market, such as wages and job-finding rates; data on firm profitability is required. The model is tractable enough to allow aggregate fluctuations to be studied easily, and a variant of it has rich predictions for wage dynamics. I find that a benchmark version of the model generates behavior of key labor market variables such as market tightness and the unemployment rate that is similar to what arises in a comparably-calibrated Mortensen-Pissarides model, although the model does have the ability to account for much richer dynamics of employment at the firm level.

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1 Introduction

The bargaining model introduced by Stole and Zwiebel (1996a,b; hereafter SZ) has in recent years become the benchmark model of bargaining in an environment with search frictions in which firms employ multiple workers. Wages are determined by bargaining between a firm and its workers over the marginal surplus that is generated by their employment relationship. A well-known feature of this model is that there is an incentive for firms to 'over-hire' workers to the point that the wage paid exceeds their marginal product; this effect arises because the presence of an additional worker strengthens the firm's bargaining position and reduces wages for all workers. In this paper I propose instead that the benchmark model of bargaining between firms and the many workers that they employ should not have this undesirable feature. In the spirit of Barro (1977), who argued against accepting uncritically the prediction of models with exogenously rigid wages that we should observe privately inefficient separations, I posit that our benchmark model of bargaining between workers and large firms should not assume that firms and workers are unable to coordinate on jointly efficient actions. I propose an alternative bargaining model that avoids this drawback, argue that it is just as tractable as the Stole and Zwiebel model, and study equilibrium in it. I show that such an equilibrium can be constrained efficient, providing a normative justification for the bargaining procedure I consider. However, it is ultimately an empirical question whether workers and firms can contract in a bilaterally efficient manner, so that I then ask whether the inefficiency that arises in the SZ model can be identified in data. Using the worker-side data that is usually used to calibrate models of labor market frictions, I show that distinguishing the two models may not be so easy.

Models where matching is frictional generate bilateral surpluses that must be split somehow between the matched parties. Even in the bilateral matching of the Mortensen-Pissarides model (Mortensen, 1982; Pissarides, 1984; hereafter MP), a quasi-rent must be split between a worker and her employer, and it is assumed that this occurs according to a constant ratio. Thus, if V^e, V^u, $J(1)$, and $J(0)$ are the Bellman values of (respectively) an employed worker, an unemployed worker, a filled job, and a job that has lost its worker, it is assumed that

$$\frac{V^e - V^u}{\eta} = \frac{J(1) - J(0)}{1 - \eta}, \tag{1}$$

where η, the (generalized Nash) bargaining power of the worker, is a structural parameter of the model. In the case where multiple workers are employed by the firm, Stole and Zwiebel generalized this by proposing that bargaining takes place over the marginal surplus. More formally, if now we denote by $J(n)$ the Bellman value of a firm that is currently employing $n \geq 0$ workers and by $V(n)$ the Bellman value of a worker employed at such a firm, then it is assumed that[1]

$$\frac{V(n) - V^u}{\eta} = \frac{J(n) - J(n-1)}{1 - \eta}. \tag{2}$$

Of course, in either the MP or the SZ model, the way in which this ex-post surplus is split does not matter for the continuation of the match between the firm and its workers, but it is important for characterizing the equilibrium since expectations about surplus-sharing affect incentives to invest in costly pre-match activities such as entry and search.

However, implicit in the SZ framework are two other assumptions about how the match surplus is split and how hiring decisions are made. Specifically, it is assumed that the incumbent workers

[1] The contribution of Stole and Zwiebel (1996*a*,*b*) is of course much greater than indicated by the reduced form equation (2). They provide a noncooperative microfoundation where firms sequentially bargain with all their employees, and they consider the implications of their bargaining framework for the theory of the firm.

belonging to the firm will be treated identically to new hires at all times, and that the right to decide on vacancy posting and hiring is allocated to the firm. That is, when a firm with $n-1$ pre-existing workers hires an additional worker, the incumbent workers each experience a capital gain of $V(n) - V(n-1)$, which will be negative in the case of a decreasing returns to scale production technology. Moreover, the right to make decisions about how many workers are hired is allocated to the firm, which chooses employment so as to maximize its own value. This generates a hold-up problem: the firm has the opportunity to impose costs upon its own incumbent workers by hiring new workers. It is this, combined with profit-maximization by the firm, that generates the over-hiring feature of the SZ model.

In this paper I suppose that firms and workers can, at the time their relationship begins, sign a contract that ensures that they will act to maximize their joint surplus,

$$S(n) \equiv [J(n) - J(0)] + n\,[V(n) - V^u]\,. \tag{3}$$

I combine this with the assumption that when a new worker is hired, bargaining takes place between, on the one side, the unit consisting of the firm and its incumbent workers, and, on the other, the new employee. In the spirit of minimal generalization of the existing literature, I wish to retain the assumption of the MP and SZ models that the marginal surplus is split in constant shares, so I assume that

$$\frac{V(n) - V^u}{\eta} = S(n) - S(n-1). \tag{4}$$

I will call the bargaining protocol embodied in (3) and (4), together with the assumption of joint surplus maximization, *privately efficient bargaining*, or *PE bargaining*.

The analysis in the paper proceeds as follows. In Section 2 I specify a benchmark model of a frictional labor market, and in Section 3 I characterize the constrained efficient allocation. I then define equilibrium under PE bargaining formally, and show that the equilibrium allocation under PE bargaining is constrained efficient provided that the Hosios condition on the bargaining power parameter η is satisfied. This is the conclusion of Section 4. Under SZ bargaining, the equilibrium is never constrained efficient in the absence of a government; this is established in Section 5. In Section 6 I consider distinguishing the two bargaining protocols using the normal labor market calibration targets familiar from papers such as Shimer (2005), and show that an econometrician unaware of whether his data generating process is an economy characterized by SZ or PE bargaining would be unable to distinguish the two. Data on profits or on factor shares is required.

Finally, in Section 7 and Section 8, I generalize the model in two directions. First, following Acemoglu and Hawkins (2010) I allow for hiring to be time-consuming (and not just costly in terms of goods, as in the benchmark model); this generates interesting implications for the dynamics of firm sizes, and potentially also for wages. I show that the basic non-identification result at the core of the paper extends also to this case. Second, I generalize the model to allow for aggregate and idiosyncratic shocks to labor productivity, and show that, again, it is impossible to distinguish PE from SZ bargaining from labor market data alone. The environment of the SZ bargaining model studied in Section 8 is almost identical to that studied by Elsby and Michaels (2010), who study the ability of a model like the one studied here to reproduce cross-sectional and cyclical features of the US aggregate labor market. However, they need to resort to numerical approximation methods to solve their model; I am able to solve exactly for the PE and SZ bargaining equilibria of a slightly less general case than the one they consider. This ability to provide an exact solution of the model is of independent interest. Contrary to the results of Elsby and Michaels (2010), I find very little evidence, in a calibrated model, of amplification or persistence in key labor market variables relative to the benchmark MP model. The similarity of my model to that of Elsby and

Michaels suggests that their results may be dependent on their approximation method; it also suggests that, while the model studied in this Section can potentially do quite well in accounting for the behavior of employment, hiring, and firing at the microeconomic (establishment) level, such a model is not particularly well-suited to generating increased amplitude of fluctuations in employment and unemployment, or in increasing the persistence of unemployment, relative to the MP model. In addition, the non-identification result I establish here shows that the behavior of a model of the type studied does not depend on the inefficiency implicit in the SZ framework, since an alternatively-parameterized model with PE bargaining is observationally equivalent in its implications for the labor market.

The efficiency of equilibria in models where wages are set by bargaining between workers and firms is a much-studied area. Of particular relevance to this paper is the seminal contribution of Hosios (1990), who established the relationship between bargaining power and the elasticity of the matching function with respect to unemployment; Acemoglu and Shimer (1999) generalize this and establish a framework I use here. In the case of firms employing multiple workers, Stole and Zwiebel (1996*a*,*b*) and Smith (1999) focus on the over-hiring effect that underlies the inefficiency of the SZ model.

Other authors using the SZ framework in recent models of large firms include Cahuc and Wasmer (2001), Cahuc, Marque and Wasmer (2008), and Mortensen (2009); none of these authors focus on the efficiency question studied here. Wolinsky (2000) and Acemoglu and Hawkins (2010) study versions of the SZ model in which hiring is time-consuming; this leads to richer predictions for wage dynamics but is more technically challenging to analyze than the basic model studied here (although in Section 7 I do also consider this case). Applications of the SZ framework have recently also included work studying wage determination (Roys, 2010), the interaction of product market regulation and the labor market (Felbermayr and Prat, 2007; Delacroix and Samaniego, 2009; Ebell and Haefke, 2009), trade (Coşar, Guner and Tybout, 2010; Helpman and Itskhoki, 2010), as well as several others; all these authors simply take the SZ model off-the-shelf as the benchmark model of bargaining between large firms and their employees. This paper can be viewed as supporting their choice somewhat, since, despite the fact that the SZ model embeds an assumption of inefficient hiring, this cannot be important for their results given the non-identification result proved here.

Hawkins (2006) and Lester (2010) study frictional labor markets with large firms and workers under directed search; it is well known (Moen, 1997; Shimer, 1996) that directed search models have different efficiency properties from models with bargained wages, so that the contributions of those papers are complementary to this study.

Most closely related to this paper, and deserving of more substantial comment, is the paper of Kaas and Kircher (2010), developed contemporaneously, which studies efficient hiring dynamics in a model closely related to the one presented here. They show how to study such a rich economic environment, very similar to the one I consider in Section 8 below, in the setting of directed search. Their analysis differs from the SZ framework in two ways: first, that they assume privately efficient contracting as I also do, and second, that they replace the random search assumption with directed search. The assumption of random search makes models with non-trivial hiring dynamics and difficult to solve in the presence of aggregate shocks (it is not accidental that in Section 7 below I do not show how to solve the case of my model with time-consuming hiring except in an aggregate steady state), so their modeling framework appears more tractable than the environment studied here in the case when one wants to make the assumption that hiring takes time. However, bargaining frameworks with random search are also of interest, and more used in the applied literature, and so the contribution of this paper is also of use for understanding what is being assumed when such models are written down. Further, as indicated in Section 8, the model of this paper is also tractable if one is willing to abstract from slow hiring at the level of the individual firm. Finally, it is worth noting that although Kaas and Kircher and I both prove that equilibria in the models we respectively study are constrained efficient, that does not mean that the resulting allocations are the same, since in the two papers, the planner faces different constraints (specifically, directed versus random search). Distinguishing between the two papers empirically seems of the same level of difficulty as distinguishing between the benchmark models of directed and random search, something that the literature has not yet satisfactorily resolved.

2 Model

In this section I introduce the benchmark model and define steady-state equilibrium under PE bargaining and under SZ bargaining. The model studied here can be significantly generalized, and I do this in Section 7 and Section 8; for now I focus on a model that is as simple as possible.

Time is continuous. All agents are risk neutral and discount the future at rate r. There are two types of agents, workers and firms. There is a fixed measure 1 of workers, who can at any time be either unemployed (in which case they produce flow unemployment income b) or employed by some firm. The flow output of a firm employing $n \geq 0$ workers is given by $y(n)$; the production function y is strictly increasing and strictly concave, and I normalize $y(0) = 0$. Employment n is a continuous variable, so that n can take any non-negative real value.

There is a large measure of potential entrant firms. At any moment a potential entrant firm has the option of paying an entry cost $\kappa > 0$ and becoming active. An active firm, at any moment, has the option of posting arbitrarily many vacancies; there is a constant marginal flow posting cost $\gamma \geq 0$ per unit measure of vacancies posted. There is a matching function $M(u, v)$ which maps the measures of unemployed workers and posted vacancies to a flow rate of new jobs; each unemployed worker is matched to a randomly-chosen vacancy at Poisson rate $M(u, v)/u$ and each vacancy is matched to a randomly-chosen unemployed worker at Poisson rate $M(u, v)/v$. I assume that $M(u, v)$ exhibits constant returns to scale in (u, v), so that if I define $\theta = v/u$, the *labor market tightness* or vacancy-unemployment ratio, then the Poisson vacancy-filling rate and the Poisson job-finding rate for a worker can be written as functions of θ alone. Specifically, I can denote these rates respectively as $q(\theta) = M(u, v)/v = M(\theta^{-1}, 1)$ and $f(\theta) = \theta q(\theta) = M(u, v)/u = M(1, \theta)$. I assume that q is monotonically decreasing in θ and satisfies the Inada conditions $\lim_{\theta \to 0} q(\theta) = +\infty$ and $\lim_{\theta \to \infty} q(\theta) = 0$. I also assume that f is monotonically increasing in θ and satisfies $f(0) = 0$ and $\lim_{\theta \to \infty} f(\theta) = \infty$. All these assumptions are satisfied if M is Cobb-Douglas in (u, v), as Petrongolo and Pissarides (2001) cannot reject in US data. Finally, since a firm's choice of how many vacancies to post is a continuous variable $h \geq 0$, I assume that a law of large numbers holds so that hiring by a firm that posts h vacancies for a duration $\tau > 0$ takes the non-stochastic value $h\tau/q(\theta)$ for sure.[2]

Existing firms are destroyed at a Poisson rate $\delta > 0$; in this case all the existing workers return to unemployment, and the firm is destroyed; there is no scrapping value. I assume that there are

[2] Out of steady state, if θ is not constant, then hiring in the time interval $[t, t + \delta t]$ is given by

$$\int_t^{t+\delta t} \frac{h}{q(\theta_t)} \, dt.$$

The usual technical issues regarding laws of large numbers applied to continua of random variables arise, so that it would be more formally satisfactory to study a discrete-time model with vanishingly short period length, and also to assume that h can take on only a countable number of possible values, perhaps non-negative integer multiples of some minimum hiring level $\underline{h} > 0$. This complicates the algebra without changing anything of economic importance, so as is customary in economics, I ignore these issues.

no other separations allowed (in particular, firms cannot fire workers).[3]

The unemployment rate is a state variable of the model; its law of motion is given by

$$\dot{U} = \delta(1 - U) - \theta q(\theta)U. \tag{5}$$

For the majority of the paper, I consider only steady-state allocations; in Section 8 I generalize to the case of aggregate shocks.

I commence the analysis of the model by characterizing efficient allocations; I then study the equilibria of two bargaining models.

3 Efficient allocations

Suppose that there is a benevolent planner for the economy; this planner takes as given the frictional matching process of the economy and the production technology, and subject to these constraints, attempts to maximize social welfare. Since all agents are risk-neutral, this is equivalent to maximizing the present discounted value of output. The planner's choice variables are a vacancy-posting strategy for each active firm, and a time path for the flow rate at which potential firms become active in order to maximize the present discounted value of output. This problem is high-dimensional, but characterization of the optimum is fortunately fairly simple, at least in steady state. This is for two reasons.

First, all firms optimally use the same target employment level $n^* > 0$, ceasing posting vacancies once this target is reached. This is because the production function $y(\cdot)$ is common to all firms, and is strictly concave.

Second, the vacancy-posting strategy of the firm optimally takes a 'bang-bang' form, so that only new entrants post any vacancies, and they post enough vacancies so that their employment level increases from 0 to n^* in a vanishingly-short duration. To see why this is optimal, observe that a firm that posts v_i vacancies for a duration τ succeeds in hiring $v_i\tau/q(\theta)$ workers; so does a firm that posts ζv_i vacancies for a duration τ/ζ, where $\zeta > 0$ is arbitrary. Now, consider the sequence of allocations generated by keeping constant the labor-market tightness and the target size n^* of all firms, but assuming that the measure of vacancies posted by any firm posting postively many vacancies is $\bar{v}$. To reach the target employment level n^*, the firm must post $\bar{v}$ vacancies for a total duration of $T(\bar{v}) \equiv n^*/(q(\theta)\bar{v})$, which is of course decreasing in $\bar{v}$. Employment after any duration less than this is given by $n_t = t\bar{v}q(\theta)$. The total discounted value of the output produced by such a firm, net of entry and vacancy-posting costs, is given by

$$-\kappa + \int_0^{T(\bar{v})} e^{-(r+\delta)t} \left[y(n_t) - \gamma\bar{v}\right] dt + \frac{e^{-(r+\delta)T(\bar{v})}}{r+\delta} y(n^*).$$

It is easy to verify that this is increasing in $\bar{v}$ and is maximized as $\bar{v} \to \infty$ by the value

$$-\kappa - n^* \frac{\gamma}{q(\theta)} + \frac{y(n^*)}{r+\delta}.$$

This reduces the planner's problem to a choice of sequences of flow entry rates e_t of new firms, target employment levels n_t^* for these firms, the labor market tightness θ_t, and the unemployment rate u_t. Since I am trying to characterize optimal steady state allocations only, I assume that in fact all these sequences are constant. The maximization problem for the planner then takes the form of maximizing the present discounted value of output, which I record as the following proposition.

[3] This is relaxed in Section 8. Before that point, it is without loss of generality since firms would not want to fire workers at any non-negative cost in the models considered before that.

Proposition 1 (Efficient allocations)**.** *The constrained efficient allocation solves*

$$\max_{e,n^*,\theta,u} \int_0^\infty e^{-rt}\left(\left[\frac{y(n^*)}{r+\delta} - n^*\frac{\gamma}{q(\theta)} - \kappa\right]e + bu\right)dt,$$

subject to

$$n^*e = \theta q(\theta)u = f(\theta)u \qquad \text{and} \qquad \dot{u} = \delta(1-u) - \theta q(\theta)u.$$

The two constraints are that hiring by entrant firms be consistent with the outflow from unemployment, and the law of motion for the unemployment rate. Denote by λ the Lagrange multiplier on the law of motion for unemployment; then the first-order conditions for the optimal allocation can be quickly manipulated (eliminating e using the constraint to reduce the optimization to a problem over n^*, θ, and u alone):

$$y'(n^*) = \frac{1}{n^*}\left[y(n^*) - (r+\delta)\kappa\right] \qquad [n^*] \qquad (6)$$

$$r\lambda = \left[\frac{y(n^*)}{r+\delta} - n^*\frac{\gamma}{q(\theta)} - \kappa\right]\frac{f(\theta)}{n^*} + b - \lambda(\delta + \theta q(\theta)) \qquad [u] \qquad (7)$$

$$0 = \left[\frac{y(n^*)}{r+\delta} - n^*\frac{\gamma}{q(\theta)} - \kappa\right]\frac{u}{n^*}f'(\theta) - en^*\gamma\frac{d\frac{1}{q(\theta)}}{d\theta} - \lambda f'(\theta)u \qquad [\theta] \qquad (8)$$

These first-order conditions are intuitive. The first-order condition with respect to n^* requires that the marginal additional production from hiring an additional worker be equal to the average additional production, taking account of the entry cost.[4] The first-order condition for u relates the marginal benefit of having more unemployed workers (that it increases the rate of firm entry consistent with labor market tightness and target firm size, $e = \theta q(\theta)u/n^* = f(\theta)u/n^*$, and that it generates additional unemployment income) with the costs (that it reduces the number of unemployed in future via increased matching, and also since fewer employed workers today mean less firm destruction, also leading to lower unemployment in future). The first-order condition for θ requires that the gains from a tighter labor market (that greater entry is feasible, ceteris paribus, and that vacancy posting is cheaper - note that $-d\frac{1}{q(\theta)}/d\theta > 0$) equal the costs (that more unemployed workers will find jobs, reducing unemployment).

I now move to considering the equilibrium of a version of the model where firms and workers must bargain among themselves in a decentralized manner. I first consider the framework of privately efficient (PE) bargaining considered in the introduction; I then study the Stole and Zwiebel (SZ) framework.

4 Equilibrium under Privately Efficient (PE) bargaining

I now characterize the decentralized equilibrium under PE bargaining. First, generalizing equations (3) and (4) to the setting of continuous employment considered here, I assume that bargaining satisfies

$$\eta S'(n) = V(n) - V^u \qquad (9)$$

where the $V(n)$ is the (Hamilton-Jacobi-)Bellman value of a worker newly-hired by a firm with n workers,[5] V^u is the HJB value of an unemployed worker, and $S(n)$ is defined as the surplus a

[4] This makes it apparent why it was necessary to assume a non-zero entry cost κ: if $\kappa = 0$, optimal firm size is $n^* = 0$, and the model reduces to the MP limit.

[5] Whether n includes the new hire is unimportant when employment is a continuous variable; however, consistently with SZ, I assume it is inclusive.

firm and n workers are generating over the outcome where the firm is destroyed and the n workers returned to unemployment. What is this surplus? Of course, the surplus $S(n^{PE})$ associated with a firm that is at its target employment level is just the discounted value of its production above the flow value its workers would be generating if unemployed; that is,

$$S(n^{PE}) = \int_0^\infty e^{-(r+\delta)t} \left[y(n^{PE}) - n^{PE} r V^u \right] dt = \frac{y(n^{PE}) - n^{PE} r V^u}{r+\delta}. \tag{10}$$

For $n > n^{PE}$, because there are no separations, the optimal vacancy posting strategy is not to post any vacancies. This means that the surplus in this case is

$$S(n) = \frac{y(n) - n r V^u}{r+\delta} \qquad \text{for } n > n^{PE}. \tag{11}$$

Since the optimal strategy of a firm and its incumbent workers with employment less than n^{PE} is to post a very large number of vacancies for a very short time (for the same reason that a strategy of this type was also optimal for the planner), an active firm with current employment $n \in [0, n^{PE})$ will hire $n^{PE} - n$ workers immediately. Thus the only reason that its value is less than $S(n^{PE})$ is that first, a vacancy posting cost of $\gamma/q(\theta)$ must be paid per unit measure of workers hired, and second, that some surplus must be paid to newly-hired workers according to (9). The marginal surplus associated with a firm remaining matched with its existing workers is the combination of these two things. It follows immediately that, for $n \in (0, n^{PE})$,

$$S'(n) = \frac{1}{1-\eta} \frac{\gamma}{q(\theta)}. \tag{12}$$

(The factor $\frac{1}{1-\eta}$ arises since share η of the marginal surplus is paid to the worker, so that the remaining $1-\eta$ share must equal the vacancy posting costs that must be paid to increase n, of $\gamma/q(\theta)$ per unit measure increase in n.) Solving the rather trivial differential equation (12) with initial condition (10) gives that for $0 \le n \le n^{PE}$,

$$S(n) = -(n^{PE} - n) \frac{1}{1-\eta} \frac{\gamma}{q(\theta)} + S(n^{PE}). \tag{13}$$

The first order condition for the optimal choice of n^{PE} is a smooth pasting condition, that the right derivative $\lim_{n \to (n^{PE})^+} S'(n)$ equal the left derivative $\lim_{n \to (n^{PE})^-} S'(n)$. Comparing (11) and (12), it follows that n^{PE} satisfies

$$\frac{y'(n^{PE}) - rV^u}{r+\delta} = \frac{1}{1-\eta} \frac{\gamma}{q(\theta)}. \tag{14}$$

Next, observe that according to (9) and (12), the value gain associated with being hired by a firm is independent of how many workers the firm has already hired, and satisfies

$$\frac{\eta}{1-\eta} \frac{\gamma}{q(\theta)}. \tag{15}$$

This means that the HJB equation for an unemployed worker can be written[6]

$$rV^u = b + f(\theta) \frac{\eta}{1-\eta} \frac{\gamma}{q(\theta)} = b + \frac{\eta}{1-\eta} \gamma\theta. \tag{16}$$

[6] Had $V(n) - V^u$ depended on n, it would have been necessary to take an expectation over n in (16). The 'bang-bang' feature of the vacancy-posting strategy ensures that this equation is simpler than the corresponding equation in Acemoglu and Hawkins (2010), and corresponds closely to the assumption made by Elsby and Michaels (2010). See also Section 7 below.

The HJB equation for an employed worker is not actually needed in the analysis of equilibrium, but will be useful for calibration. To determine it, note first that $V(n) = V(n^{PE})$ for all $n < n^{PE}$. Then if I assume that firms with n^* workers pay a constant wage w to all their employees, then w, $V(n^{PE})$, and V^u are related according to

$$rV(n^{PE}) = w + \delta \left[V^u - V(n^{PE})\right]. \tag{17}$$

Using (15) and (16) to eliminate $V(n^{PE})$ and V^u from this equation, it follows immediately that

$$w = b + \frac{\eta}{1-\eta}\gamma \left[\theta + \frac{r+\delta}{q(\theta)}\right]. \tag{18}$$

To close the model, observe that the free entry condition takes the form that $S(0) = \kappa$. According to (10) and (13), it follows that

$$\kappa = S(0) = \frac{y(n^{PE}) - n^{PE} rV^u}{r+\delta} - n^{PE}\frac{1}{1-\eta}\frac{\gamma}{q(\theta)}. \tag{19}$$

Equations (14), (16), and (19) relate the three key endogenous variables of the model, and are necessary conditions for equilibrium. The following proposition establishes the existence and uniqueness of equilibrium.[7]

Proposition 2 (Existence and uniqueness under PE bargaining)**.** *There is a unique equilibrium under PE bargaining.*

Next, the following proposition establishes that the hiring target n^{PE} arising in the equilibrium with PE bargaining is in fact identical to the hiring target n^* chosen by the planner in the constrained efficient allocation.

Proposition 3 (Efficient hiring under PE bargaining)**.** $n^{PE} = n^*$.

Proposition 3 implies that any discrepancy between the equilibrium allocation under PE bargaining and the constrained efficient allocation arise from differences in market tightness. But conditional on the intensive margin of how many workers to hire being chosen optimally, the model resembles the benchmark MP model, in the sense that the key variable to determine is the labor market tightness θ. A natural conjecture, therefore, is that the Hosios condition for constrained efficiency in that model should be necessary and sufficient here as well. This conjecture is correct.

Proposition 4 (Constrained efficiency under PE bargaining)**.** *The equilibrium under PE bargaining is constrained efficient if and only if*

$$\eta = -\frac{\theta q'(\theta)}{q(\theta)}. \tag{20}$$

In the statement of Proposition 4, the value of θ referred to is the constrained efficient value (which coincides with the PE equilibrium value under the assumption of the proposition).

The intuition for Proposition 4 is, as is familiar from the MP model, that an intermediate value of the bargaining power parameter η strictly between 0 and 1 needs to be chosen to ensure that two potential sources of inefficiency counterbalance exactly. First, if η is too high, workers extract a large amount of surplus at hiring (ceteris paribus). This reduces the value of entry $S(0)$, since firms receive a smaller share of output. This tends to reduce entry, and render the market tightness θ inefficiently low. On the other hand, since firms do not internalize the congestion externality that their entry and vacancy posting behavior imposes on other firms, efficiency requires that their share of the ex post surplus not be too high. The proof of the proposition is a fairly simple algebraic manipulation, following Acemoglu and Shimer (1999). Note that $\theta q'(\theta)/q(\theta) = u/M(u,v) \times \frac{\partial}{\partial u}M(u,v)$ is just the elasticity of the matching function with respect to unemployment, as usual.

[7]The proof of Proposition 2, together with all other omitted proofs, can be found in the Appendix.

5 Equilibrium under Stole and Zwiebel (SZ) bargaining

In this section I find the equilibrium allocation under SZ bargaining and compare it with the efficient allocation. The key difference between the PE and SZ bargaining protocols is that under SZ, incumbent workers can be held up by their employer as it hires new workers (and that the right to determine hiring remains with the firm, so that this hold-up mechanism is operative).

Denote by $J(n)$ the HJB value of a firm employing n workers, and $V(n)$ the value of a worker employed at such a firm.[8] Then the appropriate version of (2) in the continuous employment setting is

$$\eta J'(n) = (1-\eta)\left[V(n) - V^u\right]. \tag{21}$$

To relate this to the determination of wages and optimal employment per firm, it is necessary to consider what the value function of a firm employing a different number of workers than its target is. In the setup here where arbitrarily many workers can be hired instantaneously at any time, this value is not well-determined.[9] Therefore, consider for a moment a variant of the model in which firms can only hire workers when they have a 'hiring opportunity;' assume that these arrive for active firms at Poisson rate $\alpha > 0$. When a hiring opportunity arrives, the firm can hire arbitrarily many workers at constant marginal cost, as in the benchmark model, which is the limit as $\alpha \to \infty$ of this variant model. In this case the HJB equation for a firm is given by

$$(r+\delta)J_\alpha(n) = y(n) - nw_\alpha(n) + \alpha\left[J_\alpha(n_\alpha^{SZ}) - J_\alpha(n) - (n_\alpha^{SZ} - n)\frac{\gamma}{q(\theta_\alpha)}\right]. \tag{22}$$

(I index the endogenous variables by α to indicate that the hiring friction affects them.) When a hiring opportunity arrives, the firm instantaneously adjusts its employment level to its target value n_α^{SZ}, paying the appropriate vacancy-posting cost. The corresponding HJB equation for a worker is

$$rV_\alpha(n) = w_\alpha(n) + \delta\left[V_\alpha^u - V_\alpha(n)\right] + \alpha\left[V_\alpha(n_\alpha^{SZ}) - V_\alpha(n)\right]. \tag{23}$$

Differentiate (22) with respect to n, use the analog of (21) to eliminate $\eta J'_\alpha(n) = (1-\eta)\left[V_\alpha(n) - V_\alpha^u\right]$, and then use (23) to eliminate $V_\alpha(n)$ to see that

$$w_\alpha(n) + \eta n w'_\alpha(n) = \eta y'(n) + (1-\eta) r V_\alpha^u.$$

This implies that[10]

$$w_\alpha(n) = (1-\eta) r V_\alpha^u + n^{-\frac{1}{\eta}}\left[c + \int_0^n \nu^{\frac{1-\eta}{\eta}} y'(\nu)\, d\nu\right]. \tag{24}$$

[8]Note that unlike in the case of PE bargaining, it is unnecessary to specify further that this is the value of a newly-hired worker, since by assumption all workers are treated symmetrically under SZ bargaining.

[9]One can verify that the SZ bargaining equation, together with the condition for optimal hiring, determines the derivative of the wage at the target hiring level n^{SZ} as a function of the level of the wage and of that target hiring level, but in order to pin down the wage, it's necessary to assume that a firm with $n \neq n^{SZ}$ workers must produce using only those workers for some positive expected duration. If firms 'believe' that away from their target hiring level, the wages that they would have to pay would change very rapidly, then a range of possible target hiring levels can be supported. An imprecise analogy can be made to the fact that beliefs about off-equilibrium path type distributions are not pinned down simply by Bayes' rule in dynamic games of imperfect information; equilibrium refinements are needed for this. The heuristic argument for wage determination given below plays a similar role here.

[10]I assume that the integral on the right side of (24) is finite. In the case that $y(n) = An^\alpha$ is Cobb-Douglas, a necessary and sufficient condition to ensure this is that $\frac{1}{\eta} + \alpha > 1$, so that $\nu^{\frac{1-\eta}{\eta}} y'(\nu) = A\nu^{\frac{1}{\eta}+\alpha-2}$ is integrable near $\nu = 0$. Since $\eta < 1$ and $\alpha > 0$, the condition is always satisfied in this case.

Assuming that the wage bill for a small firm is finite, so that $nw_\alpha(n)$ remains finite as $n \to 0^+$, it is immediate that the constant of integration c in (24) is zero, so that in fact

$$w_\alpha(n) = (1-\eta)rV_\alpha^u + n^{-\frac{1}{\eta}} \int_0^n \nu^{\frac{1-\eta}{\eta}} y'(\nu)\, d\nu. \tag{25}$$

Since (25) takes a form that is independent of α, I therefore impose that

$$w(n) = (1-\eta)rV^u + n^{-\frac{1}{\eta}} \int_0^n \nu^{\frac{1-\eta}{\eta}} y'(\nu)\, d\nu. \tag{26}$$

as the key wage-determination equation of the SZ bargaining procedure, taken as given by firms and workers.[11] An alternative formulation of this equation that may be more intuitive is that

$$w(n) = (1-\eta)rV^u + \eta \frac{\int_0^n \nu^{\frac{1-\eta}{\eta}} y'(\nu)\, d\nu}{\int_0^n \nu^{\frac{1-\eta}{\eta}}\, d\nu};$$

that is, the wage of a worker at a firm employing n workers is a weighted average of the flow outside option, rV^u, and a term that is itself a weighted average of all the inframarginal products, $y'(\nu)$, for $\nu \in (0, n)$. Differentiating (26) with respect to n immediately implies also that

$$w(n) + \eta n w'(n) = \eta y'(n) + (1-\eta)rV^u. \tag{27}$$

Note finally that (21) continues to hold.

As in the constrained efficient and PE models, it is apparent that the optimal hiring policy of a firm takes a bang-bang form where all vacancy posting is done at entry. Denote by n^{SZ} the target employment level. An analogous argument to that from the PE case in Section 4 shows that

$$J(n) = \begin{cases} -(n^{SZ} - n)\frac{\gamma}{q(\theta)} + J(n^{SZ}) & \text{if } n < n^{SZ} \\ \frac{y(n) - nw(n)}{r+\delta} & \text{if } n \geq n^{SZ}. \end{cases} \tag{28}$$

The target hiring level n^{SZ} is chosen by firms to maximize their HJB value $J(n)$, taking into account the vacancy posting costs. Optimality requires the smooth pasting condition

$$J'(n^{SZ}) = \frac{\gamma}{q(\theta)}. \tag{29}$$

Using (28) this immediately implies that

$$\frac{\gamma}{q(\theta)} = \frac{y'(n^{SZ}) - w(n^{SZ}) - n^{SZ} w'(n^{SZ})}{r+\delta}. \tag{30}$$

When setting n^{SZ}, the firm takes account of the effect of its marginal hiring on wages, shown by the presence of the $n^{SZ} w'(n^{SZ})$ term.

Next, note that the relevant HJB equations for the worker take the form

$$rV(n^{SZ}) = w(n^{SZ}) + \delta \left[V^u - V(n^{SZ})\right] \tag{31}$$

$$rV^u = b + f(\theta)\left[V(n^{SZ}) - V^u\right]. \tag{32}$$

[11] Note that this wage equation takes the same form as in other papers using the SZ framework, such as Acemoglu and Hawkins (2010), Cahuc, Marque and Wasmer (2008), and Elsby and Michaels (2010). However, in those papers, unlike here, the wage equation can be proved to have this form as an implication of the (21) since firms do spend positive amounts of time with their workforce at a level different from its target, so that the heuristic derivation given here is unnecessary. (The same will apply in Section 7 when I generalize to the case where hiring is time-consuming.)

This is because even though a worker may not be the last hired at their new employer, they anticipate that $V(n) = V(n^{SZ})$ for all $n \in [0, n^{SZ}]$ since additional workers will be hired immediately without them being compensated for this.

Finally, the free entry condition that $J(0) = \kappa$ closes the model. According to (28), this takes the form

$$\kappa = J(0) = \frac{y(n^{SZ}) - n^{SZ} w(n^{SZ})}{r + \delta} - n^{SZ} \frac{\gamma}{q(\theta)}. \tag{33}$$

Equations (26), (30), (31), (32), and (33) are sufficient to characterize the key endogenous variables n^{SZ}, $w(n^{SZ})$, θ, $V(n^{SZ})$, and V^u. This is the substance of the folllowing proposition.

Proposition 5 (Existence and uniqueness under SZ bargaining)**.** *There is a unique equilibrium under SZ bargaining.*

Contrasting to the case of PE bargaining, under SZ bargaining firms do not set the efficient hiring target. More formally, in the following proposition I show that if all the other endogenous variables (in particular, the market tightness θ) coincide with their constrained efficient values, then n^{SZ} does not do so.

Proposition 6 (Inefficient hiring under SZ bargaining)**.** *Assume that θ takes on the value chosen by the planner in the constrained efficient allocation. Then $n^{SZ} > n^*$.*

The intuition for this result is precisely the standard over-hiring feature of SZ bargaining: firms wish to hire excessively, ceteris paribus, in order to drive down the wage they pay. Given this intuition, it is natural to conjecture that an appropriately-designed tax that corrects this distortion would be able to restore efficiency under SZ bargaining. There are many possible ways to design such a tax scheme; the following proposition indicates one.[12]

Proposition 7 (Efficient equilibrium under SZ bargaining with taxes and transfers)**.** *Suppose a government can tax the output $y(n)$ of firms at a linear tax rate τ and rebate the proceeds lump-sum to all active firms. Then there exists a choice of τ such that the resulting equilibrium is constrained efficient if the Hosios condition holds.*

Proposition 7 establishes that the only distortion arising from SZ bargaining is at the intensive margin. (However, of course, the Hosios condition is still required so that entry can be efficient.) Note that it would not be sufficient if the social planner simply regulated firm size, so that firms were constrained to set their employment level equal to n^*, but wages were still determined by (26); in that case it is possible to construct numerical examples in which the equilibrium is not constrained efficient, even under the Hosios condition, since in that case there is no mechanism to set wages at the level required to generate efficient entry, as there is in Proposition 7 when firms optimally choose employment under the tax.

Under PE bargaining, by contrast, it is intuitive that the optimal tax rate would be zero if the Hosios condition holds: according to Proposition 4, the equilibrium is already constrained efficient with $\tau = 0$ in this case.

6 Distinguishing Privately Efficient and Stole-Zwiebel Bargaining in Data

As shown at the end of the previous section, under SZ bargaining there can exist welfare-improving distortionary tax and transfer schemes, because of the over-hiring feature of the model. By contrast,

[12]Note: I conjecture that the converse of Proposition 7 also holds, that in general there is no constrained efficient equilibrium if the Hosios condition does not hold. The proof is left for a future version of this paper.

under PE bargaining, zero taxation can be optimal. An important question, therefore, is whether an econometrician can distinguish the nature of bargaining from data. In this section, I show that if the econometrician does not have access to data on the production function or on entry costs paid by firms, the bargaining protocol is not identified. That is, given data generated by a PE bargaining economy, I can find a (differently-parameterized) SZ economy in which labor market outcomes such as the unemployment and job-finding rates, the vacancy-filling rate, and the HJB values of unemployed workers and of employed workers, are all identical.

For the sake of concreteness, in this section I will assume that the production function y takes the Cobb-Douglas form $y(n) = An^{\alpha}$. (The key result of the section, Proposition 9 below, can also be established for non-Cobb-Douglas production functions - in fact, Proposition 11 establishes it in much greater generality in Section 8 below.) Then under PE bargaining, equilibrium is characterized by the following equations (specializations of equations (14), (16), and (19)):

$$(r+\delta)\kappa = A(1-\alpha){n^{PE}}^{\alpha}$$

and

$$\begin{aligned}\frac{\gamma}{1-\eta}\left[\eta\theta + \frac{r+\delta}{q(\theta)}\right] &= A\alpha {n^{PE}}^{\alpha-1} \\ &= \frac{A^{\frac{1}{\alpha}}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{((r+\delta)\kappa)^{\frac{1-\alpha}{\alpha}}} - b\end{aligned}$$

Under SZ bargaining, on the other hand, equilibrium is characterized by the following equations (specializations of (26), (30), (31), (32), and (33)):

$$\begin{aligned} w(n) &= (1-\eta)rV^u + \eta\phi A\alpha n^{\alpha-1} \\ (r+\delta)\kappa &= (1-\eta)\phi A(1-\alpha){n^{SZ}}^{\alpha} \\ \frac{\gamma}{1-\eta}\left[\eta\theta + \frac{r+\delta}{q(\theta)}\right] &= \phi A\alpha {n^{SZ}}^{\alpha-1} \\ &= (1-\eta)^{\frac{1-\alpha}{\alpha}}\phi^{\frac{1}{\alpha}}\frac{A^{\frac{1}{\alpha}}\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{((r+\delta)\kappa)^{\frac{1-\alpha}{\alpha}}} - b\end{aligned}$$

Here $\phi = (\alpha\eta + 1 - \eta)^{-1}$; observe that although $\phi > 1$, we have that $(1-\eta)\phi < 1$ and $(1-\eta)^{\frac{1-\alpha}{\alpha}}\phi^{\frac{1}{\alpha}} < 1$. This implies the following proposition.

Proposition 8 (Comparison of PE and SZ economies with unchanged parameters)**.** *Conditional on all parameters, the economy with PE bargaining and the economy with SZ bargaining can be compared as follows.*

- $n^{SZ} > n^{PE}$;
- θ *is lower under SZ bargaining than under PE bargaining.*

This is a 'theorist's comparative static.' Conditional on changing only the bargaining protocol from PE to SZ, but keeping all parameters the same, employment per firm increases, but labor market tightness decreases. (The first effect is reminiscent of the over-hiring predicted by Proposition 6, but note that that Proposition conditions on θ being efficient. In fact θ falls since the inefficient hiring process reduces the amount of firm entry consistent with the free entry condition.)

As argued in the introduction to this section, a perhaps more important question is whether an econometrician can identify which bargaining protocol applies. The following Proposition suggests that the ability to do this using worker data alone is limited.

Proposition 9 (Non-identification of PE and SZ bargaining)**.** *Suppose that an econometrician has access to data on all parameters of the model with the exception of A and κ. Then wages, unemployment, market tightness, employment per firm, and the replacement ratio for unemployed workers are not sufficient information to distinguish whether the underlying economy used PE or SZ bargaining.*

Proof. Suppose that the parameters A and κ of the PE bargaining economy consistent with the listed set of moments are A_{PE} and κ_{PE}. Define A_{SZ} and κ_{SZ} according to

$$A_{SZ} = \frac{1}{\phi} A_{PE} = (1 - \eta + \alpha\eta) A_{PE} \qquad \text{and} \qquad \kappa_{SZ} = (1-\eta)\kappa_{PE}.$$

Then it is immediate that

$$(A/\kappa)_{PE} = (1-\eta)\phi(A/\kappa)_{SZ} \tag{34}$$

and

$$\left(\frac{A^{\frac{1}{\alpha}}}{\kappa^{\frac{1-\alpha}{\alpha}}}\right)_{PE} = (1-\eta)^{\frac{1-\alpha}{\alpha}} \phi^{\frac{1}{\alpha}} \left(\frac{A^{\frac{1}{\alpha}}}{\kappa^{\frac{1-\alpha}{\alpha}}}\right)_{SZ} \tag{35}$$

Then the moments mentioned in the statement of the Proposition will coincide exactly in data generated by a model with PE bargaining and parameters A_{PE} and κ_{PE}, and in a model with SZ bargaining and parameters A_{SZ} and κ_{SZ} (and other parameters unchanged). A formal proof will be provided in the Appendix in a later version of the paper. □

[It is worth remarking that extending the proof of Proposition 9 extends easily to the case of non-Cobb-Douglas production functions; this will be done formally in the next draft of this paper.]

Equation (34) requires that A/κ be higher under SZ bargaining than under PE bargaining. This is intuitive: the hiring distortion endemic to SZ bargaining reduces productive efficiency; to be consistent with observed wages and market tightness data, the econometrician estimating a model with SZ bargaining would conclude that A/κ is higher than would her counterpart estimating a PE bargaining model on the same dataset. Equation (35) then gives the appropriate adjustment to κ to ensure that the free entry condition holds in both models.

It is worth remarking that the standard calibration strategy for frictional labor market models, as in Shimer (2005), gives rise to the kind of identification problem mentioned in Proposition 9. This calibration strategy is described in more detail in Subsection 8.5 below, especially in Table 1.

Proposition 9 is somewhat distressing for the normative implications of the model. If the bargaining protocol cannot be distinguished from labor market data, policy-makers have no guidance on whether distortionary taxation would be welfare-reducing or potentially welfare-enhancing. One might therefore wonder what kind of data would aid in distinguishing the two models. Simply, we need information on productivity or entry costs directly, or on firm's profitability, or on the share of output paid in entry costs or wages. The problem of identification relies on being able to identify A or κ from additional information beyond the targets mentioned in Table 1. Information on A or κ directly clearly suffices. Information on the share of output paid in entry costs suffices also (because it can be shown that this equals $1 - \alpha$ under PE bargaining but is greater under SZ bargaining). Another possible source of identification of the bargaining protocol would come from the response of the model to changes in distortionary tax rates.

In the next two sections of the paper, I generalize the basic model studied up to now for two reasons. The first is to show how the PE bargaining environment may be applied to more general situations than the very simple environment studied thus far. Thus, in Section 7 I allow for time-consuming hiring, following Acemoglu and Hawkins (2010), while in Section 8 I allow for aggregate and idiosyncratic productivity shocks, following Elsby and Michaels (2010). The second is to show that the non-identification result mentioned above is in fact much more general than what is already established.

7 Generalization 1: Time-to-Hire

An important and interesting generalization of the current model arises from following Acemoglu and Hawkins (2010) by considering the case where hiring is time-consuming as well as expensive in terms of goods. This allows the implications for wage dynamics to be considered, and is more comparable to papers such as Hawkins (2006), Acemoglu and Hawkins (2010), and Kaas and Kircher (2010). It is also of interest to show that the main result of the paper, Proposition 9 extends to this case: otherwise one might conjecture that the result is somehow special to the assumption of 'bang-bang' hiring made in the benchmark model. For example, it could be the case that the growth paths of firms differ between the cases of PE and SZ bargaining, even if the target firm size is the same. In this section I show that this is not the case, and that in fact the non-identification result of Proposition 9 generalizes to this case also.

The analysis in this section will be briefer than in the preceding sections, since it parallels them closely, together with the analysis from Acemoglu and Hawkins (2010).

Following the analysis in that paper, I assume that a firm can post v vacancies at any time at flow cost $c(v)$; this results in a flow rate of hiring by the firm of $qv = q(\theta)v$ per unit time. I assume that $c(v)$ is strictly increasing, continuously differentiable, strictly convex, and satisfies the Inada conditions $\lim_{v\to 0^+} c'(v) = 0$ and $\lim_{v\to +\infty} c'(v) = +\infty$. (This ensures that the first-order conditions for the choice of v are necessary and sufficient.)

I also allow for additional generality in this section by assuming that at Poisson rate s, independent across workers and over time, each worker is hit by a shock that forces them to leave the firm and return to unemployment. The firm and its other workers continue their relationship.

7.1 SZ bargaining

First I consider the case of SZ bargaining, since it parallels more closely the environment developed in Acemoglu and Hawkins (2010). The Hamilton-Jacobi-Bellman equation for the value of a firm employing n workers is

$$(r+\delta)J(n) = y(n) - nw(n) + \max_v \left[-c(v) + qvJ'(n)\right] - snJ'(n). \tag{36}$$

The free-entry condition requires that $J(0) = \kappa$. The first-order condition for the optimal choice of how many vacancies to post is

$$c'(v) = qJ'(n); \tag{37}$$

denote the solution (which is unique conditional on J by the convexity of c) by $v(n)$. Then differentiating the HJB equation with respect to n and applying the envelope theorem, it follows that

$$(r+\delta+s)J'(n) = y'(n) - w(n) - nw''(n) + \left[qv(n) - sn\right] J''(n). \tag{38}$$

The HJB equation for a worker employed at such a firm is

$$rV(n) = w(n) + \left[qv(n) - sn\right] V'(n) + (\delta+s)\left[V^u - V(n)\right],$$

or equivalently

$$(r+\delta+s)\left[V(n) - V^u\right] = w(n) - rV^u + \left[qv(n) - sn\right] V'(n). \tag{39}$$

If we assume the usual SZ bargaining equation $\eta J'(n) = (1-\eta)\left[V(n) - V^u\right]$ and its corollary $\eta J''(n) = (1-\eta)V'(n)$, it follows immediately from (38) and (39) that

$$\eta\left[y'(n) - w(n) - nw'(n)\right] = (1-\eta)\left[w(n) - rV^u\right],$$

or equivalently

$$w(n) + \eta n w'(n) = \eta y'(n) + (1-\eta) r V^u. \tag{40}$$

(40) takes the same form for the wage equation that arose in Section 5 above, and under the same assumptions on parameters made in that section, the unique solution is

$$w(n) = (1-\eta) r V^u + n^{-\frac{1}{\eta}} \int_0^n \nu^{\frac{1-\eta}{\eta}} y'(\nu)\, d\nu. \tag{41}$$

For comparison with the case of PE bargaining to be studied below, it's useful to substitute from the wage equation (41) into the firm's HJB equation (36) to observe that

$$(r+\delta)J(n) = y(n) - n^{1-\frac{1}{\eta}} \int_0^n \nu^{\frac{1}{\eta}-1} y'(\nu)\, d\nu - n(1-\eta) r V^u + \max_v \left[-c(v) + qvJ'(n)\right] - snJ'(n). \tag{42}$$

To close the model, one needs the HJB equation for an unemployed worker. Unlike the case considered in Section 5, this equation does not take the simple form given by (32), since the value of being hired by a firm depends on the number of incumbent employees at that firm, rather than equalling $V(n^{SZ})$ for sure. Instead, denoting the distribution of firms over employment sizes by $G(n)$, we have that

$$rV^u = b + \theta q(\theta) \frac{\int \left[V(n) - V^u\right] v(n)\, dG(n)}{\int v(n)\, dG(n)}.$$

The form of this equation arises because a worker contacts a firm of size n in proportion to the product of the measure of vacancies posted by such firms, $v(n)$, with the measure of such firms in the population, $G(n)$; when a contact occurs, the gain to the worker is $V(n) - V^u$. It is useful to use the SZ bargaining equation to substitute for this capital gain term as equal to $\frac{\eta}{1-\eta} J'(n)$, so that the previous equation becomes

$$rV^u = b + \theta q(\theta) \frac{\int \frac{\eta}{1-\eta} J'(n) v(n)\, dG(n)}{\int v(n)\, dG(n)}. \tag{43}$$

Finally, to complete the characterization of the equilibrium, one needs to characterize the firm size distribution. Following Acemoglu and Hawkins (2010), one can show that the steady-state distribution $G(\cdot)$ admits a continuously differentiable density $g(\cdot)$, which satisfies

$$\frac{g'(n)}{g(n)} = \frac{s - \delta - qv'(n)}{qv(n) - sn} \tag{44}$$

on the interval $[0, n^*]$, where n^* is defined to be that n at which $qv(n) = sn$, at which the firm's hiring is just equal to its loss of workers to exogenous separation. (If $s = 0$ then $n^* = +\infty$.) This completes the set of equations necessary to characterize of the equilibrium under SZ bargaining.

For a proof of existence and uniqueness of equilibrium under SZ bargaining, the reader is referred to Acemoglu and Hawkins (2010).

7.2 PE bargaining

The second part of the analysis is to solve for the equilibrium of the model under PE bargaining. The flow path of payments to a worker is not precisely determined under PE bargaining; what is important are present values, rather than their implementation through potentially stochastic time paths of payments. In the case of degenerate hiring dynamics, I resolved this issue by assuming that wages were constant within an employment relationship. Since all workers, no matter whether hired first or last, were in that case paid the same surplus over unemployment (that is, $V(n)$ was independent of $n \in [0, n^*]$), this meant that the firm paid a single constant wage to all its workers (a wage which, under the hypotheses of Proposition 9, equalled the wage paid by a firm in a differently-parameterized SZ equilibrium). In the case considered now, where hiring is time-consuming, workers hired by small firms are able to extract a greater gain since $S(n)$ is concave in n. It follows that if the payment of promised present values to workers was implemented by a constant wage payment over time, then wages would have to be different across workers in the firm. Rather than making this assumption, I instead assume that a flow transfer payment $t_h(n)$ is made by the firm to each of its n existing workers when a hire occurs to pay them the difference between their promised value $V(n)$ and the promised value of the newly hired workers, $V(n+\varepsilon) - V(n)$. A similarly-calculated payment $t_s(n)$ is made when a separation occurs. In this way, tenure at the firm (or equivalently, the size of the firm when a given worker was hired) is rendered no longer a state variable for the worker's HJB value, so that the only state variable for a firm or for the workers it employs is its current employment level, n. This gives rise to the HJB equations

$$(r+\delta)J(n) = y(n) - nw(n) + \max_v \left[-c(v) + qv(J'(n) - nt_h(n))\right] - sn(J'(n) + nt_s(n)) \tag{45}$$

$$rV(n) = w(n) + qv(n)\left[V'(n) + t_h(n)\right] - sn(V'(n) - t_s(n)) + \delta\left[V^u - V(n)\right] \tag{46}$$

which together imply that, where $S(n) = J(n) + n\left[V(n) - V^u\right]$,

$$(r+\delta)S(n) = y(n) - nrV^u - c(v(n)) + (qv(n) - sn)(1-\eta)S'(n) \tag{47}$$

(To establish this equation, it is necessary to observe that because of the PE bargaining assumption that $V(n) - V^u = \eta S'(n)$, it follows that $S'(n) = (J(n) + n(V(n) - V^u))' = J'(n) + nV'(n) + (V(n) - V^u) = J'(n) + nV'(n) + \eta S'(n)$, so that $(1-\eta)S'(n) = J'(n) + nV'(n)$.)

The first-order condition characterizing optimal vacancy posting is

$$c'(v(n)) = q(1-\eta)S'(n). \tag{48}$$

There is a free-entry condition $S(0) = \kappa$. Also, analogously to the case of SZ bargaining, closing the model requires knowing the HJB equation for an unemployed worker, which can be written

$$\begin{aligned} rV^u &= b + \frac{\int \left[V(n) - V^u\right] v(n)\, dG(n)}{\int v(n)\, dG(n)} \\ &= b + \frac{\int \eta S'(n) v(n)\, dG(n)}{\int v(n)\, dG(n)}. \end{aligned} \tag{49}$$

Finally, as in the case of SZ bargaining, equation (44) characterizes the density of the firm size distribution.

An analysis of existence and uniqueness can be given along the same lines as that for the equilibrium under SZ bargaining, given in Acemoglu and Hawkins (2010); the details are omitted. [DETAILS TO BE PROVIDED IN APPENDIX IN LATER DRAFT]

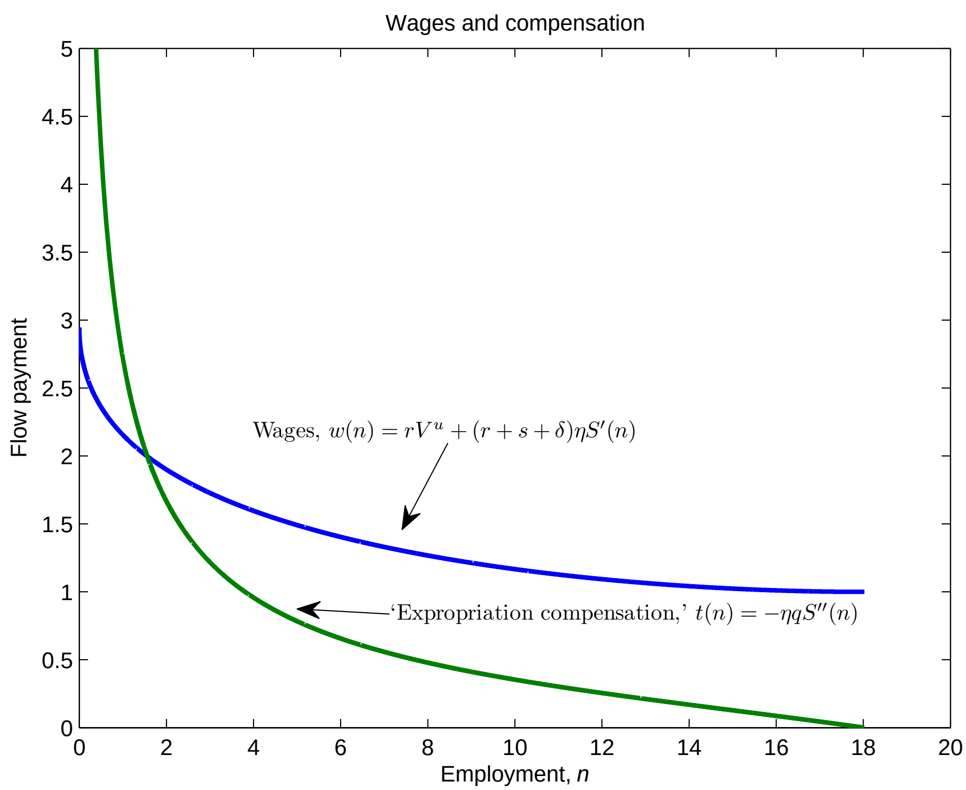

Figure 1: Wages

To complete the discussion of the model with PE bargaining, I note that the differential equation system implied by (47), together with the other equations above, can be solved to generate a prediction for wages. [DETAILED EXPLANATION TO BE ADDED IN THE APPENDIX IN A LATER DRAFT] This is illustrated in Figure 1. Note that under PE bargaining, the value of a newly-hired worker is determined, but the time path of payments is not. Figure 1 shows two possibilities.

First, one might want to assume that each worker is paid a constant wage within his employment spell at a firm. In this case the wage of a worker hired when employment at the firm was n is indicated by the value $w(n)$ shown on the flatter curve in Figure 1, labeled 'Wages.' Workers hired when the firm was smaller are paid constant, higher wages forever.

An alternative arrangement is that transfers are used so as to make incumbent workers always have the same Bellman value as do new hires, as considered in the analysis above. In this case the flow payment to a worker at a firm which currently has n workers consists of two components: the wage indicated in the previous paragraph, $w(n)$, together with an additional transfer to compensate the worker for the fact that he is losing his bargaining power and wages are declining over time. Thus the flow payment to a worker is given by the sum of $w(n)$ and $t(n)$, labeled 'Expropriation compensation, $t(n)$' in the Figure. Note that in this case, as the firm reaches its target hiring size, $t(n) \to 0$. Note also that both components of the wage are declining over time as n increases as the firm hires more workers.

There are of course other possible schemes for distributing the payment of wages over time, while realizing the same promised value (nothing is special about the two methods considered above, other than ease of explanation).

7.3 Non-Identification

The similarity between the form of the equations defining equilibrium under SZ and under PE bargaining is notable. In fact, this similarity is enough to establish the following result, which generalizes Proposition 9 above to the case of time-consuming hiring considered in this section.

Proposition 10 (Non-identification of PE and SZ bargaining with time-consuming hiring)**.** *Suppose that an econometrician has access to data on all parameters of the model with the exception of the production function $y(\cdot)$ and the entry cost κ. Then wages, unemployment, market tightness, employment per firm, and the replacement ratio for unemployed workers are not in general sufficient information to distinguish whether the underlying economy used PE or SZ bargaining.*

The proof of Proposition 10 in the Appendix is constructive, in the sense that given an equilibrium in an economy with PE bargaining, I change only the production function and the entry cost, according to

$$y_{SZ}(n) = y_{PE}(n) - n^{1-\frac{1}{\eta}} \int_0^n \nu^{\frac{1}{\eta}-1} y'_{PE}(\nu)\, d\nu \kappa_{SZ} \qquad = (1-\eta)\kappa_{PE}.$$

and generate a new economy in which the original allocation is now an equilibrium under SZ bargaining.

Note that because of the non-uniqueness of the prediction of the PE bargaining model for the time path of wages within an employment relationship, strictly speaking Proposition 10 guarantees only that the Hamilton-Jacobi-Bellman values of workers should be the same in the two economies. However, of course, under PE bargaining, the time path of wages is indeterminate: because of the full commitment assumption, all that matters for the behavior of all other endogenous variables in the economy is the behavior of HJB values, and not of the path of the actual payments that generate these values. Deducing promised values from data of course requires that the econometrician have a model for backing out these unobserved values from data on wages themselves. This means that any attempt to distinguish the SZ and PE bargaining models based on wage data also requires auxiliary assumptions on the mapping between wages and HJB values in the PE bargaining case. Any test must therefore also be a joint test of the validity of the model for wages that is applied. It is thus probably easier to interpret Proposition 10 as guaranteeing that the behavior of unemployment, labor market tightness, and the firm size distribution are not sufficient to identify the type of bargaining that is occurring, because of this difficulty in modeling the dynamics of wages.

[THIS SECTION TO BE EXPANDED]

8 Generalization: Aggregate and Idiosyncratic Productivity Shocks

This section provides another variation on the theme, commenced in the preceding section, of showing that the conclusion of Proposition 9 does not depend on the special assumptions made there for the sake of simplicity. In this section, I generalize the environment considered in a different way, and allow for idiosyncratic shocks to firms' productivity, and, in Proposition 11 below, I show that an analogous non-identification result again holds.

One can view the results to be presented here simply as a verification that the intuition provided by Proposition 9 is more general than the specific environment studied there. However, the model studied in this section is also of independent interest since I show how to solve tractable cases of a model with bargaining between workers and large firms when there are productivity shocks nearly in closed form. By comparison, previous authors working with models of this type, for example, Elsby and Michaels (2010), have needed to resort to numerical approximation methods (Krusell and Smith, 1998) to solve their models. This allows me to re-examine the question of whether models with large firms employing multiple workers generate larger fluctuations in key aggregate labor market variables such as market tightness and the unemployment rate in response to aggregate productivity shocks. Elsby and Michaels (2010) assert that this is the case. Preliminary results suggest that their results are somewhat fragile.

The environment is the same as in the benchmark model, with two alterations, one technical and one substantive.

The substantive modification is that firms' labor productivity is affected by both aggregate and idiosyncratic shocks. When the aggregate productivity shock is p and the idiosyncratic productivity shock is π, a firm with n employees produces flow output of $y(n; p, \pi)$, assumed to be increasing in p and in π. (For example, think of the case $y(n; \pi, p) = \pi p \hat{y}(n)$ for a fixed strictly increasing, strictly concave function $\hat{y}(\cdot)$ satisfying the usual Inada conditions.)

I allow the aggregate productivity shock p to take m_p possible values, $p_1 < p_2 < \ldots < p_{m_p}$. When aggregate productivity is p_i, the probability that its value in the following period is $p_{i'}$ is denoted $\lambda_{i,i'}$. Of course, $\sum_{i'=1}^{m_p} \lambda_{i,i'} = 1$ for all i. Analogously, I allow the idiosyncratic productivity shock π to take m_π possible values, $\pi_1 < \pi_2 < \ldots < \pi_{m_\pi}$. Conditional on the current idiosyncratic shock π_j, the probability the idiosyncratic shock takes the value $\pi_{j'}$ in the following period is denoted $\mu_{j,j'}$.[13] Again, $\sum_{j'=1}^{m_\pi} \mu_{j,j'} = 1$ for all j. I assume that both processes are ergodic; denote the invariant distributions by $F_p(\cdot)$ and $F_\pi(\cdot)$.

Since firms are now subject to aggregate and idiosyncratic productivity shocks, they may wish to lay workers off in the event of a sufficiently negative shock. I therefore assume that firms are

[13] Note that for the sake of simplifying the notation, it is implicitly assumed here that the process for the idiosyncratic productivity shock is independent of the value of the aggregate productivity shock. No difficulty is created if this is not true: for example, the argument carries through unchanged if $\mu_{j,j'}$ in fact is a function of i.

able to lay workers off at constant marginal cost $c \geq 0$. (I allow for $c = +\infty$, in which case firing is impossible.)

The technical modification is that I now assume that time is discrete. The assumption of discrete time is needed for a technical reason that I comment further on below. Denote the discount factor by β. I need to specify the order of events within each period, which is as follows. First, the aggregate productivity shock p and the idiosyncratic productivity shocks π for each firm are drawn and publicly observed. Next, each firm decides how many vacancies to post (at constant marginal cost γ per vacancy) and how many workers to fire (at constant marginal cost c per worker fired). Next, matching occurs; a firm that posted v vacancies hires v/q workers, where q is the ratio of the measures of total vacancies and total unemployed workers. Finally, production occurs.

It's necessary to specify what the initial idiosyncratic productivity draw of a new entrant firm is. I assume, somewhat arbitrarily, that the initial idiosyncratic productivity shock p is drawn from the ergodic distribution $F_\pi(p)$.[14]

Denote by $\theta_i(t)$ the market tightness at date t when the aggregate productivity shock is i; I use this redundant notation to prepare for the special case in Subsection 8.4 below in which the dynamics will be independent of t. Write $q_i(t)$ for $q(\theta_i(t))$.

8.1 PE bargaining

Denote by $J_{ij}(n,t)$ the Bellman value of a firm with n employed workers at date t when the aggregate productivity shock is p_i and its idiosyncratic productivity shock is π_j. Denote by $V_{ij}(n,t)$ the Bellman value of a worker at such a firm. Denote by $V_i^u(t)$ the Bellman value of an unemployed worker. The Bellman equation governing the law of motion of the match surplus generated by a firm and n employed workers, $S_{ij}(n) \equiv J_{ij}(n,t) + n\left[V_{ij}(n,t) - V_i^u(t)\right]$, is most easily seen by writing the Bellman equations for the firm and for the workers separately. Rather than keep track of the distribution of promised values for existing workers, I assume that the firm pays transfers to each of its incumbent workers whenever its aggregate or idiosyncratic productivity changes and whenever it hires or fires workers. Denote the transfer made when aggregate productivity changes from p_i to p_i' and idiosyncratic productivity from π_j to $\pi_{j'}$ by $d^p(i,j,i',j',n,t)$. Denote the transfer made whenever a new worker is hired by the firm by $d^h(i,j,n,t)$, and that made whenever a new worker is fired by the firm by $d^f(i,j,n,t)$. I assume these transfers are chosen such that at each time, incumbent workers have, after the transfer, the same Bellman value as a newly-hired worker would. (This is parallel to the treatment of wage payments in Section 7, and the same comments made there about non-uniqueness of wage dynamics also apply here; as in that analysis, this is a convenient assumption on the time path of payments from firms to workers to make for notational purposes, and under the full commitment assumption, makes no difference to the dynamics of real variables such as labor market tightness or entry.) This ensures that the notation I introduced at the beginning of the paragraph, implying that the current employment level at a firm is a sufficient statistic for its Bellman value and for those of its employees, is appropriate.

Denote by $\bar{n}_{ij}(t)$ the current 'firing target' of a firm with productivity state (p_i, π_j) at date t, in the sense that a firm with greater than $\bar{n}_{ij}(t)$ workers will instantaneously fire all additional workers to reduce its employment to this level. ($\bar{n}_{ij}(t)$ is infinite if $c = +\infty$.) Denote by $n_{ij}^*(t)$ the current 'hiring target' for a firm in state (p_i, π_j) at date t, in the sense that a firm with fewer than $n_{ij}^*(t)$ workers will instantaneously hire enough workers to reach this level. The existence of such targets follows from the constant marginal costs of firing and hiring.

[14] An alternative would be to suppose the initial productivity shock was drawn randomly from some other distribution supported on $\{p_i\}$; this would complicate the notation without affecting anything substantial. (For example, I could assume, following Mortensen and Pissarides (1994), that the initial productivity draw is p_m for sure.)

At a firm that is neither hiring nor firing (so that $n^*_{ij}(t) \leq n \leq \bar{n}_{ij}(t)$), the Bellman equations for the value of the firm and of its employees are

$$J_{ij}(n,t) = y_{ij}(n) - nw_{ij}(n,t) + \beta(1-\delta)\sum_{i'}\sum_{j'}\lambda_{i,i'}\mu_{j,j'}\left[J_{i'j'}(n,t+1) - nd^p(i,j,i',j',n,t)\right] \quad (50)$$

and

$$V_{ij}(n,t) = w_{ij}(n,t) + \beta\sum_{i'}\lambda_{i,i'}\left[\delta V^u_{i'}(t+1) + (1-\delta)\sum_{j'}\mu_{j,j'}\left[V_{i'j'}(n,t+1) + d^p(i,j,i',j',n,t)\right]\right] \quad (51)$$

Adding the previous two equations – of course, the wage payments and transfers cancel – generates the Bellman equation for the joint surplus, $S_{ij}(n,t) \equiv J_{ij}(n,t) + n\left[V_{ij}(n,t) - V^u_i(t)\right]$:

$$S_{ij}(n,t) = y_{ij}(n,t) - n\left[V^u_i(t) - \beta\sum_{i'}\lambda_{i,i'}V^u_{i'}(t+1)\right] + \beta(1-\delta)\sum_{i'}\sum_{j'}\lambda_{i,i'}\mu_{j,j'}S_{i'j'}(n,t+1). \quad (52)$$

Note that the term $V^u_i(t) - \beta\sum_{i'}\lambda_{i,i'}V^u_{i'}(t+1)$ is just the 'rental' value of a single period of being an unemployed worker. That is, it is the discrete-time analog of the rV^u in equations like (10).[15]

For $n < n^*_{ij}(t)$, the bang-bang hiring process ensures that

$$\begin{aligned}
J_{ij}(n,t) &= -\frac{1}{1-\eta}\frac{\gamma}{q_i(t)}(n^*_{ij}(t) - n) - nd^h(i,j,n,t) + J_{ij}(n^*_{ij}(t)) \\
V_{ij}(n,t) &= d^h(i,j,n,t) + V_{ij}(n^*_{ij}(t)) \\
S_{ij}(n,t) &= -\frac{1}{1-\eta}\frac{\gamma}{q_i(t)}(n^*_{ij}(t) - n) + S_{ij}(n^*_{ij}(t))
\end{aligned} \quad (53)$$

while for $n > \bar{n}_{ij}(t)$, the bang-bang firing process similarly establishes that

$$\begin{aligned}
J_{ij}(n,t) &= -c(n - \bar{n}_{ij}(t)) - nd^f(i,j,n,t) + J_{ij}(\bar{n}_{ij}(t)) \\
V_{ij}(n,t) &= d^f(i,j,n,t) + V_{ij}(\bar{n}_{ij}(t)) \\
S_{ij}(n,t) &= -c(n - \bar{n}_{ij}(t)) + S_{ij}(\bar{n}_{ij}(t))
\end{aligned} \quad (54)$$

Note that, unlike in the model without shocks studied in the first part of this paper, firms with employment greater than the target value associated with the current productivity shock arise in equilibrium. Firm hiring dynamics are as follows: an entrant firm immediately grows to the target size $n^*_{ij}(t)$ associated with the current productivity state (p_i, π_j). Then, whenever productivity changes such that the hiring target size shifts to a value $n^*_{i'j'}(t)$ greater than the firm's current number of workers n, it posts another burst of vacancies in order to grow instantaneously to the new target size. Whenever a sufficiently bad shock $(p_{i''}, \pi_{j''})$ occurs, the firm fires a measure of workers to reduce its employment to the firing target $\bar{n}_{i''j''}(t)$. The two targets are not equal because of the asymmetry of hiring and firing: hiring an additional worker when aggregate productivity is $q_i(t)$ has a marginal cost of $\frac{1}{1-\eta}\frac{\gamma}{q_i(t)}$, while firing a worker has marginal cost c. This gives rise to an (s,S) band of inaction $[n^*_{ij}(t), \bar{n}_{ij}(t)]$. If in fact firing had a marginal *benefit*, with $c = -\frac{1}{1-\eta}\frac{\gamma}{q_i(t)}$, the two targets would indeed be equal and the band of inaction would shrink to a single point.

[15] In the absence of shocks, so that $\lambda_{i,i} = 1$ and $\lambda_{i,i'} = 0$ for $i' \neq i$, this term would simplify to $(1-\beta)V^u_i(t)$, which makes the correspondence clearer. Out of steady state, the continuous-time version would be $rV^u(t) - \dot{V}^u(t)$.

The first-order condition for optimal hiring is, similarly to the version in the non-stochastic version of the model,

$$S'_{ij}(n^*_{ij}(t), t) = \frac{1}{1-\eta}\frac{\gamma}{q_i(t)}. \tag{55}$$

(I abuse notation slightly and denote differentiation with respect to the first argument using a prime.) The first order condition for optimal firing is, analogously,

$$S'_{ij}(\bar{n}_{ij}(t), t) = -c. \tag{56}$$

In the discrete time model with the timing assumption made above, the appropriate form of the privately efficient bargaining assumption (4) is the following

$$V_{ij}(n,t) - \left[b + \beta \sum_{i'} \lambda_{i,i'} V^u_{i'}(t+1)\right] = \eta S'_{ij}(n,t) \tag{57}$$

The term $b + \beta \sum_{i'} \lambda_{i,i'} V^u_{i'}(t+1)$ is the value of a worker who remains unemployed for all of the current period, and then begins to search for a job as of tomorrow. This is the appropriate outside option once the labor market has closed for the current period. Despite the heterogeneity of labor productivity, note that S' takes the same value, $\frac{1}{1-\eta}\frac{\gamma}{q_i(t)}$, at all firms which post any vacancies. This implies that $V_{ij}(n,t)$ is independent of the idiosyncratic productivity shock j.

The Bellman equation for unemployed workers is

$$V^u_i(t) = \theta_i(t) q_i(t) \mathbf{E}_j V_{ij}(n,t) + (1 - \theta_i(t) q_i(t)) \left[b + \beta \sum_{i'} \lambda_{i,i'} V^u_{i'}(t+1)\right], \tag{58}$$

where the expectation is taken over the productivity types of firms that post vacancies j. However, because $V_{ij}(n,t)$ does not depend on j, the expectation is unnecessary. Substituting from (57) into (58) and rearranging yields that

$$V^u_i(t) - \beta \sum_{i'} \lambda_{i,i'} V^u_{i'}(t+1) = b + \frac{\eta}{1-\eta}\gamma\theta_i(t). \tag{59}$$

Again, this simple form arises despite the heterogeneity of labor productivity because S' takes the same constant value, $\frac{1}{1-\eta}\frac{\gamma}{q_i(t)}$, at all firms which post any vacancies.

Substituting from (59) into (52) then establishes a simpler formula for the joint surplus $S_{ij}(n,t)$.

$$S_{ij}(n,t) = y_{ij}(n,t) - n\left[b + \frac{\eta}{1-\eta}\gamma\theta_i(t)\right] + \beta(1-\delta)\sum_{i'}\sum_{j'} \lambda_{i,i'}\mu_{j,j'} S_{i'j'}(n, t+1). \tag{60}$$

The free entry condition takes the form

$$\sum_j F(\pi_j) S_{ij}(0,t) \leq \kappa \tag{61}$$

for each aggregate productivity state p_i and each date t, with equality if there is a positive amount of entry at date t.

Denote the amount of entry in the economy by $e_i(t)$. Full definition of an equilibrium under PE bargaining in this economy should also incorporate the distribution of firm sizes, $G(n)$. Because of the simple hiring and firing dynamics, this distribution is determined only by the values of $\{n^*_{ij}\}_{ij}$ together with the (exogenous) stochastic process for productivity and (endogenous) behavior of firms to hire up to their hiring target whenever below that, fire down to their firing target whenever above that, and otherwise never change employment. I omit the details since they are standard.

8.2 SZ bargaining

Under SZ bargaining, the HJB equations for firms and workers are respectively given in the case $\geq n^*_{ij}(t) \leq n \leq \bar{n}_{ij}(t)$, by

$$J_{ij}(n,t) = y_{ij}(n) - nw_{ij}(n,t) + \beta(1-\delta)\sum_{i'}\sum_{j'}\lambda_{i,i'}\mu_{j,j'}J_{i'j'}(n,t+1) \tag{62}$$

and

$$V_{ij}(n,t) = w_{ij}(n,t) + \beta\sum_{i'}\lambda_{i,i'}\left[\delta V^u_{i'}(t+1) + (1-\delta)\sum_{j'}\mu_{j,j'}V_{i'j'}(n,t+1)\right] \tag{63}$$

The forms of these equations are identical to those which arise in the the case of privately efficient bargaining as equations (50) and (51), with the exception that no transfers are made when a change in productivity occurs.

The equation for SZ bargaining in this environment is

$$\eta J'_{ij}(n,t) = (1-\eta)\left(V_{ij}(n,t) - \left[V^u_i(t) - \beta\sum_{i'}\lambda_{i,i'}V^u_{i'}(t+1)\right]\right). \tag{64}$$

Applying an analogous argument to that made in equations (39) through (41), it follows immediately that the analogous equation to (41) holds, so that wages are given by

$$w_{ij}(n,t) = (1-\eta)\left[V^u_i(t) - \beta\sum_{i'}\lambda_{i,i'}V^u_{i'}(t+1)\right] + n^{-\frac{1}{\eta}}\int_0^n \mu^{\frac{1-\eta}{\eta}}y'(\nu)\,d\nu. \tag{65}$$

Similarly to (42), it then follows that for n in the band of inactivity, $[n^*_{ij}(t), \bar{n}_{ij}(t)]$, such that neither hiring nor firing occurs,

$$\begin{aligned} J_{ij}(n,t) = y_{ij}(n) - n^{1-\frac{1}{\eta}}\int_0^n \nu^{\frac{1}{\eta}-1}y'_{ij}(\nu)\,d\nu - n(1-\eta)\left[V^u_i(t) - \beta\sum_{i'}\lambda_{i,i'}V^u_{i'}(t+1)\right] \\ + \beta(1-\delta)\sum_{i'}\sum_{j'}\lambda_{i,i'}\mu_{j,j'}J_{i'j'}(n,t+1). \end{aligned} \tag{66}$$

For $n < n^*_{ij}(t)$, the bang-bang hiring process ensures that

$$\begin{aligned} J_{ij}(n,t) &= -\frac{\gamma}{q_i(t)}(n^*_i - n) + J_{ij}(n^*_{ij}(t),t) \\ \text{and} \qquad V_{ij}(n,t) &= V_{ij}(n^*_{ij}(t),t), \end{aligned}$$

while for $n > \bar{n}_{ij}(t)$, it similarly follows that

$$\begin{aligned} J_{ij}(n,t) &= -c(n - \bar{n}_{ij}(t)) + J_{ij}(\bar{n}_{ij}(t),t) \\ \text{and} \qquad V_{ij}(n,t) &= V_{ij}(\bar{n}_{ij}(t),t). \end{aligned}$$

The first-order condition for optimal hiring is, as in the non-stochastic version of the model,

$$J'_{ij}(n^*_{ij}(t),t) = \frac{\gamma}{q_i(t)}. \tag{67}$$

The first-order condition for optimal firing is, analogously,

$$J'_{ij}(\bar{n}_{ij}(t), t) = -c. \tag{68}$$

The Bellman equation for unemployed workers takes the same form as in the case of PE bargaining, equation (58). An similar argument to the PE case then establishes that, analogously to (59) in the case of PE bargaining,

$$V_i^u(t) - \beta \sum_{i'} \lambda_{i,i'} V_{i'}^u(t+1) = b + \frac{\eta}{1-\eta} \gamma \theta_i(t). \tag{69}$$

The free-entry condition is

$$\sum_j F(j) J_{ij}(0, t) \leq \kappa \tag{70}$$

for all (i, t), with positive amounts of entry only if equality holds. I again omit the details of characterizing entry $e_i(t)$ and the firm size distribution $G(\cdot)$.

8.3 Non-identification

The similarity of the equations defining the equilibrium under PE bargaining with those defining the equilibrium under SZ bargaining again makes it easy to establish the following Proposition, analogous to Proposition 9 and Proposition 10. Given an equilibrium of an economy with PE bargaining, it is easy to generate an equilibrium of an economy with SZ bargaining in which all labor market variables behave the same as in the original equilibrium.

Proposition 11 (Non-identification of PE and SZ bargaining with stochastic productivity)**.** *Suppose that an econometrician has access to data on all parameters of the model with the exception of the production function $y(\cdot)$ and the entry cost κ. Then wages, unemployment, market tightness, entry, the firm size distribution and the replacement ratio for unemployed workers are not in general sufficient information to distinguish whether the underlying economy used PE or SZ bargaining.*

The proof of Proposition 11 is almost identical to that of Proposition 10.

8.4 Constructive solution procedure

Proposition 11 is non-constructive, in the sense that it does not actually help in constructing an equilibrium with either PE or SZ bargaining if one does not already have a calculated equilibrium using the other bargaining process at hand to start with. However, it turns out that a constructive method of solution is available, and in this section I outline it. This is of independent interest since the exact solution of a dynamic model with bargaining and large firms is a contribution of this paper.

I solve for an equilibrium of an economy with PE bargaining; as pointed out by Proposition 11, constructing an equilibrium of a model with SZ bargaining can be done by constructing the equilibrium for a differently-parameterized economy with PE bargaining, so this is without loss of generality. In particular, the ability of the model to account for aggregate fluctuations will be the same in either case.

I look for an equilibrium in which market tightness $\theta_i(t)$ depends only on aggregate productivity $p = p_i$ and the hiring and firing targets $n^*_{ij}(t)$ and $\bar{n}_{ij}(t)$ depend only on aggregate productivity $p = p_i$ and idiosyncratic productivity $\pi = \pi_j$. One trivial case which ensures this will be the case is if there is no aggregate uncertainty (that is, $m_p = 1$), in which case any steady-state equilibrium has this property. In the presence of aggregate shocks, it is less clear that such an equilibrium exists. The problematic case arises when aggregate productivity $p = p_i$ changes to a new value $p_{i'}$ such that existing firms wish immediately to grow to larger sizes (for example, because $n^*_{i'j} > n^*_{ij}$ for each j). This means that incumbent firms post a positive measure of new vacancies in order to reach their desired target sizes instantaneously. If the number of vacancies required to do this is very large (say, because the productivity shock itself is very large), then market tightness will overshoot its conjectured value θ_i. However, if aggregate productivity moves only by a small amount, this problem need not arise. (Because a fraction δ of the existing firms are destroyed at the same time that the shock arrives, it is possible for all the remaining incumbents to grow somewhat. Thus, intuitively, an equilibrium of the conjectured form can exist only if aggregate productivity does not change too rapidly.)[16]

The basic equation required to characterize the economy is (60). In an equilibrium of the conjectured type, the time indices can be dropped to write

$$S_{ij}(n) = y_{ij}(n) - n\left[b + \frac{\eta}{1-\eta}\gamma\theta_i\right] + \beta(1-\delta)\sum_{i'}\sum_{j'}\lambda_{i,i'}\mu_{j,j'}S_{i'j'}(n). \tag{71}$$

I specialize for concreteness to the case of a Cobb-Douglas production function with multiplicative productivity shocks,

$$y_{ij}(n) = Ap_i\pi_j n^\alpha.$$

Substituting this functional form and differentiating (71) with respect to n establishes that if n lies within the range of inaction $[n^*_{ij}, \bar{n}_{ij}]$, then

$$S'_{ij}(n) = Ap_i\pi_j\alpha n^{\alpha-1} - \left[b + \frac{\eta}{1-\eta}\gamma\theta_i\right] + \beta(1-\delta)\sum_{i'}\sum_{j'}\lambda_{i,i'}\mu_{j,j'}S'_{i'j'}(n). \tag{72}$$

In addition, the bang-bang hiring and firing behavior of firms give that

$$S_{ij}(n) = -\frac{1}{1-\eta}\frac{\gamma}{q_i}(n^*_{ij}-n)+S_{ij}(n^*_{ij}) \text{ if } n \le n^*_{ij} \qquad \text{and} \qquad S_{ij}(n) = -c(n-\bar{n}_{ij})+S_{ij}(\bar{n}_{ij}) \text{ if } n > \bar{n}_{ij} \tag{73}$$

and the first-order conditions for optimality of n^*_{ij} and $\bar{n}_{ij}$ imply that

$$S'_{ij}(n) = \frac{1}{1-\eta}\frac{\gamma}{q_i} \text{ if } n \le n^*_{ij} \qquad \text{and} \qquad S'_{ij}(n) = -c \text{ if } n > \bar{n}_{ij}. \tag{74}$$

Supposing that the values of market tightness, q_i are known for each i. Using the matching function, one can solve for θ_i; for concreteness, I assume that the matching function $M(u,v)$ is also Cobb-Douglas, with $M(u,v) = Zu^\phi v^{1-\phi}$; in this case, $\theta_i = (Z/q_i)^{1/\phi}$. For each i and each k in $\{1, 2, \ldots, m_p\}$ and for each j and each l in $\{1, 2, \ldots, m_\pi\}$, substitute $n = n^*_{kl}$ and $n = \bar{n}_{kl}$

[16] Note that the difficulty arising here comes about because the firm size distribution – and in particular, the measure of already-active firms – is a state variable of the model. By contrast, in the benchmark Mortensen-Pissarides model with one-to-one matching between workers and firms, vacancies are only posted by firms which do not yet have any employees, and the measure of such vacancies is a jump variable which can respond instantaneously to aggregate shocks. Thus Shimer (2005) is able to construct the analogous equilibrium in the MP model to the one I study here no matter what his assumption on aggregate shocks.

Note also that this is the reason why it is important that I write the model in this section in discrete time, rather than in continuous time. In continuous time, with a jump process for aggregate productivity, an equilibrium of the type I consider will never exist. In continuous time, one would need to allow for smoothly changing productivity, but this requires a continuum of aggregate states, rather than the finite number I allow for.

into equations (72) and (74). This generates a system of $2m_p^2m_\pi^2 + 2m_pm_\pi$ linear equations in the $2m_p^2m_\pi^2 + 2m_pm_\pi$ variables $\left\{S'_{ij}\left(n^*_{kl}\right)\right\}_{i,j,k,l}$, $\left\{S'_{ij}\left(\bar{n}_{kl}\right)\right\}_{i,j,k,l}$, $\left\{{n^*_{kl}}^{\alpha-1}\right\}_{k,l}$, and $\left\{{\bar{n}_{kl}}^{\alpha-1}\right\}_{k,l}$. (There are $m_p^2m_\pi^2$ (i,j,k,l)-tuples, each of which generates precisely two equations, one using n^*_{kl} and one using $\bar{n}_{kl}$, except that an additional two equations is generated for tuples of the form (i,j,i,j) for which both (72) and (74) provide valid equations.) However, in order to generate the correct equations, it is necessary to know for each (i,j,k,l) whether $n^*_{kl} < n^*_{ij}$ (in which case a firm which begins a period with n^*_{kl} workers in productivity state (p_i,π_j) immediately hires a positive measure of workers and the first part of (74) applies), whether $n^*_{kl} \in [n^*_{ij}, \bar{n}_{ij}]$ (in which case neither hiring nor firing occurs and (72) applies), or whether $n^*_{kl} > \bar{n}_{ij}$ (in which case the firm fires a positive measure of workers and the second part of (74) applies). That is, the form of the equations is a function of the rank order of the terms of the tuple $L \equiv (n^*_{11}, n^*_{12}, \ldots, n^*_{1m_\pi}, n^*_{21}, \ldots, n^*_{2m_\pi}, \ldots, n^*_{m_pm_\pi}, \bar{n}_{11}, \bar{n}_{12}, \ldots, \bar{n}_{1m_\pi}, \bar{n}_{21}, \ldots, \bar{n}_{2m_\pi}, \ldots, \bar{n}_{m_pm_\pi})$.

Note that the solution to the set of equations defined above is unique conditional on the rank order of the elements of L. To see this, first suppose that for some (k,l), the value of n^*_{kl} is known and consider the set of m_pm_π equations given by (71) for $i = 1,2,\ldots m_p$ and $j = 1,2,\ldots m_\pi$ with $n = n^*_{kl}$. The assumption that the processes for p and π are ergodic ensures that in this set of linear equations in $\{S_{ij}(n^*_{kl})\}_{ij}$, the coefficient matrix is invertible. It also implies that for each (i,j) and each (i',j'), $S_{i'j'}(n^*_{kl})$ is increasing in $Ap_i\pi_j\alpha {n^*_{kl}}^{\alpha-1} - \left[b + \frac{\eta}{1-\eta}\gamma\theta_i\right]$, and therefore strictly decreasing in n^*_{kl}. For any (i,j) for which $n^*_{kl} \notin [n^*_{ij}, \bar{n}_{ij}]$, then replace the equation arising from (71) for the term $S'_{ij}(n^*_{kl})$ with the correct equation arising from (74). This equation is of the form $S'_{ij}(n^*_{kl}) = x$ for some constant x, and so adducing it instead of the equation arising from (71) does not affect the full rank of the coefficient matrix, nor the property that $S_{i'j'}(n^*_{kl})$ is strictly decreasing in n^*_{kl} for any (i',j') such that $n^*_{kl} \in [n^*_{i'j'}, \bar{n}_{i'j'}]$. Repeating this step as many times as necessary establishes that there is a unique solution for $\{S_{ij}(n^*_{kl})\}_{ij}$, with the same comparative static property in n^*_{kl} just mentioned. In particular, note that the value of $S'_{kl}(n^*_{kl})$ is decreasing in n^*_{kl}. Also, note that this solution depends only on the rank order of L and the value of n^*_{kl}. One can then proceed to use (74) to obtain the additional equation, so far unused, that $S'_{k,l}(n^*_{kl}) = \frac{1}{1-\eta}\frac{\gamma}{q_i}$, which is independent of n^*_{kl}; thus it follows that there is also a unique solution for n^*_{kl}. A similar argument then applies to establish that there is also a unique solution for $\left\{S'_{ij}(\bar{n}_{kl})\right\}_{ij}$ and for $\bar{n}_{kl}$. Repeating this argument for all possible tuples (k,l) establishes the claimed uniqueness property.

At this point the values of n^*_{ij} are known, so one can verify whether it is consistent with the original guess for the order of L. Assuming so, one can now calculate the values of $\{S_{ij}(n^*_{kl})\}_{i,j,k,l} \cup \{S_{ij}(\bar{n}_{kl})\}_{i,j,k,l}$ by applying a very similar procedure to that used already, but now applying the equations generated by (71) together with (73). This provides a system of $2m_p^2m_\pi^2$ linear equations in the $2m_p^2m_\pi^2$ variables $\{S_{ij}(n^*_{kl})\}_{i,j,k,l} \cup \{S_{ij}(\bar{n}_{kl})\}_{i,j,k,l}$. A similar argument to that given above establishes that this system of equations again has a unique solution. Moreover, it is immediate that for each (i,j,k,l), the values $S_{ij}(n^*_{kl})$ and $S_{ij}(\bar{n}_{kl})$ are strictly decreasing in each $q_{i'}$, for $i' = 1,2,\ldots,m_p$. (To see this, it is useful to recall that $\theta_{i'}$ is decreasing in $q_{i'}$.) For $\{q_{i'}\}_{i'}$ sufficiently small, the values of $S_{ij}(n^*_{kl})$ and $S_{ij}(\bar{n}_{kl})$ can be made large and positive for all (i,j,k,l), while if $\{q_{i'}\}_{i'}$ are sufficiently large, the values of $S_{ij}(n^*_{kl})$ and $S_{ij}(\bar{n}_{kl})$ can be made arbitrarily close to zero for all (i,j,k,l). If $m_p = 1$, the intermediate value theorem applied to this observation and the free entry condition (61) establishes global existence and uniqueness of equilibrium, and for $m_p > 1$ a proof of existence can be given using a version of the Brouwer fixed point theorem for monotone functions (Persson, 2005) (details to come). Independent of m_p, it establishes local uniqueness.

To summarize, the algorithm used is as follows.

1. Guess values for market tightness q_i for each $i = 1, 2, \ldots, m_p$; calculate $\{\theta_i\}_i$.

2. Guess the rank order of the elements of L.

3. Solve the $2m_p^2 m_\pi^2 + 2m_p m_\pi$ equations given by (72) and (74) for $\left\{S'_{ij}\left(n^*_{kl}\right)\right\}_{i,j,k,l}$, $\left\{S'_{ij}\left(\bar{n}_{kl}\right)\right\}_{i,j,k,l}$, $\left\{{n^*_{kl}}^{\alpha-1}\right\}_{k,l}$, and $\left\{{\bar{n}_{kl}}^{\alpha-1}\right\}_{k,l}$.

4. Verify whether the order of the solution for $\left\{n^*_{ij}\right\}_{ij} \cup \left\{\bar{n}_{ij}\right\}_{ij}$ is consistent with the guessed order from step 2. If not, guess a new order and return to step 3.

5. Solve the $2m_p^2 m_\pi^2$ equations given by (71) and (73) for $\left\{S_{ij}\left(n^*_{kl}\right)\right\}_{i,j,k,l}$, $\left\{S_{ij}\left(\bar{n}_{kl}\right)\right\}_{i,j,k,l}$.

6. Verify whether the free entry condition (61) holds with equality for every i. If not, guess a new set of market tightnesses $\{q_i\}_i$ and return to step 1.

It would be possible to search through every possible rank order of the elements of L; however, for $m_p + m_\pi$ greater than 3, this takes significant computational time (the number of possible rank orders of the $2(m_p + m_\pi)$ elements of L is, of course, $(2(m_p + m_\pi))!$, although a large number of these are not economically sensible – for example, we know that $n^*_{ij} < \bar{n}_{ij}$ for every (i, j), and it is also reasonable to conjecture that if productivity is persistent, in the sense that the expected value of idiosyncratic productivity next period is strictly increasing in idiosyncratic productivity this period, then $n^*_{ij} < n^*_{ij'}$ whenever $j < j'$). However, fortunately in practice it seems that the simple algorithm of using for a new guess in step 4 of the algorithm the order which is consistent with the solution for $\left\{n^*_{ij}\right\}_{ij} \cup \left\{\bar{n}_{ij}\right\}_{ij}$ found in step 3 quickly converges to an ordering that is consistent with the solution it generates. Next, in step 6, the m_p functions given by $(q_1, q_2, \ldots, q_{m_p}) \mapsto \sum_j F(\pi_j) S_{ij}(0, t) - \kappa$ are smooth and strictly increasing in each coordinate element, so that standard numerical algorithms for solution-finding work well (for example, MATLAB's `fsolve` command with the default algorithm).

8.5 Calibrated solution

I now calibrate the model described in the previous subsection and report its micro- and macroeconomic properties. The results described in this Section are preliminary and will surely change in later versions of this paper.

For the sake of comparability with the literature, I borrow parameters as closely as possible from the benchmark paper in the recent literature on the quantitative study of fluctuations in search and matching models, which is Shimer (2005).

First, the level of market tightness is intrinsically meaningless in the model. This choice is a normalization: multiplying the scale parameter in the matching function, Z, by $x^{\phi-1}$ and dividing the vacancy-posting cost parameter γ by x leads to an equilibrium which differs only in that for each i, the equilibrium value of θ_i is multiplied by x and the equilibrium value of q_i is divided by x. This leaves the matching rate for workers, $\theta_i q_i$, the cost of hiring for firms, $\frac{1}{1-\eta}\frac{\gamma}{q_i}$, and the flow value of unemployment, $b + \frac{\eta}{1-\eta}\gamma\theta_i$) unchanged. I arbitrarily choose to target a vacancy-unemployment ratio of 1. I also target an average unemployment rate of 6.87 percent, again consistent with Shimer (2005), along with a total separation rate of 0.033 at monthly frequency. (I measure unemployment at the very beginning of a period; those who find jobs during the period are counted as unemployed.) Next, following the spirit of the Shimer (2005) calibration of the Mortensen-Pissarides model, I choose b so that on average, 40 percent of the flow value of unemployment for an unemployed worker arises from the unemployment income b. I normalize the units of output so that in fact the flow value of unemployment is equal to 1 on average, which requires setting $b = 0.4$. (Note this is actually very slightly lower than the calibration used by Shimer, in which b was 40 percent of the average product of a match.) The targets just mentioned are matched by setting the efficiency parameter in the matching function as $Z = 0.467$ and choosing η and γ consistent with $\frac{\eta}{1-\eta}\gamma$ of 0.2.

I set the elasticity parameter in the matching function ζ to be 0.72, consistent with Shimer's calibration (which follows from estimating the slope of the Beveridge curve). In addition, I set the decreasing returns to labor in the production function (the parameter α in the Cobb-Douglas production function) to be $2/3 = 0.6667$. This parameterization is motivated by thinking of the only adjustable factor at the firm level as being labor, which gives a lower bound for the reasonable set of parameter values for α. (Note that this assumes, for example, that capital is totally fixed at the establishment level, which is true in the model if the only capital-like expenditure is the free-entry cost, but probably unreasonable as a characterization of the data. In a later version of the paper I will consider the effect of alternative strategies for choosing the value of α. (One possibility is to consider the fixed factor as managerial talent, as in a Lucas span of control model, which would suggest choosing α much closer to 1.)

The parameterization of the aggregate and idiosyncratic productivity shock processes are similar. Aggregate productivity takes m_p possible values. I set the transition probabilities so that when a productivity shock arrives, productivity changes by a single grid point; thus, $\lambda_{i,i'} = 0$ for $|i - i'| > 1$. I also set $\lambda_{i,i-1} = (i-1)\Delta_p$ and $\lambda_{i,i+1} = (m_p - i)\Delta_p$ for a fixed Δ_p and for each i; this implies that $\lambda_{i,i} = 1-(m_p-1)\Delta_p$ for each i. Analogously, I assume that there is a Δ_π such that idiosyncratic productivity follows an analogous process, with $\nu_{j,j-1} = (j-1)\Delta_\pi$, $\nu_{j,j+1} = (m_\pi - j)\Delta_\pi$, $\nu_{j,j} = 1 - (m_\pi - 1)\Delta_\pi$, and $\Delta_{j,j'} = 0$ for $|j - j'| > 1$. I assume that the shock values p_i are given by $p_i = p_0 e^{id_p}$ and $\pi_j = \pi_0 e^{jd_\pi}$, with constant log increments, and such that the median values of p and of π are both equal to 1. (This ensures that the overall level of productivity is determined by the coefficient A.)

I choose the parameters of the aggregate productivity process currently for numerical convenience, setting $m_p = 3$. I assume also that $\Delta_p = 0.01$, so that the probability of a productivity shock is 2 percent at monthly frequency, and that $d_p = 0.01$ also, so that the two extremal values of aggregate productivity differ by 1 percent from the median value. These parameters are chosen for convenience; as already noted, large aggregate shocks cause the dynamics of the model to be inconsistent with the conjectured equilibrium structure. A later version of this paper will consider the robustness of the results presented with respect to the assumption on aggregate productivity.

The parameters of the idiosyncratic shock process are identified by two observations. First, greater variance in the innovations to the idiosyncratic productivity process leads firms to hire more when their idiosyncratic shock is good and fire more when it is bad. Davis, Haltiwanger and Schuh (1996) report that one-sixth of job destruction is attributable to firm shut-down. I do not attempt to match this parameter directly, since presumably at least some job destruction, particularly at smaller firms, is associated with shocks unrelated to productivity as modeled here; in the current calibration, instead, I assume that 30 percent of job destruction arises from firms subject to the exogenous destruction shock δ. In addition, I follow the calibration strategy of Elsby and Michaels (2010) in requiring that around 37 percent of firms experience zero change in their employment level each period. I choose the number of shock values $m_\pi = 21$, so that relatively dispersed values of the idiosyncratic shock are possible. With this number of grid points, matching the previously-mentioned targets requires setting $\Delta_p = 0.035$ (so that each period there is a 70 percent chance of a productivity shock) and $d_p = 0.5$ (so that productivity shocks are large, each one corresponding to a 50 percent modification in productivity; this large value arises because of

Variable	Value	Explanation
r	0.996	Annual rate of time preference 4.91%
δ	0.011	1/3 of separations from firm closure
s	0.08333	Quarterly job destruction rate 0.1
A	1.467	Average establishment size 23.8
α	0.6667	See text
m_p	3	See text
Δ_p	0.01	See text
d_p	0.01	See text
m_π	21	See text
Δ_π	0.035	See text
d_π	0.5	See text
b	0.4	Shimer (2005)
Z	0.4674	Shimer (2005); unemployment rate of 6.87%
θ	1	See text
η	0.5727	See text
γ	0.1492	See text
k	427.4	See text
ζ	0.72	Shimer (2005); slope of Beveridge curve

Table 1: Parameterization

the quite concave production function and would be smaller if α were larger.

The remaining calibration targets are that average firm size equal 23.8 workers per establishment, as reported by Davis et al. (2006), which pins down the level of productivity, $A = 1.467$, and that the free entry condition should hold at the target labor market tightness; this pins down the free entry cost κ. The key parameter values discussed above are summarized in Table 1.

To put these derived parameters in context, note that the cost incurred by a firm in posting vacancies to hire the average number of workers per establishment, 23.8, at the average labor market tightness of $\bar{q} = 0.4674$ is given by $\frac{1}{1-\eta}\frac{n^*\gamma}{\bar{q}} = 17.78$, or around one and a half years of the value of the time of a single worker, or around 4.2 percent of the value of the capital cost of firm entry, κ.

This completes the discussion of the calibration strategy. In the remainder of this Section, I discuss the dynamics of the calibrated model.

Figure 2 shows a sample of the hiring and firing behavior of a single firm. The blue line indicates the time path of the firm's number of employees, $n(t)$. The green line indicates the hiring target, n^*_{ij} corresponding to the current productivity state (i, j) at time t. The red line indicates the time path of the firing target $\bar{n}_{ij}$. As can be seen, whenever the hiring target increases to a level that exceeds the size of the firm in the preceding period, hiring occurs to the new value of the hiring target. Similarly, when the firing target falls below the current size of the firm, firing occurs so as to keep the current size no greater than that target.

Figure 3 reports a histogram of the distribution of firm sizes. As can readily be seen, the distribution does not have the fat tails required to be consistent with the empirical firm size distribution, which follows a power law. In a future version of this paper, I will assume on the distribution of the idiosyncratic productivity shock in order to be consistent with this.

The moments of greatest interest are the cyclical behavior of labor market tightness and unemployment. Figure 4 shows sample paths for these variables. The upper panel of Figure 4 shows a sample path for aggregate productivity. Corresponding to the assumption that market tightness

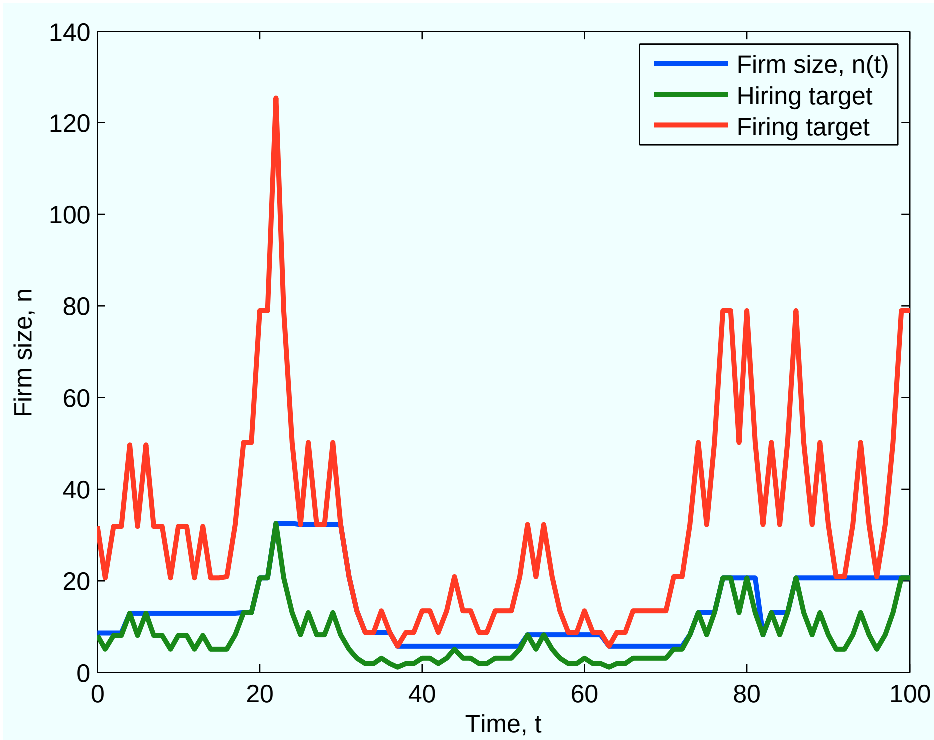

Figure 2: Firm size dynamics

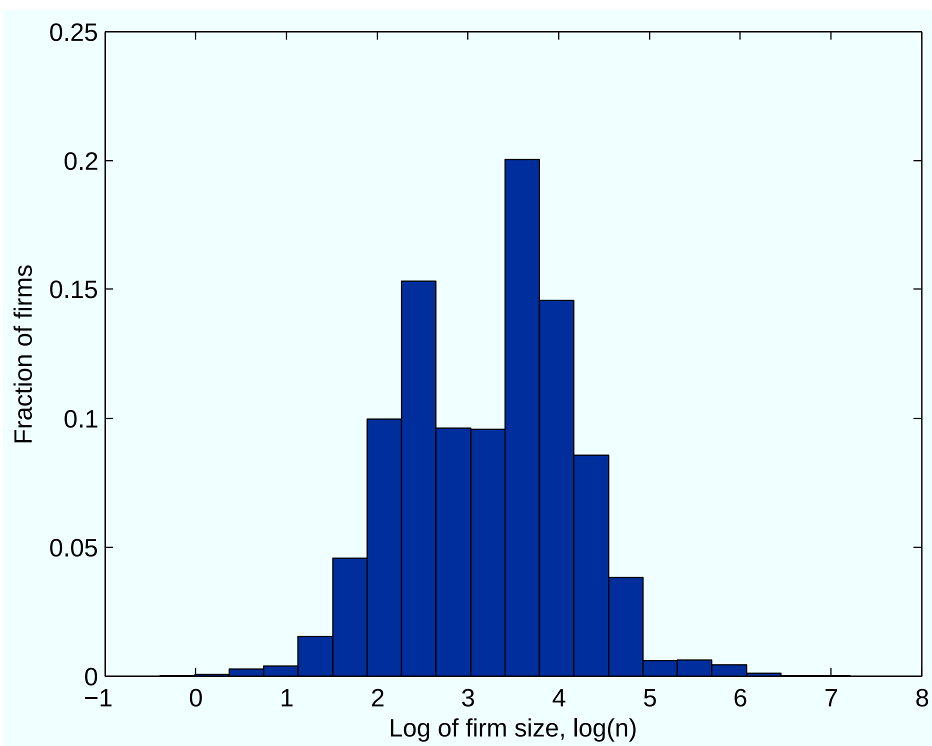

Figure 3: Firm size dynamics

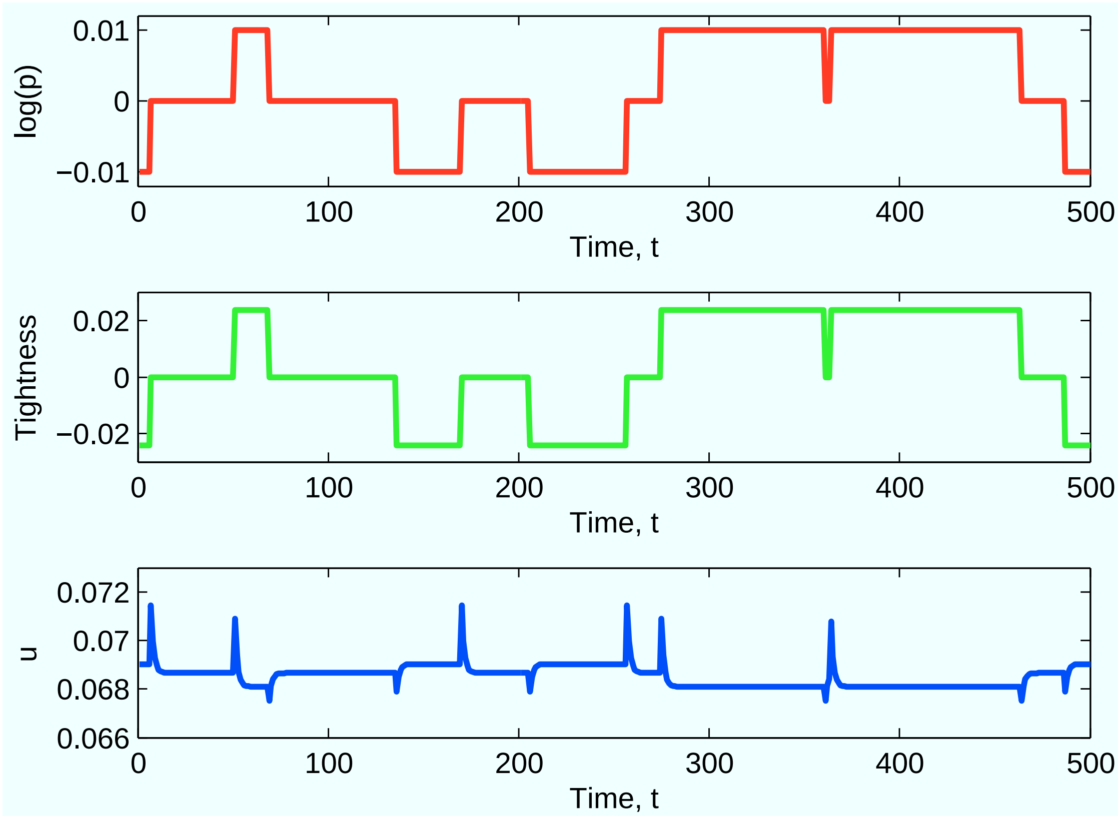

Figure 4: Dynamics of productivity, market tightness, and unemployment

depends only on aggregate productivity, the middle panel of the Figure shows that the dynamics of this variable are qualitatively identical to those for aggregate productivity. In the bottom panel are shown the dynamics of unemployment, which are slightly more complicated. First, after a long time with no productivity shock, unemployment converges to a quasi-steady state value, just as in the Mortensen-Pissarides model. In the calibrated model, the values of these quasi-steady state unemployment rates, corresponding to the low, medium, and high aggregate productivity levels, are respectively 6.80, 6.86, and 6.92 percent. When a negative productivity shock arrives, however, there is initially a fall in unemployment; after this initial decrease, unemployment converges exponentially to its quasi-steady state value. Note that the speed of convergence for unemployment to its quasi-steady state value is just as rapid as in the Mortensen-Pissarides model (it is very close to exponential with rate of convergence equal to the sum of the worker separation rate and job finding rate, and therefore has half-life only around 6 weeks in the calibrated model). Thus, the volatility of unemployment is slightly greater than it would have been if the transitional dynamics between quasi-steady states are ignored. However, this effect is very small (the standard deviation of the unemployment rate (measured as a percentage) is 0.0488 in the full model, but would be only 0.0424 if unemployment was always at its quasi-steady state value).

In summary, the preliminary results presented in this Section give little support to claims that allowing for decreasing returns to labor should necessarily increase the amplitude of aggregate fluctuations in employment and unemployment. There are several important caveats to this conclusion. First, the calibration presented here is preliminary, and it is possible that once a distribution of firm-level idiosyncratic productivity shocks more consistent with the micro evidence is allowed for, the results will change. There seems no reason that this should be so, however. Second, the solution method developed here relies on small aggregate shocks, such that the burst of hiring that arises after an aggregate shock can be consistent with a labor market tightness that depends only on aggregate productivity. However, there is no reason to conjecture that even in such a case, the volatility of unemployment or labor market tightness will increase substantially (rather, the difficulty is more technical than substantive). Third, the model was solved under free entry of firms. Results from Acemoglu and Hawkins (2010) suggest that the dynamics of models in this family in which the adjustment process of the firm size distribution is time-consuming may exhibit qualitatively different dynamics than models with free entry, and the same needs to be investigated here. Thus, the reason for the variation between the preliminary results presented here and those of Elsby and Michaels (2010), who solve a model similar to the one presented here and find significantly greater amplitude and persistence of fluctuations in unemployment and market tightness, needs to be a subject of further work.

9 Conclusion

The question of how bargaining occurs between workers and firms is key for understanding both empirical predictions and normative conclusions of models of frictional labor markets. In this paper, I propose a model of bargaining that avoids the failure of firms and workers to contract on privately efficient actions, contrary to the principle espoused in Barro (1977). I show that, unsurprisingly, this model has superior efficiency properties to the model of Stole and Zwiebel (1996*a*,*b*). However, it is not possible to distinguish the two using data on workers' experiences in the labor market; data on firm profitability or productivity is required for this. The model is relatively tractable, and a versions of it with time-consuming hiring and with aggregate fluctuations can readily be solved.

Appendix

This Appendix contains statements of some definitions and results that are not formalized in the text, together with all technical proofs. Some proofs are not yet complete in this preliminary version of the paper.

Proof of Proposition 2. Substitute from (16) into (14) and (19) to eliminate rV^u. Subtracting the resulting two equations yields that

$$y'(n^{PE}) = \frac{1}{n^{PE}}\left[y(n^{PE}) - (r+\delta)k\right]. \tag{75}$$

That is, n^{PE} satisfies (6). But if y is strictly concave, it is immediate that the solution to (6) is unique (since $\frac{d}{dn}\left[y(n) - ny'(n)\right] = -ny''(n) > 0$).

Next, use (16) to eliminate rV^u from the first order condition that determines n^{PE}, equation (14). After rearrangement, this yields the equation

$$\frac{\gamma}{1-\eta}\left[\eta\theta + \frac{r+\delta}{q(\theta)}\right] = y'(n^{PE}) - b. \tag{76}$$

The left side of (76) is monotonically increasing in θ, while the right side is independent of θ according to (75). Finally, (16) then determines rV^u uniquely. □

Proof of Proposition 3. This result was proved during the proof of Proposition 2 above. □

Proof of Proposition 4. Rearrange (8), replace $f'(\theta) = q(\theta) + \theta q'(\theta)$, and divide through by the common factor u to see that

$$\left[\left(\frac{y(n^*)}{r+\delta} - k\right)\frac{1}{n^*} - \lambda\right](q(\theta) + \theta q'(\theta)) = \gamma.$$

Substitute from (7) to eliminate

$$\lambda = \frac{\left[\frac{y(n^*)}{r+\delta} - \frac{n^*\gamma}{q(\theta)} - k\right]\frac{\theta q(\theta)}{n^*} + b}{r+\delta+\theta q(\theta)}$$

from the previous equation, and rearrange to obtain

$$\frac{y(n^*)}{n^*(r+\delta)} - \frac{k}{n^*} - \frac{b}{r+\delta} = \frac{\gamma}{r+\delta}\left(\frac{r+\delta+\theta q(\theta)}{q(\theta)+\theta q'(\theta)} - \theta\right). \tag{77}$$

On the other hand, eliminate rV^u from (19) using (16) to see that in the equilibrium under PE bargaining,

$$\frac{y(n^{PE})}{n^{PE}(r+\delta)} - \frac{k}{n^{PE}} - \frac{b}{r+\delta} = \frac{\gamma}{(1-\eta)q(\hat{\theta})}\left(1 + \frac{\beta\hat{\theta}q(\hat{\theta})}{r+\delta}\right). \tag{78}$$

(In (78), the notation $\hat{\theta} \equiv \theta^{PE}$ is intended to remind the reader that the value of θ potentially differs from that in the efficient allocation, as in (77).) By Proposition 3, it follows that the left sides of (77) and (78) coincide, so that a necessary and sufficient condition for the PE equilibrium allocation to be efficient is that the right sides coincide. After rearrangement, this condition is equivalent to

$$\frac{r+\delta-\theta^2 q'(\theta)}{q(\theta)+\theta q'(\theta)} = \frac{r+\delta+\eta\hat{\theta}q(\hat{\theta})}{(1-\eta)q(\hat{\theta})}. \tag{79}$$

First, suppose the PE equilibrium allocation is constrained efficient, so that $\hat{\theta} = \theta$; then cross-multiply (79) by $(1-\eta)q(\theta)(q(\theta)+\theta q'(\theta))$ and rearrange to deduce that

$$(r+\delta+\theta q(\theta))(\eta q(\theta) + \theta q'(\theta)) = 0.$$

Since $\theta q(\theta) = f(\theta) \geq 0$ and $r + \delta > 0$, it follows that $\eta q(\theta) + \theta q'(\theta) = 0$, which is the claimed condition (20). Conversely, suppose that (20) is satisfied; then substituting this condition into both the numerator and denominator transforms (79) into

$$\frac{r+\delta+\eta\hat{\theta}q(\theta)}{(1-\eta)q(\theta)} = \frac{r+\delta+\eta\hat{\theta}q(\hat{\theta})}{(1-\eta)q(\hat{\theta})}.$$

Since $x \mapsto \frac{r+\delta+\eta f(x)}{(1-\eta)q(x)}$ is strictly increasing in x (because by assumption f is strictly increasing and q is strictly decreasing), it follows immediately that $\hat{\theta} = \theta$.

Finally, since $n^* = n^{PE}$ by Proposition 3, the result of the Proposition is now immediate. □

Proof of Proposition 5. First use (21) and (29) to obtain that

$$V(n^{SZ}) - V^u = \frac{\eta}{1-\eta}J'(n^{SZ}) = \frac{\eta}{1-\eta}\frac{\gamma}{q(\theta)}. \tag{80}$$

It follows immediately from (32) that

$$rV^u = b + \frac{\eta}{1-\eta}\gamma\theta. \tag{81}$$

Also, subtract (32) from (31) to see that

$$(r + \delta + \theta q(\theta)) \left[V(n^{SZ}) - V^u\right] = w(n^{SZ}) - b;$$

from (80) it follows that

$$w(n^{SZ}) = b + \frac{\eta}{1-\eta}\frac{\gamma}{q(\theta)}(r + \delta + \theta q(\theta)).$$

Substituting this into the free entry condition (33) gives, after rearrangement,

$$\frac{y(n^{SZ}) - (r+\delta)k}{n^{SZ}} = b + \frac{\gamma}{1-\eta}\left(\eta\theta + \frac{r+\delta}{q(\theta)}\right). \tag{82}$$

On the other hand, substituting from (26) into the smooth-pasting condition (30) establishes that

$$(1-\eta)\frac{-rV^u + \frac{1}{\eta}\left(n^{SZ}\right)^{-\frac{1}{\eta}} \int_0^{n^{SZ}} \nu^{\frac{1-\eta}{\eta}} y'(\nu)\, d\nu}{r+\delta} = \frac{\gamma}{q(\theta)},$$

or equivalently, after substituting for rV^u from (81), that

$$\frac{1}{\eta}\left(n^{SZ}\right)^{-\frac{1}{\eta}} \int_0^{n^{SZ}} \nu^{\frac{1-\eta}{\eta}} y'(\nu)\, d\nu = b + \frac{\gamma}{1-\eta}\left(\eta\theta + \frac{r+\delta}{q(\theta)}\right). \tag{83}$$

Equating the left sides of (82) and (83) and rearranging implies that n^{SZ} solves

$$y(n) - \frac{1}{\eta} n^{\frac{\eta-1}{\eta}} \int_0^n \nu^{\frac{1-\eta}{\eta}} y'(\nu)\, d\nu = (r+\delta)k. \tag{84}$$

The derivative of the left side of the previous equation with respect to n is

$$-\frac{1-\eta}{\eta}\left[y'(n) + \frac{1}{\eta}n^{-\frac{1}{\eta}} \int_0^n \nu^{\frac{1-\eta}{\eta}} y'(\nu)\, d\nu\right],$$

which is negative since $\eta \in (0,1)$ and $y(\cdot)$ is strictly increasing. The right side is constant in n. Thus (84) uniquely determines n^{SZ} in terms of exogenous variables.

Noting that the right side of (82) or (83) is strictly increasing in θ then determines θ uniquely in terms of n^{SZ}, and therefore uniquely. Finally, (81) then determines rV^u. □

Proof of Proposition 6. Rearrange the free entry condition (33) to see that

$$\frac{1}{n^{SZ}}\left[y(n^{SZ}) - (r+\delta)k\right] = w(n^{SZ}) + (r+\delta)\frac{\gamma}{q(\theta)}.$$

From a comparison of this equation to (6), the equation characterizing efficient hiring, it follows immediately that if $n^{SZ} = n^*$ then

$$\frac{y'(n^{SZ}) - w(n^{SZ})}{r+\delta} = \frac{\gamma}{q(\theta)}.$$

Comparing this to (30), the first-order condition characterizing n^{SZ}, yields that this occurs only if $w'(n^{SZ}) = 0$. But (26) immediately implies that this can occur only if

$$\int_0^{n^{SZ}} n^{\frac{1}{\eta}} y''(\nu)\, d\nu = 0,$$

which is impossible if the production function y is weakly concave and not everywhere linear. □

Proof of Proposition 7. Denote the marginal tax rate on firms' output by τ, and the lump-sum rebate to active firms by T. Denote the value of an unemployed worker by V_τ^U, that of a worker employed at a firm with n employees by $V_\tau(n)$, the wage at such a firm by $w_\tau(n)$, and the the value of such a firm by $J_\tau(n)$. As in the model without taxes, it is elementary to show that hiring takes a bang-bang form, with entrant firms immediately growing to their target size; denote this by n_τ^{SZ}. Denote the market tightness by θ_τ.

The equations characterizing the equilibrium with the tax and transfer scheme are the following. First, the wage-bargaining equation (26) becomes

$$w_\tau(n) = (1-\eta) r V_\tau^u + (1-\tau) n^{-\frac{1}{\eta}} \int_0^n \nu^{\frac{1-\eta}{\eta}} y'(\nu)\, d\nu. \tag{85}$$

The firm's value function becomes

$$J_\tau(n) = \begin{cases} -(n_\tau^{SZ} - n)\frac{\gamma}{q(\theta_\tau)} + J_\tau(n_\tau^{SZ}) & \text{if } n < n^{SZ} \\ \frac{(1-\tau)y(n) - n w_\tau(n)}{r+\delta} & \text{if } n \geq n^{SZ}. \end{cases} \tag{86}$$

The smooth-pasting condition (29) is unchanged in form, as are the HJB equations for the worker, equations (31) and (32). The free-entry condition (33) becomes

$$\frac{(1-\tau)y(n^{SZ}) + T - n^{SZ} w(n^{SZ})}{r+\delta} - n^{SZ}\frac{\gamma}{q(\theta_\tau)} = k; \tag{87}$$

however, government budget balance, together with the symmetry of all firms, implies that $\tau y(n^{SZ}) = T$, so that (87) reduces to an equation of the same form as (33):

$$\frac{y(n^{SZ}) - n^{SZ} w(n^{SZ})}{r+\delta} - n^{SZ}\frac{\gamma}{q(\theta_\tau)} = k; \tag{88}$$

Combining (29) and (86), it follows that

$$\frac{(1-\tau)y'(n_\tau^{SZ}) - w_\tau(n_\tau^{SZ}) - n_\tau^{SZ} w'(n_\tau^{SZ})}{r+\delta} = \frac{\gamma}{q(\theta_\tau)}. \tag{89}$$

Multiply this last equation by n_τ^{SZ} and subtract from the free-entry condition (88) to see that

$$\frac{y(n_\tau^{SZ}) - (1-\tau) n_\tau^{SZ} y'(n_\tau^{SZ}) + \left(n_\tau^{SZ}\right)^2 w'(n_\tau^{SZ})}{r+\delta} = k. \tag{90}$$

Now, suppose that the equilibrium with taxes and transfers is efficient. Then, in particular, $n_\tau^{SZ} = n^*$, and satisfies (6), so that $\frac{y(n^*) - n^* y'(n^*)}{r+\delta} = k$. A comparison of this expression with (90) shows that

$$\tau y'(n^*) + n^* w'(n^*) = 0,$$

from which it follows by differentiating (85) that the tax rate τ must satisfy

$$y'(n^*) = \frac{1-\tau}{\eta} n^*$$

Using (85), this implies that

$$\frac{\frac{1-\eta}{\eta}(1-\tau)\left(n_\tau^{SZ}\right)^{-\frac{1}{\eta}} \int_0^{n_\tau^{SZ}} \nu^{1-\eta}\eta y'(\nu)\, d\nu - (1-\eta) r V_\tau^u}{r+\delta} = \frac{\gamma}{q(\theta_\tau)}. \tag{91}$$

A necessary condition for the equilibrium with taxes to be constrained efficient is that this equation must hold if $n_\tau^{SZ} = n^*$, $\theta_\tau = \theta$, and $V_\tau^U = V^u$, the constrained efficient values, arising under PE bargaining. This immediately determines the only possible candidate optimum tax rate. I complete the proof by showing that for this choice of tax rate, (n^*, θ, V^u) solve the equilibrium equations above. The only non-obvious equation is the free-entry condition (33). To establish that this holds, first substitute from (91) into (85) to see that

$$w_\tau(n^*) = rV^u + \frac{\eta}{1-\eta}\frac{\gamma(r+\delta)}{q(\theta)}.$$

Manipulate the two worker HJB equations to obtain that

$$(r + \delta + \theta q(\theta)) r V^u = (r+\delta)b + \theta q(\theta) w(n^*),$$

and combine these two equations. This shows that

$$rV^u = b + \frac{\eta}{1-\eta}\gamma\theta$$

and

$$w(n^*) = b + \frac{\eta}{1-\eta}\frac{\gamma}{q(\theta)}(r + \delta + \theta q(\theta)).$$

(Note that this equals the wage determined for PE bargaining in (18).) A comparison with (19) now makes in clear that the free-entry condition (33) holds. This completes the proof. □

Proof of Proposition 10. Consider the data generated by an economy with PE bargaining, with production function $y_{PE}(\cdot)$ and entry cost κ_{PE}. Define a new production function $y_{SZ}(\cdot)$ by

$$y_{SZ}(n) = y_{PE}(n) - n^{1-\frac{1}{\eta}} \int_0^n \nu^{\frac{1}{\eta}-1} y'_{PE}(\nu)\, d\nu$$

and a new entry cost by

$$\kappa_{SZ} = (1-\eta)\kappa_{PE}.$$

Then if $S(\cdot)$, $G(\cdot)$, θ, and rV^u are the values arising in an equilibrium under PE bargaining, then an equilibrium under SZ bargaining can be generated by setting $J(n) \equiv (1-\eta)S(n)$, with the values of the firm-size distribution $G(\cdot)$, the labor market tightness θ, and the HJB value of an unemployed worker unchanged. The proof is nearly trivial. Observe that under this definition of $y_{SZ}(\cdot)$, $J(\cdot)$ so defined solves (42) because $S(\cdot)$ solves (47) and because the vacancy posting decisions of firms given by (37) and (48) coincide. Furthermore, $S(\cdot)$ solves the free entry condition, and the firm size distribution is identical in the two allocations. It then follows from a comparison of (43) and (49) that the HJB values of an unemployed worker coincide in the two economies, and then from (39) and (46) it follows that the HJB values of employed workers coincide also. It remains to observe that the first-order conditions are sufficient to characterize the equilibrium, for which it is necessary and sufficient that $y_{SZ}(\cdot)$ be strictly increasing, strictly concave, and satisfy the appropriate Inada conditions. While it is difficult to give general conditions under which this holds, observe that if $y_{PE}(n) \equiv A_{PE}n^{\alpha}$ is Cobb-Douglas, then as in the proof of Proposition 9, one can verify that $y_{SZ}(n) = A_{SZ}n^{\alpha}$ is also Cobb-Douglas with the same power α and $A_{SZ} = (1-\eta+\alpha\eta)A_{PE}$; in this case, as observed in footnote 10, the result holds for all $\alpha > 0$ and $\eta \in (0,1)$. □

Proof of Proposition 11. Analogously to the proof of Proposition 10, consider the data generated by an economy with PE bargaining, with production function $y_{PE}(n;p,\pi) =$ and entry cost κ_{PE}. Define a new production function $y_{SZ}(n;p,\pi)$ by

$$y_{SZ}(n;p,\pi) = y_{PE}(n;p,\pi) - n^{1-\frac{1}{\eta}} \int_0^n x^{\frac{1}{\eta}-1} y'_{PE}(x;p,\pi)\, dx$$

and a new entry cost by

$$\kappa_{SZ} = (1-\eta)\kappa_{PE}.$$

Then if $S(\cdot)$, $G(\cdot)$, θ, and rV^u are the values arising in an equilibrium under PE bargaining, then an equilibrium under SZ bargaining can be again be generated by setting $J_{ij}(n,t) \equiv (1-\eta)S_{ij}(n,t)$, with the values of the firm-size distribution $G(\cdot)$, the labor market tightness θ, and the HJB value function of unemployed workers unchanged. Observe that under this definition of $y_{SZ}(\cdot)$, $J_{ij}(\cdot)$ so defined solve (66) because $S(\cdot)$ solves (52) and because the hiring and firing targets of firms given by (67) and (68) coincide with those given by (55) and (56). Furthermore, $S(\cdot)$ solves the free entry condition (61), so $J_{ij}(\cdot)$ solves (70); the firm size distribution is identical in the two allocations. The HJB equation for unemployed workers, (69) holds because (59) does, so from (51) it follows that so too does (63), the HJB equation for employed workers. □

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