Robust Predictions for Bilateral Contracting with Externalities*

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1 Introduction

This paper studies bilateral contracting between one principal and N agents in the presence of externalities among agents. Examples of such situations include (i) vertical contracting where sales by an upstream firm to a downstream firm reduce the downstream price received by other downstream firms (Hart-Tirole [1990], McAfee-Schwartz [1994], Rey-Tirole [1996]), (ii) nonexclusive insurance, where the contract between an insured and and and insurer affects the level of care taken by the insured and hence the profits of other insurers (Pauly [1974], Bernheim and Whinston [1986]), Kahn and Mookherjee [1998]), Bisin and Gottardi [1998], Bisin et al. [1999]), (iii) lending with default, where the debt owed by the principal to one agent affects the expected repayment to other agents (Bizer and deMarzo [1992], Dubey et al. [1996], Rajan and Parlour [1998]), and many others (see Segal [1999]).

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The existing literature on such contracting situations is divided into two branches. One branch studies specific noncooperative bilateral contracting games. In the two most-studied games, one of the two sides makes take-it-or-leave-it offers to the other. The game in which the principal makes offers to the agents, which we refer to below as the *offer game*, has been studied, for example, by Hart and Tirole [1990], McAfee and Schwartz [1994], Rey and Tirole [1996], and Segal [1999]. The game in which the agents make offers to the principal, which we refer to below as the *bidding game*, appears in models of common agency (Bernheim and Whinston [1986], Martimort and Stole [1998]). As well, some specific dynamic contracting games have been considered (deMarzo and Bizer [1993], Kahn and Mookherjee [1998]).

While this literature has generated important insights about the nature of the inefficiency caused by contracting externalities, the games studied in this literature generate a wide range of equilibrium outcomes, both within a given game and across different games. This diversity of equilibrium outcomes can be traced to differences in agents' beliefs about the principal's contracts with other agents. For example, in the offer game an agent who observes an out-of-equilibrium offer by the principal can hold arbitrary beliefs about the principal's offers to other agents, and these beliefs can sustain a large set of equilibrium outcomes.

The other branch of the literature studies the case with a large number of agents and postulates competitive equilibrium (Dubey et al. [1996], Bisin and Gottardi [1998]). For example, in the lending and common agency applications, the competitive concept demands that each lender/agent take the principal's total borrowing/insurance, and hence the principal's actions, as given. The problem with this approach is that it has not been justified as a limit of outcomes of noncooperative contracting games with N agents as $N \to \infty$ (the way competitive equilibrium in economies without externalities is justified by Cournot-style competitive limit results).[1]

This paper considers a family of noncooperative games of contracting with externalities, which we call *bilateral contracting games*, and which includes the offer and bidding games described above as special cases. We identify properties of equilibrium outcomes that are robust in the sense that they must be satisfied by all equilibria of all bilateral contracting games. In a setting with specialized payoffs, we show that when a competitive equilibrium exists, all bilateral contracting outcomes converge to it as $N \to \infty$. We also

[1] An exception is Bisin et al. [1999].

examine equilibrium contracting outcomes when a competitive equilibrium fails to exist due to nonconvexities, and their limit behavior as $N \to \infty$. In particular, we show that the "convexification approach" used in general equilibrium theory to ensure the existence of competitive equilibrium is incorrrect - asymptotically all contracting outcomes may be very different from the convexified competitive equilibrium. To illustrate the relation between our necessary conditions and the equilibrium outcomes of particular contracting games, we also characterize the equilibrium outcomes of offer and bidding games.

A second sense in which we identify outcome that are robust is that we allow for fully general contracts between the principal and an agent. A key idea behind our results is that the set of equilibrium outcomes is dramatically affected when the parties are allowed to offer each other menus from which the principal can then choose, rather than point contracts. From an agent's viewpoint, a menu can separate the different "types" of the principal corresponding to different trades with other agents. For example, in the offer game, the principal can offer an agent a menu to signal her trade with other agents, which is similar to Maskin and Tirole's [1992] analysis of signaling by an informed principal. According to Myerson's "inscrutability principle" and Maskin and Tirole's analysis, there exists a menu giving all types of the principal their maximum payoff among all menus that the agent must accept regardless of his beliefs. This menu is called the Rothschild-Stiglitz-Wilson (RSW) menu by Maskin and Tirole. In bilateral contracting games in which an agent makes an offer to the principal (such as the bidding game), the agent can use the same RSW menu (plus a lump-sum payment) to screen the principals of different types.

The requirement that deviations in which the parties offer each other an RSW menu be unprofitable imposes a significant bound on the set of equilibrium outcomes. Namely, if the equilibrium bilateral surplus of the principal and an agent i were too low, there would exist a bilateral contract (consisting of the RSW menu and a lump-sum transfer) that would guarantee each of them of a higher payoff. For a specialized payoff setting in which a competitive equilibrium exists, we find that one RSW menu is the "competitive menu," which allows the principal to buy an arbitrary quantity at the competitive equilibrium price. The lower bound on bilateral surplus resulting from this menu implies the above-mentioned competitive limit result as $N \to \infty$. We also construct an RSW menu in some cases in which a competitive equilibrium does not exist, and use it to study contracting outcomes.

The rest of the paper is organized as follows. In Section 2 we exposit the key ideas of the paper in a simple example of vertical contracting. Section 3 defines a general family of bilateral contracting games and characterizes their equilibria using the concept of an RSW menu. Sections 4-6 apply the general characterizations of Section 3 to a specialized payoff setting. Section 4 bounds the set of equilibrium outcomes of all bilateral contracting games. Section 5 identifies RSW menus for specific settings and characterizes equilibrium outcomes of offer games. Section 6 characterizes equilibrium outcomes of bidding games. Section 7 concludes.

2 A Simple Example

In this section we consider a simple example of contracting with externalities. To be specific, we focus on the setting of vertical contracting, where one manufacturer (the "principal") sells its output to N retailers ("agents") (Hart-Tirole [1990], McAfee-Schwartz [1994]). The retailers then resell their purchases in the downstream market.[2]

For simplicity, we assume that the retail technology converts each unit of the manufacturer's product into a unit of the final good at a zero marginal cost. We let the manufacturer's cost function be $c(X) = \alpha X + \beta X^2$, and the downstream inverse demand function be $P(X) = a - bX$. We assume that $a > \alpha > 0$, $b > 0$, and $b + 2\beta > 0$.

The efficient outcome for the vertical structure is to sell the monopoly quantity:

$$X^* = Arg \max_{X \geq 0} P(X)X - c(X) = \frac{a - \alpha}{2b + 2\beta}.$$

On the other hand, if the vertical structure were to behave as a price-taker, it would sell the competitive quantity X^c at which the marginal cost equals price,

$$c'(X^c) = P(X^c) \Rightarrow X^c = \frac{a - \alpha}{b + 2\beta},$$

provided that its profit from doing so is nonnegative. This is so (and so such a competitive equilibrium exists) if and only if $\beta \geq 0$. The efficient quantity

[2] For simplicity we will not allow retailers to withdraw the good from the market.

X^* and the competitive quantity X^c will serve as benchmarks against which to compare the outcomes of bilateral contracting.

We will consider bilateral contracting games in which each retailer contracts with the manufacturer without observing other retailers' contracts with the manufacturer. A retailer's behavior in such a game depends on his beliefs about the other retailers' contracts. The scope of possible beliefs will determine the possible contracting outcomes.

Here we illustrate the role of beliefs in the *offer game*, in which the manufacturer makes simultaneous offers to the downstream firms, who then accept or reject. Suppose that the manufacturer offers each retailer i a contract (x_i, t_i), where $x_i \geq 0$ is the delivery of the product and t_i is the retailer's payment.[3] After observing the offer, retailer i forms beliefs about other retailers' contracts. One form of of beliefs considered in the literature is passive beliefs: after observing a deviation, each retailer continues to believe that other retailers receive their equilibrium offers. Let $\left(\widehat{x}_1, ..., \widehat{x}_N, \widehat{t}_1, ..., \widehat{t}_N\right)$ denote the equilibrium purchase profile. If retailer i receives offer $(x_i, t_i) \neq \left(\widehat{x}_i, \widehat{t}_i\right)$, he still believes that other retailers make their equilibrium purchases $\widehat{x}_{-i}$, and he will accept the offer if and only if $P(\widehat{X}_{-i} + x_i)x_i \geq t_i$, where $\widehat{X}_{-i} = \sum_{j \neq i} \widehat{x}_i$. Given this, the manufacturer's equilibrium sale $\widehat{x}_i$ to retailer i must satisfy

$$\widehat{x}_i \in Arg \max_{x_i \geq 0} P(\widehat{X}_{-i} + x_i)x_i - c(\widehat{X}_{-i} + x_i). \tag{1}$$

Condition (1) can be interpreted as saying that it is impossible to increase the *bilateral surplus* between the manufacturer and any retailer i (this is the total surplus $P(X)X - c(X)$, less the gross surplus of retailers $j \neq i$, $\sum_{j \neq i} P(X)x_j$) given the trades of all other retailers $j \neq i$. In the present example, the unique profile of trades satisfying (1) is the symmetric profile $x^p = \left(\widehat{X}_N^p/N, ..., \widehat{X}_N^p/N\right)$, where

$$\widehat{X}_N^p = \frac{a - \alpha}{(1 + 1/N)b + 2\beta}.$$

Note that $\widehat{X}_1^p = X^*$ (the efficient quantity), and that $\widehat{X}_N^p \uparrow X^c$ (the competitive quantity) as $N \to \infty$.

[3]The contracting outcomes would not change if the manufacturer could offer a menu from which the retailer would choose without observing other contracts, as in Hart and Tirole [1990].

While the restriction to passive beliefs provides a simple story for the inefficiency of bilateral contracting and competitive convergence, it has at least two difficulties. First, a passive beliefs equilibrium need not exist. For instance, in the presetn example any passive beliefs equilibrium must involve an aggregate trade that converges to the competitive quantity X^c as $N \to \infty$. But, we have already seen that if $\beta < 0$ this would involve negative profits for the manufacturer, and so cannot be an equilibrium outcome.[4]

Second, and more fundamentally, it is clearly an *ad hoc* restriction on beliefs that may not be particularly compelling in many circumstances. To take an extreme example, if the manufacturer has only $\overline{X}$ units for sale, i.e., $c(X) = \infty$ for $X > \overline{X}$, then a retailer who is offered $\overline{X}$ units is assured that other retailers will not get any, regardless of what the equilibrium allocation was supposed to be. More generally, for our payoff specifications with $\beta \neq 0$, retailers should be aware that the manufacturer's optimal contract offer to one retailer depends on his contract with other retailers.

Once one does not assume passive beliefs, the concept of weak Perfect Bayesian Equilibrium, which allows retailers to hold arbitrary beliefs after observing out-of-equilibrium offers, sustains a large set of equilibrium outcomes. For example, as noted by McAfee and Schwartz [1994], the efficient outcome X^* can be sustained for any N by endowing retailers with *symmetry beliefs*, under which each retailer believes that the manufacturer offers all retailers the same contract. Therefore, with symmetry beliefs, competitive convergence does *not* obtain. More generally, for any N, we can support with suitable beliefs any equilibrium aggregate quantity $\widehat{X} \geq 0$ such that the total surplus $P(\widehat{X})\widehat{X} - c(\widehat{X})$ is nonnegative (for example, by letting each retailer believe that $X_{-i} > a/b$ following any observed deviation).

A key idea of this paper is that the manufacturer may be able to avoid this problem of negative inferences by retailers by designing her contracts cleverly, and in particular, by using contracts that specify a menu of possible trades from which the manufacturer can choose. For example, suppose that $\beta > 0$ and that the manufacturer deviates from an equilibrium by offering retailer i a contract which gives the manufacturer the right to choose from the *competitive menu* $C = \{(x, t) : x \geq 0, t = p^c x\}$. Observe that retailer i is *guaranteed a zero payoff if he accepts the offer, regardless of his belief about*

[4] The cause of this non-existence can be traced to the possibility of deviations by the principal to several agents at once (here, she will prefer to offer no contracts at all). See Rey and Verge [1997] for another example of nonexistence of a passive beliefs equilibrium due to multilateral deviations.

the aggregate quantity X_{-i} sold to other retailers. Indeed, if $X_{-i} < X^c$, then the manufacturer will optimally choose $x_i = X^c - X_{-i}$ from the menu, and the retailer's payoff is $P(X^c)x_i - p^c x_i = 0$. On the other hand, if $X_{-i} > X^c$, then the manufacturer will choose $x_i = 0$ from the menu, and the retailer's payoff is again zero. By paying in addition an arbitrarily small amount $\varepsilon > 0$ to the retailer, the manufacturer can convince him to accept the menu regardless of his beliefs.

This idea of using menus is inspired by Maskin and Tirole's [1990] analysis of signaling by an informed principal. By choosing from a menu, the manufacturer (principal) can signal to retailer i her "type", which in this case consists of her sales X_{-i} to other retailers; the only difference from Maskin and Tirole's analysis is that the principal's type is chosen by her rather than by nature.

To see an implication of this observation, suppose that $\left(\widehat{x}_1, ..., \widehat{x}_N, \widehat{t}_1, ..., \widehat{t}_N\right)$ with $\widehat{X} = \sum_i \widehat{x}_i \leq X^c$ is a weak Perfect Bayesian Equilibrium outcome of the offer game in which the manufacturer can offer menus (in Section 4 we rule out equilibria with aggregate trades above X^c). Consider the manufacturer's deviation in which she offers retailer i menu C (plus a small payment $\varepsilon > 0$) and then chooses $x_i = X^c - \widehat{X}_{-i}$ from it, while keeping her contracting with other agents fixed. For such a deviation to be unprofitable, we must have

$$\sum_j \widehat{t}_j - c(\widehat{X}) \geq \sum_{j \neq i} \widehat{t}_j + p^c(X^c - \widehat{X}_{-i}) - c(X^c).$$

On the other hand, by the retailer's individual rationality constraint, $P(\widehat{X})\widehat{x}_i \geq \widehat{t}_i$. Adding with the previous inequality this yields

$$P(\widehat{X})\widehat{x}_i - c(\widehat{X}) \geq p^c(X^c - \widehat{X}_{-i}) - c(X^c).$$

Averaging across all agents, we obtain

$$\left[\frac{1}{N}P(\widehat{X}) + \frac{N-1}{N}p^c\right]\widehat{X} - c(\widehat{X}) \geq p^c X^c - c(X^c).$$

For our parametrized inverse demand and cost functions, this is a quadratic inequality in $\widehat{X}$, whose solutions can be calculated to be

$$\widehat{X} \in [\underline{X}_N, X^c], \text{ where } \underline{X}_N = \frac{a - \alpha}{(b/[\beta N] + 1)(b + 2\beta)}.$$

Note that $\underline{X}_N \uparrow X^c$ as $N \to \infty$. Therefore, any weak Perfect Bayesian Equilibrium of the offer game in which the manufacturer can offer menus, with any specification of the retailers' out-of-equilibrium beliefs, must converge to the competitive equilibrium as the number of retailers goes to infinity. In particular, the efficient (monopoly) aggregate quantity X^*, which obtained with symmetry beliefs when the manufacturer could not offer menus, cannot be sustained now when $N > 2+b/\beta$. On the other hand, the outcome $\left(\widehat{X}_N^p/N, ..., \widehat{X}_N^p/N\right)$ can be sustained for all N, since $\widehat{X}_N^p \in (\underline{X}_N, X^c)$.

While we have so far focused on the offer game, the main insight is more general. For example, an alternative bargaining process that has been studied in the literature on vertical contracting is a *bidding (* or *common agency) game,* in which retailers simultaneously offer menus to the manufacturer, from which the manufacturer then chooses bundles (see, for example, Bernheim and Whinston [1998] and O'Brien and Shaffer [1997]).[5,6] By offering such a menu, retailer i can "screen" the different "types" X_{-i} of the manufacturer. (The idea that an agent's contract with other principals in the common agency game constitutes his "type" has been developed by Martimort and Stole [1998].) By considering deviations in which a retailer offers the manufacturer the competitive menu C described above, we can establish the competitive convergence result for bidding games. More generally, competitive convergence obtains in a large family of contracting games in which the manufacturer can make offers to some retailers and some retailers can make offers to the manufacturer. Moreover, the insight that allowing the manufacturer to use menus extends to settings in which a competitive equilibrium does not exist.

In the next section, we begin the main analysis of the paper by developing this approach to contracting with externalities in a more general setting.

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[5]The standard terminology in common agency games is that the manufacturer is an "agent" and retailers are "principals"; however, we stick to our previous terminology for the sake of consistency across games.

[6]Bidding games have received less attention than have offer games in settings in which contracting externalities are present. The only papers that we are aware of that have studied such cases are Bernheim and Whinston [1986] and [1998].

3 Characterization of Contracting Outcomes

We consider bilateral contracting games between one principal and N agents (N will also denote the set of agents). The trade between the principal and agent i will be represented by $x_i \in \mathcal{X}_i$. In addition, the agent can make a monetary transfer t_i to the principal. Let $0 \in \mathcal{X}_i$ denote the default trade between the principal and agent i. The default outcome between them is then $(x_i, t_i) = (0, 0)$.

The parties' utilities are quasilinear in money: the principal's payoff is $\sum_i t_i - c(x)$, and each agent i's payoff is $u_i(x) - t_i$, where $x = (x_1, ..., x_N) \in \mathcal{X}_1 \times \ldots \times \mathcal{X}_N$ is the agents' trade profile. We assume that $c(\cdot)$ is lower semicontinuous, which allows e.g. for fixed costs.[7] Note that each agent's utility can depend on all agents' trades: thus, we allow for externalities among agents. We assume, however, that agents' reservation utilities do not depend on others' trades: $u_i(0, x_{-i}) = 0$ for all $x_{-i} \in \mathcal{X}_{-i}$. (In the terminology of Segal [1999], there are no externalities on non-traders.)

A bilateral contract between the principal and each agent i takes the form of a menu, i.e., a subset $M_i \subset \mathcal{X}_i \times \Re$. After the contract is signed, the principal chooses a bundle $(x_i, t_i) \in M_i$. Her optimal choice in general depends on her contracts with other agents, which are not observed by agent i.[8] We restrict menus to be compact sets, to ensure that for any collection of menus $(M_1, \ldots, M_N)$, the principal has an optimal choice: $\arg\max_{(x,t) \in M_1 \times \ldots \times M_N} \left[\sum_i t_i - c(x)\right] \neq \emptyset$.[9]

We consider the following class of *bilateral contracting games*: A game lasts for K periods. In each period $k = 1, ..., K$, a subset $A_k \subset N$ of agents simultaneously make offers to the principal, and simultaneously the principal makes offers to a subset $P_k \subset N$ of agents (with $P_k \cap A_k = \emptyset$). Then the principal and agents simultaneously decide whether to accept offers extended to them, after which the principal chooses (x_i, t_i) from all contracts signed

[7] The function $c(\cdot)$ is lower semi-continuous if $x^n \to x$ implies that $\liminf_{n \to \infty} c(x^n) \geq c(x)$. Note that this assumption allows a fixed cost.

[8] We could allow more complicated contracts where agent i as well as the principal send messages determining (x_i, t_i). However, since agents possess no private information, allowing such contracts would not generate new equilibrium outcomes.

[9] If a collection of menus from which the principal does not have an optimal choice could be offered and accepted in a bilateral contracting game, then there would be no continuation equilibrium following this information set, hence the game would not have an equilibrium.

at this stage.[10] Then the game proceeds to the next period. The principal observes all history, while each agent observes only menus offered to him and by him as well as the principal's choices from any of these menus that are accepted. We assume that $\cup_{k=1}^{K}(P_k \cup A_k) = N$, i.e., each agent has at least one chance to contract with the principal. The outcome (x_i, t_i) between the principal and agent i is the principal's last choice from the last contract signed with agent i. The parties receive payoffs at the end of the game on the basis of this outcome (there is no discounting). Two examples of bilateral contracting games are given by the offer and bidding games discussed in Section 2.

3.1 Acceptable Menus and Neccessary Conditions

We will examine some properties that all pure-strategy weak Perfect Bayesian Equilibrium outcomes of any bilateral contracting game must satisfy. To fix ideas, consider first the offer game. It is clear that any weak PBE outcome $\left(\widehat{x}, \widehat{t}\right)$ of the game must satisfy agents' participation constraints:

$$u_i(\widehat{x}) - \widehat{t}_i \geq 0 \text{ for all } i \in N. \tag{2}$$

Indeed, if this were not the case, an agent i would profitably deviate by rejecting the principal's offer.

Another necessary condition on weak PBE outcomes of the offer game follows from considering the principal's deviations to menus that must be accepted by agents regardless of their beliefs about the principal's trade with other agents. Formally,

Definition 1 *A menu $M_i \subset \mathcal{X}_i \times \Re$ is acceptable to agent i if for any $\overline{x}_{-i} \in \mathcal{X}_{-i}$ and any $\left(\overline{x}_i, \overline{t}_i\right) \in \arg\max_{(x_i,t_i)\in M_i} \left[t_i - c(x_i, \overline{x}_{-i})\right]$ we have $u_i(\overline{x}_i, \overline{x}_{-i}) - \overline{t}_i \geq 0$. The set of compact menus that are acceptable to agent i is denoted by $\mathcal{A}_i \subset 2^{\mathcal{X}_i \times \Re}$.*

Consider the principal's deviation in the offer game where she offers the acceptable menu $M_i \in \mathcal{A}_i$ to agent i, while following her equilibrium contracting with other agents. The principal's maximum profit from this deviation

[10] Our analysis is not affected if the principal instead chooses at the end of the game from the menus then in effect.

is

$$\Pi_i^{M_i}(\widehat{x}_{-i}) = \max_{(x_i,t_i)\in M_i} \left[t_i - c(x_i, \widehat{x}_{-i})\right].^{11}$$

By paying agent i an arbitrarily small extra $\varepsilon > 0$, the principal can ensure that this deviation is accepted. Since the deviation must not be profitable, any weak PBE outcome $\left(\widehat{x}, \widehat{t}\right)$ must satisfy

$$\widehat{t}_i - c(\widehat{x}) \geq \Pi_i^{M_i}(\widehat{x}_{-i}).$$

Summing this inequality with agent i's participation constraint (2), we see that any weak PBE outcome must satisfy

$$u_i(\widehat{x}) - c(\widehat{x}) \geq \Pi_i^{M_i}(\widehat{x}_{-i}) \text{ for all } i \in N.$$

This condition puts a lower bound on the bilateral surplus $u_i(\widehat{x}) - c(\widehat{x})$ of the principal and each agent i in any weak PBE equilibrium of the offer game. It turns out, moreover, that this condition must hold in *any* bilateral contracting game. If it did not, then either the principal or agent i would deviate by offering each other menu M_i plus a lump-sum transfer, so that (i) the agent is better off regardless of his beliefs, and (ii) the principal is better off if she keeps her equilibrium contracting with other agents. This deviation would be accepted and would be profitable. A formalization of this idea yields

Proposition 1 *If $M_i \in \mathcal{A}_i$, then any weak PBE $\left(\widehat{x}, \widehat{t}\right)$ of a bilateral contracting game must satisfy*

$$u_i(\widehat{x}) - c(\widehat{x}) \geq \Pi_i^{M_i}(\widehat{x}_{-i}) \text{ for all } i \in N. \quad (3)$$

Proof. Suppose in negation that there exists an equilibrium in which the inequality is reversed. Let $\overline{k}$ be the largest k for which $i \in P_k \cup A_k$ (i.e., the principal and agent i contract).

Suppose first that $i \in P_{\overline{k}}$. Consider a deviation by the principal in contracting with agent i in which she rejects all of agent i's offers, offers the null contract $\{(0,0)\}$ to the agent in periods $k < \overline{k}$ and offers him M_i plus a small payment $\varepsilon > 0$ in period $k = \overline{k}$. Agent i will accept this contract for any beliefs about the principal's contracting with other agents. Suppose that the

[11] We define this profit not including the (fixed) transfer from agents $j \neq i$, $\sum_{j\neq i} t_j$.

principal uses her equilibrium strategy in contracting with other agents, in which case her payoff from the deviation is

$$\sum_{j\neq i}\widehat{t}_j+\Pi_i^{M_i}(\widehat{x}_{-i})-\varepsilon>\sum_{j\neq i}\widehat{t}_j+u_i(\widehat{x})-c(\widehat{x})\geq\sum_j\widehat{t}_j-c(\widehat{x}),$$

where the first inequality holds because condition (3) is violated, and the second follows from agent i's participation constraint (2). Thus, the deviation makes the principal better off.

Suppose now that instead $i\in A_{\overline{k}}$. Consider a deviation by the agent in which he uses the equilibrium strategy in periods $k<\overline{k}$ and offers M_i in period $\overline{k}$ minus a payment

$$\Delta=\Pi_i^{M_i}(\widehat{x}_{-i})-\left[\widehat{t}_i-c(\widehat{x})\right]-\varepsilon,$$

where $\varepsilon>0$ is small. By construction, this deviation will make the principal better off than in equilibrium even if she continues to use her equilibrium strategy with other agents. (She might be able to do even better by changing her strategies with other agents.) Since rejecting the agent's deviation cannot make the principal better off than in equilibrium (this option was available to her in equilibrium), she will strictly prefer to accept the deviation. Since M_i is acceptable, the agent's payoff from this deviation will be at least Δ for any beliefs. Since in equilibrium $\widehat{t}_i\leq u_i(\widehat{x})$, we see that

$$\Delta\geq\Pi_i^{M_i}(\widehat{x}_{-i})-[u_i(\widehat{x})-c(\widehat{x})]-\varepsilon>0$$

when condition (3) is violated and ε is sufficiently small. Thus, the agent's deviation is profitable. ■

Observe that the necessary condition (3) is stronger the larger is its right-hand side, i.e., the higher the profit the principal receives from the compact acceptable menu M_i to which she deviates. This leads to the following corollary to Proposition 1:

Corollary 1 *Any pure-strategy weak PBE outcome* $(\widehat{x},\widehat{t})$ *of a bilateral contracting game must satisfy*

$$u_i(\widehat{x})-c(\widehat{x})\geq\sup_{M_i\in\mathcal{A}_i}\Pi_i^{M_i}(x_{-i})\equiv\overline{\Pi}_i(x_{-i}). \tag{4}$$

In principle, identifying the bound in (4) involves identifying a distinct profit-maximizing compact acceptable menu for each type x_{-i} (assuming that such an optimal menu exists – i.e. that the sup is actually attained). In fact, however, it turns out that there is often a *single* compact acceptable menu that maximizes the principal's profit for *every* type that she may have. This point goes back to Myerson's [1983] "inscrutability principle" and Maskin and Tirole's [1990] analysis of signaling by an informed principal. Formally, we call such a menu an *RSW menu*, in accord with the terminology introduced by Maskin and Tirole:

Definition 2 *A menu $R_i \in \mathcal{A}_i$ is an RSW menu for agent i if $\Pi_i^{R_i}(x_{-i}) = \overline{\Pi}_i(x_{-i})$ for all $x_{-i} \in \mathcal{X}_{-i}$.*

When there is an RSW menu R_i, we shall refer to the profit function $\overline{\Pi}_i(x_{-i})$ as the *RSW profit function.* If an RSW menu exists, it suffices for purposes of attaining the bound in (4) to consider the principal's deviations to this menu, regardless of her type. At first it may appear that the existence of an RSW menu would be an unlikely coincidence. However, it turns out that this existence is quite natural. It is based on the following observation:

Lemma 1 *The union of acceptable menus is an acceptable menu.*

Proof. Suppose $M = \cup_{s\in S} M_s$, where M_s is an acceptable menu to agent i for each $s \in S$. Take any $\overline{x}_{-i} \in \mathcal{X}_{-i}$, and any $\left(\overline{x}_i, \overline{t}_i\right) \in \arg\max_{(x_i,t_i)\in M} \left[t_i - c(x_i, \overline{x}_{-i})\right]$. By construction, $\left(\overline{x}_i, \overline{t}_i\right) \in M_s$ for some $s \in S$, and $\left(\overline{x}_i, \overline{t}_i\right) \in \arg\max_{(x_i,t_i)\in M_s} \left[t_i - c(x_i, \overline{x}_{-i})\right]$. Since M_s is acceptable, we must have $u_i(\overline{x}_i, \overline{x}_{-i}) - \overline{t}_i \geq 0$. ■

Lemma 1 suggests that a natural candidate for an RSW menu is the union $\Omega_i = \cup\mathcal{A}_i \equiv \cup_{M_i\in\mathcal{A}_i} M_i$ of all compact acceptable menus to agent i. Indeed, by definition of Ω_i if optimal choices from Ω_i exist for all x_{-i} then we have

$$\begin{aligned}
\Pi_i^{\Omega_i}(x_{-i}) &= \max_{(x_i,t_i)\in\cup\mathcal{A}_i} \left[t_i - c(x_i, x_{-i})\right] = \sup_{M_i\in\mathcal{A}_i} \max_{(x_i,t_i)\in M_i} \left[t_i - c(x_i, x_{-i})\right] \\
&= \sup_{M_i\in\mathcal{A}_i} \Pi_i^{M_i}(x_{-i}) = \overline{\Pi}_i(x_{-i}).
\end{aligned}$$

A technical complication may arise when the acceptable menu Ω_i is not compact and optimal choices do not exist for some x_{-i}, which may happen since the set $\mathcal{A}_i$ of compact acceptable menus is usually infinite. In our cases of interest, however, there will always exist a compact RSW menu $R_i \subset \Omega_i$.[12] (We will find it convenient to consider the smallest possible RSW menu, by eliminating all the bundles that are never optimal for the principal.)

Is condition (4) also sufficient for a trade profile $\widehat{x}$ to emerge in a weak PBE of a given bilateral contracting game? The answer is no: the set of equilibrium outcomes in a bilateral contracting game may be a proper subset of those satisfying (2) and (4). The reasons for additional restrictions on outcomes differ from game to game. We illustrate this point in the next two subsections by characterizing weak PBE outcomes of two particular bilateral contracting games: the offer game and the bidding game.

3.2 Necessary and Sufficient Conditions in Offer Games

In the offer game, even when the principal cannot raise profit by deviating to an RSW menu to one agent, she may be able to deviate profitably by offering RSW menus to several agents. This can lead to further restrictions on the set of sustainable outcomes.

To consider the profitability of such deviations, we need to extend the concept of an RSW profit function to sets of agents:

Definition 3 *The RSW profit function for a set $D \subset N$ is*

$$\overline{\Pi}_D(x_{-D}) = \sup_{M_i \in \mathcal{A}_i \text{ for } i \in D} \; \max_{(x_D, t_D) \in \prod_{i \in D} M_i} \left[\sum_{i \in D} t_i - c(x_D, \hat{x}_{-D}) \right].$$

Note that when an RSW menu R_i exists for each agent i, the RSW profit function above can be written as

$$\overline{\Pi}_D(x_{-D}) = \max_{(x_D, t_D) \in \prod_{i \in D} R_i} \left[\sum_{i \in D} t_i - c(x_D, \hat{x}_{-D}) \right].$$

Consideration of such multilateral deviations gives rise to the following characterization of weak PBE outcomes:

[12] Were this not the case, we would proceed by approximating the RSW profit arbitrarily closely using compact acceptable subsets of Ω_i. Maskin and Tirole [1990] indeed perform such approximation in a model where the sets $\mathcal{X}_i$ are finite. The RSW profit in their model is obtained with a menu on the boundary of Ω_i, in which some but not all of the principal's optimal choices satisfy the agent's participation contraint. Maskin and Tirole use the existence of an arbitrarily close (compact) acceptable menu in which the principal has a unique optimal choice.

Proposition 2 *Suppose that $u_i(\cdot)$ is bounded below for all i. Then $\left(\widehat{x}, \widehat{t}\right)$ is a weak PBE outcome of the offer game if and only if agents' participation constraints (2) hold, and*

$$\sum_{i \in D} \hat{t}_i - c(\hat{x}) \geq \overline{\Pi}_D(\widehat{x}_{-D}) \text{ for all } D \subset N. \tag{5}$$

Proof. Necessity: If (2) did not hold, an agent would profitably deviate by rejecting the principal's offer. If (5) did not hold, the principal could profitably deviate to a set of agents $D \subset N$ by offering each agent $i \in D$ a menu $M_i \in \mathcal{A}_i$ minus a small payment $\varepsilon > 0$. All agents from D would accept, and the principal would guarantee herself a payoff arbitrarily close to $\sum_{i \notin D} \widehat{t}_i + \overline{\Pi}_D(x_{-D}) > \sum_i \hat{t}_i - c(\hat{x})$.

Sufficiency: A weak PBE sustaining $\left(\widehat{x}, \widehat{t}\right)$ is described by the following strategies and beliefs. The principal's strategy is to offer the point contract $\widehat{M}_i = \left\{\left(\widehat{x}_i, \widehat{t}_i\right)\right\}$ to each agent i. Following any accepted menu profile $(M_1, ..., M_N)$ in which only one agent, say j, accepts a non-degenerate menu, the principal chooses from among those elements of $(x, t) \in \arg\max_{(x,t) \in \prod_i M_i} \left[\sum_i t_i - c(x)\right]$ that have the lowest value of $u_j(x) - t_j$.[13] Each agent i's strategy is to accept $\widehat{M}_i$ and all menus from $\mathcal{A}_i$, and to reject all other menus. For any menu $M_i \notin \mathcal{A}_i$ we can find $\overline{x}_{-i}(M_i) \in \mathcal{X}_{-i}$ such that there exists $(x_i', t_i') \in \arg\max_{(x_i,t_i) \in M_i} [t_i - c(x_i, \overline{x}_{-i}(M_i))]$ with $u_i(x_i', \overline{x}_{-i}(M_i)) - t_i' < 0$. Let agent i believe following any menu $M_i \neq \widehat{M}_i$ such that $M_i \notin \mathcal{A}_i$ that the principal's offer to any other agent $j \neq i$ is $M_j = \left\{\left(\overline{x}_j(M_i), \overline{t}\right)\right\}$, where $\overline{t} < \inf_{x \in \mathcal{X}_1 \times ... \times \mathcal{X}_N} u_j(x)$ for all j.

Agents' participation constraints (2) ensure that agents have no profitable deviations. The principal's deviation to any menu $M_i \notin \mathcal{A}_i$ will be rejected by agent i given his beliefs and the principal's strategy. The principal's deviation to any menus $M_i \in \mathcal{A}_i$ (including the null contract) to agents $i \in D$ cannot give her a higher payoff than $\sum_{i \notin D} \widehat{t}_i + \overline{\Pi}_D(x_{-D})$, which by (5) does not exceed her equilibrium payoff. ■

3.3 Necessary and Sufficient Conditions in Bidding Games

Finally, we characterize the equilibria of the bidding game. Recall that in the bidding game, agents simultaneously offer menus to the principal, who then

[13] We can specify any optimal choices by the principal for all other profiles of accepted contracts.

chooses which menus to accept and makes choices from the accepted menus. Note that in contrast to the offer game, in a bidding game we need to consider only unilateral deviations, just as we did in developing the necessary condition in Proposition 1. The additional constraints on equilibrium outcomes in bidding games relative to Proposition 1 come from a different source: since an agent in the bidding game knows the equilibrium menus offered by other agents, he can predict the principal's choices from them following his deviation. Thus, his beliefs about the principal's trades with other agents are not arbitrary, which creates the possibility for more profitable deviations, and hence fewer sustainable outcomes.

One issue that arises in characterizing equilibrium outcomes in bidding games is that in situations in which the principal is indifferent among a number of trade profiles she may be able to prevent an agent from reducing his transfer by (effectively) threatening to choose a less desirable outcome from among these profiles. This possibility is ruled out if the principal's choice is invariant to additive transformations in the menus she chooses from; that is, if the addition of lump-sum payments (of either sign) to a set of menus does not alter the principal's optimal choices from these menus.

Proposition 3 *Suppose the agents use strategies* $M = (M_1, \ldots, M_N)$*. For* $S \subset N$ *and* $x_{-S} \in \prod_{j \in N \setminus S} \mathcal{X}_j$*, define*

$$\pi_S(x_{-S}) = \max_{(x_S, t_S) \in \prod_{j \in S} [M_j \cup (0,0)]} \left[\sum_{j \in S} t_j - c(x_S, x_{-S}) \right].$$

The outcome $\left(\widehat{x}, \widehat{t}\right)$ *can arise in a subgame perfect Nash equilibrium*[14] *in which agents use strategies* M *only if*

(i)

$$\sum_j \widehat{t}_j - c(\widehat{x}) = \pi_N,$$

(ii) For all $i \in N$*,*

$$u_i(\widehat{x}) - \widehat{t}_i \geq 0,$$

[14] Observe that the bidding game is a game of perfect information. Therefore the concept of weak PBE reduces to that of subgame perfect Nash equilibrium.

(iii) For all $i \in N$ and $x_i \in \mathcal{X}_i$ there exists

$$(x_{-i}, t_{-i}) \in \arg \max_{(x_{-i},t_{-i}) \in \prod_{j \in N|i}[M_j \cup (0,0)]} \left[\sum_{j \neq i} t_j - c(x_i, x_{-i}) \right]$$

such that

$$u_i(x_i, x_{-i}) + \pi_{-i}(x_i) \leq u_i(\widehat{x}) + \sum_{j \neq i} \widehat{t}_j - c(\widehat{x}).$$

If, in addition, the principal's choice is invariant to additive transformations in the menus she chooses from, then

(iv) For all $i \in N$,

$$\sum_j \widehat{t}_j - c(\widehat{x}) = \pi_{-i}(0).$$

Moreover, if conditions (i)-(iv) hold, then $\left(\widehat{x}, \widehat{t}\right)$ can be supported as a subgame perfect Nash equilibrium outcome in which the agents use strategies M.[15]

Proof. Necessity: (i) says that the principal's choice from agents' equilibrium menus is optimal, which is necessary for subgame perfection. If condition (ii) did not hold for agent i then i would profitably deviate by offering $(0, 0)$. If (iii) did not hold for some i and x_i, agent i would deviate to $M_i' = \{(x_i, t_i)\}$, where $t_i = \pi_{-i}(0) - \pi_{-i}(x_i) + \varepsilon$ for a small $\varepsilon > 0$. This deviation will be accepted by the principal, and let (x_{-i}, t_{-i}) denote the agent's choice from other agents' menus following it. Then agent i's payoff will be

$$\begin{aligned} u_i(x_i, x_{-i}) - t_i &= u_i(x_i, x_{-i}) - \pi_{-i}(0) + \pi_{-i}(x_i) - \varepsilon \\ &> u_i(\widehat{x}) + \sum_{j \neq i} \widehat{t}_j - c(\widehat{x}) - \pi_{-i}(0) \geq u_i(\widehat{x}) - \widehat{t}_i, \end{aligned}$$

[15]Observe that conditions (iii) and (iv) imply condition (ii) of Proposition 3: using (iii) with $x_i = 0$ and (iv), we can write

$$u_i(\widehat{x}) + \sum_{j \neq i} \widehat{t}_j - c(\widehat{x}) \geq \pi_{-i}(0) = \sum_j \widehat{t}_j - c(\widehat{x}),$$

which implies (ii).

where the strict inequality is by the violation of (iii), and the weak inequality is by (i). Therefore, the deviation would make agent i better off. Finally, if the principal's choice is invariant to additive transformations in the menus she chooses from and if (ii) did not hold, then agent i would deviate to offer menu M_i minus a small $\varepsilon > 0$. The principal would accept the deviation and choose $(x_i, t_i) = (\widehat{x}_i, \widehat{t}_i - \varepsilon)$ and $(x_{-i}, t_{-i}) = (\widehat{x}_{-i}, \widehat{t}_{-i})$, making agent i better off.

Sufficiency: (i) ensures that the principal does not want to deviate. Let the principal's strategy in response to a deviation M_i' by agent i be to chose $(x_i, t_i) \in \arg\max_{M_i'} [t_i + \pi_{-i}(x_i)]$ and

$$(x_{-i}, t_{-i}) \in \underset{(x_{-i},t_{-i})\in\arg\min_{(x_{-},t_{-i})\in\prod_{j\in N\setminus i}\left[M_j\cup(0,0)\right]}\left[\sum_{j\neq i}t_j - c(x_i,x_{-i})\right]}{\arg\min} [u_i(x_i, x_{-i}) - t_i] .$$

If agent i's deviation M_i' is accepted by the principal and she chooses $(x_i, t_i) \in M_i'$, we must have $t_i \geq \pi_{-i}(0) - \pi_{-i}(x_i)$. But then, agent i's payoff from the deviation is

$$u_i(x_i, x_{-i}) - t_i \leq u_i(x_i, x_{-i}) + \pi_{-i}(x_i) - \pi_{-i}(0) \leq u_i(\widehat{x}) + \sum_{j\neq i} \widehat{t}_j - c(\widehat{x}) - \pi_{-i}(0) = u_i(\widehat{x}),$$

where the second inequality uses (iii) and the last equality uses (iv). Therefore, the deviation cannot be profitable. ■

Condition (iii) of Proposition 3 captures the fact that an agent i can predict the principal's reaction to his deviation based on his knowledge of the menus offered in equilibrium by other agents. It can be verified that condition (iii) implies the necessary condition developed in Proposition 1. We will see in Section 6, however, that it can lead to further restrictions in the set of sustainable outcomes beyond this necessary condition.

4 Application: Necessary Conditions and Competitive Convergence

In the remainder of the paper we illustrate the application of the results of Section 3 to a particular class of payoff functions with externalities. In particular, we suppose that $\mathcal{X}_i \subset \mathbb{R}_+$ such that $[0, X^c] \subset \mathcal{X}_i$ for each agent i, and that the agent's payoff is $u_i(x_i, x_{-i}) - t_i = \alpha(X)x_i - t_i$, where $X = \sum_{i=1}^N x_i$.

The principal's payoff is $\sum_{i=1}^{N} t_i - c(X)$.[16] We assume that $\alpha(\cdot)$ is bounded from below and that $c(\cdot)$ is lower semicontinuous. We also normalize $c(0) = 0$. Note that this setting includes as a special case the vertical contracting example discussed in Section 2, studied by Hart and Tirole [1990] and McAfee and Schwartz [1994]. Other models whose structure falls into this class include the insurance with moral hazard model studied by Pauly [1974], Kahn and Mookerjee [1998], and Bisin et al. [1998, 1999] (see Segal [1999] for other examples).

For example, in the insurance model, a single risk-averse individual with constant absolute risk aversion $r \geq 0$ and initial wealth W (the principal in our terminology) may contract with any of N risk-neutral firms. The individual expends the unobservable monetary level of care e, which determines the probabilty $\pi(e)$ of a loss of $L > 0$. Letting $x_i \geq 0$ denote the payment promised by agent i in the event of a loss, the certainty equivalent of the principal's payoff is

$$\max_e (W + \sum_i t_i - e) - \frac{1}{r} \ln[1 + \pi(e)(\exp\{-r(X - L)\} - 1)].$$

The principal chooses care $e(X)$ to maximize this expression. By the Monotone Selection Theorem of Milgrom and Shannon [1994], $e(X)$ is nonincreasing in X. The payoff of each agent i is then $-\pi(e(X))x_i - t_i$.

Our aim in this section is to derive certain properties of outcomes of bilateral contracting games using Proposition 1. Among the results we establish is a "competitive limit" result as the number of agents grows infinitely large. For this purpose it is helpful to define the following notion of competitive equilibrium:

Definition 4 *A price-quantity pair* $(p^c, X^c) \in \mathbb{R}^2$ *is a competitive equilibrium if (i)* $p^c = \alpha(X^c)$ *and (ii)* $X^c \in \arg\max_{X \in \mathbb{R}_+} p^c X - c(X)$. *It is a strict competitive equilibrium if in addition* $\arg\max_{X \in \mathbb{R}_+} [p^c X - c(X)] = \{X^c\}$.

Condition (ii) of Definition 4 says that X^c is the principal's optimal supply given the market price p^c, while condition (i) implies that agents are willing to demand an aggregate quantity of X^c if each agent takes price p^c and the aggregate trade X^c as given. Figure 1 in Section 2 depicts a competitive equilibrium in the vertical contracting example. Figure 2 depicts

[16] These payoffs corresponds to the case of Condition L with no externalities on non-traders in Segal [1999].

a competitive equilibrium in the insurance with moral hazard model when $e \in \{0,1\}$. Observe that the shaded area A must be at least as large as the cross-hatched area B for (p^c, X^c) to be a competitive equilibrium. A competitive equilibrium (p^c, X^c) is *strict* if X^c is the principal's unique optimal trade when facing price p^c.

We will also study some situations in which a competitive equilibrium does not exist. In these situations, we shall use the following weaker notion of competitive equilibrium:

Definition 5 *A price-quantity pair $(p^c, X^c) \in \mathbb{R}^2$ is a partial competitive equilibrium relative to $\overline{X} \geq 0$ if (i) $p^c = \alpha(X^c)$ and (ii) $X^c \in \arg\max_{X \in [\overline{X}, +\infty)} p^c X - c(X)$. It is a strict partial competitive equilibrium relative to $\overline{X} \geq 0$ if in addition $\arg\max_{X \in [\overline{X}, +\infty)} [p^c X - c(X)] = \{X^c\}$.*

Figures 3(a) and (b) depict partial competitive equilibria in the vertical contracting and insurance examples respectively. Observe that in Figure 3(b), area A exceeds area B, but is smaller than areas B and C combined, and so while (p^c, X^c) is a partial competitive equilibrium relative to $\overline{X}$, it is not a competitive equilibrium. Note also from the definition that (p^c, X^c) is a (strict) partial competitive equilibrium relative to $\overline{X} = 0$ if and only if (p^c, X^c) is a (strict) competitive equilibrium.

Recall that Proposition 1 provides a condition that a contracting outcome must satisfy for deviations to a given acceptable menu to be unprofitable. In what follows, we shall focus on one particular menu, the *competitive menu* $C = \{(x,t) : t = p^c x \text{ and } x \in [0, X^c]\}$. Our first result establishes conditions under which this menu is acceptable:

Lemma 2 *Suppose that (p^c, X^c) is a strict partial competitive equilibrium relative to $\overline{X} \geq 0$. Suppose also that $p^c X - c(X)$ is decreasing on $\left[0, \overline{X}\right] \cup [X^c, +\infty)$.*[17] *Then the competitive menu $C = \{(x, p^c x) : x \in [0, X^c]\}$ is acceptable, and the associated profit function is*

$$\Pi^C(X_{-i}) \quad = \quad \begin{cases} -c(X_{-i}) & \text{when } X_{-i} \leq \widetilde{X} \text{ or } X_{-i} \geq X^c, \\ p^c(X^c - X_{-i}) - c(X^c) & \text{when } X_{-i} \in \left(\widetilde{X}, X^c\right), \end{cases} \tag{6}$$

[17] The condition can be replaced with the assumptions that $p^c X - c(X)$ is nonincreasing for $X > X^c$ and $\alpha(X) \geq \alpha(X^c)$ for all $X \in [0, \overline{X})$. In this case, if the principal does not choose $X \in \{X^c, X_{-i}\}$, she must choose $X < \overline{X}$, and the agent is again assured a nonnegative payoff.

$$\textit{where } \widetilde{X} \;=\; \begin{cases} 0 \textit{ if } (p^c, X^c) \textit{ is a strict competitive equilibrium,} \\ \max\left\{X \in [0, \overline{X}] : p^c X - c(X) \geq p^c X^c - c(X^c)\right\} \textit{ otherwise.} \end{cases}$$

Proof. Note first that the maximum in the definition of $\widetilde{X}$ exists by lower semicontinuity of $c(\cdot)$. Given the menu C, the principal chooses the aggregate trade to solve $\max_{X \in [X_{-i}, X^c]} p^c X - c(X)$. Under the assumptions, this maximization is achieved by choosing $X = X_{-i}$ when $X_{-i} > X^c$ (otherwise the principal would gain by reducing X), by choosing $X = X^c$ when $X_{-i} \in (\widetilde{X}, X^c]$ (by the definition of $\widetilde{X}$), by choosing $X = X_{-i}$ if $X_{-i} = \widetilde{X}$ and $p^c \widetilde{X} - c(\widetilde{X}) > p^c X^c - c(X^c)$, by choosing $X \in \{X_{-i}, X^c\}$ if $X_{-i} = \widetilde{X}$ and $p^c \widetilde{X} - c(\widetilde{X}) = p^c X^c - c(X^c)$, and by choosing $X = X_{-i}$ when $X_{-i} \leq \widetilde{X}$ (choosing $X > \widetilde{X}$ is ruled out by the above arguments, and if $X \in \left(X_{-i}, \widetilde{X}\right)$ the principal would gain by reducing X). This implies that the profit function associated with C is given by 6. Since the principal in all cases chooses $X \in \{X_{-i}, X^c\}$, the agent necessarily receives a zero payoff. Hence, C is an acceptable menu. ■

Note that when $c(\cdot)$ is differentiable, the assumption that $p^c X - c(X)$ is decreasing on $\left[0, \overline{X}\right] \cup [X^c, +\infty)$ means that the marginal cost exceeds p^c on these intervals.

Since C is an acceptable menu, Proposition 1 implies that any weak PBE outcome $(\widehat{x}, \widehat{t})$ of a bilateral contracting game must satisfy

$$\alpha(\widehat{X})\widehat{x}_i - c(\widehat{X}) \geq \Pi^C(\widehat{X}_{-i}) \text{ for all } i \in N. \tag{7}$$

Condition (7) gives a lower bound on the *bilateral surplus* of the principal and agent i, $\alpha(\widehat{X})\widehat{x}_i - c(\widehat{X})$. One might wonder whether considering deviations to acceptable menus other than C might lead to tighter restrictions on the set of equilibrium trades. In the next section we identify conditions under which the competitive menu is in fact an RSW menu, and so (7) represents the tightest bound that we can get using Proposition 1. (This is true, for example, when the marginal cost $c'(\cdot)$ is either increasing or U-shaped.) In this section, we show that condition (7) gives rise to significant restrictions on the set of contracting outcomes.

We begin with a result providing conditions under which we can be assured that the aggregate trade in any weak PBE of a bilateral contracting game cannot exceed X^c.

Proposition 4 *Suppose that the assumptions of Lemma 2 hold and that* $\alpha(X) \leq p^c$ *for all* $X \geq X^c$. *Then any weak PBE outcome* $(\widehat{x}, \widehat{t})$ *of a bilateral contracting game must have an aggregate trade level* $\widehat{X} \leq X^c$.[18]

Proof. Suppose not and consider an agent i with $\widehat{x}_i > 0$. If $\widehat{X}_{-i} > X^c$, then

$$\begin{aligned}\alpha(\widehat{X})\widehat{x}_i - c(\widehat{X}) &\leq p^c\widehat{x}_i - c(\widehat{X}) = [p^c\widehat{X} - c(\widehat{X})] - p^c\widehat{X}_{-i} \\ &< [p^c\widehat{X}_{-i} - c(\widehat{X}_{-i})] - p^c\widehat{X}_{-i} = -c(\widehat{X}_{-i}) \leq \Pi^C(\widehat{X}_{-i}),\end{aligned}$$

where the first inequality follows from the assumption that $\alpha(X) \leq \alpha(X^c)$ for all $X > X^c$, the second from the assumption that $p^cX - c(X)$ is decreasing on $[X^c, +\infty)$, and the third from (6). But this violates condition (3) for deviations to the acceptable menu C to agent i.

If, on the other hand, $\widehat{X}_{-i} \leq X^c$, then, using (6), we have

$$\Pi^C(\widehat{X}_{-i}) \geq p^c(X^c - \widehat{X}_{-i}) - c(X^c) > p^c\widehat{x}_i - c(\widehat{X}) \geq \alpha(\widehat{X})\widehat{x}_i - c(\widehat{X}),$$

where the strict inequality follows from the definition of strict competitive equilibrium, and the weak inequality from the assumption that $\alpha(X) \leq \alpha(X^c)$ for all $X > X^c$. But this again violates condition (3). ■

Since $\widehat{X}_{-i} \leq \widehat{X} \leq X^c$ for all i, we must have $\Pi^C(\widehat{X}_{-i}) \geq p^c(X^c - \widehat{X}_{-i}) - c(X^c)$. Thus, condition (7) implies that any weak PBE outcome $(\widehat{x}, \widehat{t})$ of a bilateral contracting game must satisfy

$$\alpha(\widehat{X})\widehat{x}_i - c(\widehat{X}) \geq p^c(X^c - \widehat{X}_{-i}) - c(X^c) \text{ for all } i \in N. \tag{8}$$

Condition (8) can be interpreted as a constraint on the *bilateral surplus* of the principal and agent i, $\alpha(X)x_i - c(X)$. Defining the function $B(X, X_{-i}) = \alpha(X)(X - X_{-i}) - c(X)$, giving the bilateral surplus when the aggregate trade with agents $j \neq i$ is X_{-i} and the aggregate trade is X (and so the trade with agent i is $x_i = (X - X_{-i})$), condition (8) can be written as

$$B(X, X_{-i}) \leq B(X^c, X_{-i}) \text{ for all } i = 1, ..., N. \tag{9}$$

[18] If $\alpha(X) > \alpha(X^c)$ for $X > X^c$, equilibrium outcomes with $X > X^c$ are possible. For example, when $\alpha(\cdot)$ is increasing, the efficient aggregate trade X^* exceeds X^c and is sustainable by trading X^* with one agent and zero with all others (because of the nonnegativity of trades).

Figure 3 depicts this constraint: the shaded region in the figure is the bilateral surplus $B(X, X_{-i})$ at the equilibrium outcome, while the cross-hatched region is the bilateral surplus $B(X^c, X_{-i})$.

In Section 2 we already observed that condition (8) imposes a significant restriction on the set of equilibrium outcomes. Here we show that whenever a strict competitive equilibrium exists, under mild additional assumptions, as the number of agents N grows large, this condition *determines completely* the equilibrium aggregate trade: it must approach the competitive equilibrium trade X^c.

Proposition 5 *Suppose that a strict competitive equilibrium* (p^c, X^c) *exists, that* $p^c X - c(X)$ *is decreasing for* $X \geq X^c$*, and that* $\alpha(X) \leq p^c$ *for* $X \geq X^c$*. Suppose that, in addition,* $\alpha(X)$ *is continuous at* $X = X^c$*, and the aggregate surplus* $W(X) = \alpha(X)X - c(X)$ *is bounded above. If* $\{\widehat{X}^N\}_{N=1}^{\infty}$ *is a sequence of weak PBE aggregate trades of a bilateral contracting game with* N *agents, then* $\widehat{X}^N \to X^c$ *as* $N \to \infty$.

Proof. Adding up condition (8) over all $i \in N$, and letting $\widehat{X} = \widehat{X}^N$, we obtain

$$\alpha(\widehat{X}^N)\widehat{X}^N - Nc(\widehat{X}^N) \geq p^c \left(NX^c - (N-1)\,\widehat{X}^N\right) - Nc(X^c). \qquad (10)$$

Dividing by $N-1$, the inequality can be rewritten as

$$\begin{aligned}
[p^c\widehat{X}^N - c(\widehat{X}^N)] &\geq \frac{N}{N-1}[p^c X^c - c(X^c)] - \frac{1}{N-1}W(\widehat{X}^N) \\
&\geq \frac{N}{N-1}[p^c X^c - c(X^c)] - \frac{1}{N-1}\sup_X W(X) \\
&\to p^c X^c - c(X^c) \text{ as } N \to \infty.
\end{aligned}$$

Since $c(\cdot)$ is lower semi-continuous, this implies that $\widehat{X}^N \to \arg\max_X [p^c X - c(X)] = X^c$ as $N \to \infty$. ■

This result provides a noncooperative foundation for the competitive equilibrium when it exists (see, e.g., Pauly [1974] and Bisin and Gottardi [1998] in the insurance model). Our next result provides a bound on the limiting behavior of aggregate trades in cases in which a competitive equilibrium does *not* exist, but a partial competitive equilibrium exists. For example, this can arise in the vertical contracting model when the manufacturer has U-shaped marginal cost, and in the insurance model with two possible care levels when the insured individual would not want to buy any insurance at a price that is actuarially fair given the low level of care.

Proposition 6 *Suppose that* (p^c, X^c) *is a strict partial competitive equilibrium relative to* $\overline{X} > 0$*, but not a competitive equilibrium. Suppose also that* $p^c X - c(X)$ *is decreasing on* $\left[0, \overline{X}\right] \cup [X^c, +\infty)$*, and that* $\alpha(X) \leq p^c$ *for* $X \geq X^c$*. Suppose that, in addition,* $\alpha(X)$ *is continuous at* $X = X^c$*, and the aggregate surplus* $W(X) = \alpha(X)X - c(X)$ *is bounded above. Let* $\{\widehat{X}^N\}_{N=1}^{\infty}$ *be a sequence of weak PBE aggregate trades. Then* $\limsup_{N\to\infty} \widehat{X}^N \leq \widetilde{X}$*, where* $\widetilde{X}$ *is defined in (6).*[19]

Proof. Suppose in negation that there exists a subsequence $\{\widehat{X}^K\}_{K=1}^{\infty} \subset \{\widehat{X}^N\}_{N=1}^{\infty}$ such that $\widehat{X}^K \to X^0 > \widetilde{X}$ as $K \to \infty$. By the same arguments as in Proposition 5, (8) implies

$$[p^c \widehat{X}^K - c(\widehat{X}^K)] \to p^c X^c - c(X^c) \text{ as } K \to \infty.$$

Take $\delta \in \left(0, X^0 - \widetilde{X}\right)$. Since (p^c, X^c) is a strict partial competitive equlibrium relative to $X^0 - \delta > \widetilde{X}$, $\widehat{X}^K > X^0 - \delta$ for K large enough, and $c(\cdot)$ is lower semicontinuous, we must then have $\widehat{X}^K \to \arg\max_{X \geq X^0 - \delta}[p^c X - c(X)] = X^c$ as $K \to \infty$.

On the other hand, since (p^c, X^c) is not a competitive equilibrium, according to Lemma 2 we must have $Arg \max_X [p^c X - c(X)] = \{0\}$, and therefore $p^c X^c - c(X^c) < 0$. By continuity of $\alpha(\cdot)$ at X^c and the lower semi-continuity of $c(\cdot)$, this implies that $\limsup_{K\to\infty} \left[\alpha(\widehat{X}^K)\widehat{X}^K - c(\widehat{X}^K)\right] < 0$. But then total surplus is negative for some K, which contradicts the fact that all agents and the principal must have nonnegative payoffs in any equilibrium. ■

A related result has been obtained in the insurance model by Kahn and Mookherjee's [1998], who call the threshold $\widetilde{X}$ the "third-best" outcome. (Similar noncompetitive outcomes have been observed by Bisin and Gottardi [1998] in the insurance model and Rajan and Parlour [1998] in the lending

[19] This statement means that for any $X' > \widetilde{X}$ there exists N' such that $\widehat{X}^N < X'$ for all $N \geq N'$.

model.) In the bilateral contracting game they consider, in equilibrium the principal obtains precisely the aggregate amount $\widetilde{X}$ of insurance. At this aggregate trade level, the principal is exactly indifferent between buying an additional amount $X^c - \widetilde{X}$ of insurance at the given price p^c and not buying any additional insurance. Proposition 6 establishes that under the appropriate assumptions, the equilibrium aggregate trade in a bilateral contracting game cannot exceed $\widetilde{X}$ when the number of agents is sufficiently large. Intuitively, if $\widehat{X} \in \left(\widetilde{X}, X^c\right)$, then there will exist a profitable bilateral deviation between the principal and an agent i whose trade with the principal is sufficiently small. In this deviation the parties will sign the competitive contract C, from which the principal then chooses quantity $X^c - \widehat{X}_{-i}$, so that the aggregate trade becomes X^c. On the other hand, the aggregate trade X^c itself cannot be sustained in equilibrium, for this would result in negative total surplus. Therefore, asymptotically we must have $\widehat{X} \leq \widetilde{X}$.

This prediction of our model should be contrasted with the "convexification" approach used in general equilibrium theory. The convexification approach considers the case of a continuum of identical principals, which ensures that their aggregate supply correspondence is convex and thus a "competitive equilibrium" at which aggregate supply equals aggregate demand exists. In economies without externalities this approach is justified, since a seller is indifferent between, say, selling 5 apples to each buyer A and buyer B, and selling all 10 apples to buyer A. However, the use of convexification in economies with externalities, such as the lending model of Dubey et al. [1996], is incorrect. Intuitively, a lender is usually not indifferent between lending \$5 to each borrower A and borrower B, and lending all \$10 to borrower A. (For example, imagine that a borrower who borrows more than \$7 might default.) For this reason, lenders will not treat borrowers as anonymous, as the convexification approach assumes, but rather contract with each borrower individually, as in our noncooperative bargaining model. Our analysis establishes that the aggregate trade in noncooperative contracting with a large number of agents must be below the "third-best" aggregate trade, and therefore quite different from the "convexified competitive equilibrium," as depicted in Figure 4.

5 Application: RSW Menus and Outcomes of Offer Games

In this section, we illustrate the use of Proposition 2 by characterizing the set of weak PBE outcomes of the offer game for the specialized payoffs specified in Section 4. We do this by first identifying an RSW menu, and using this menu to express the necessary and sufficient conditions in Proposition 2.

5.1 A Strict (Partial) Competitive Equilibrium Exists

We first provide conditions under which the competitive menu is an RSW menu:

Lemma 3 *Suppose that (p^c, X^c) is a strict partial competitive equilibrium relative to $\overline{X}$, and*

(i) $c(\cdot)$ is differentiable, with $c'(X) \leq p^c$ if and only if $X \in \left[\overline{X}, X^c\right]$, and $c'(\cdot)$ is nonincreasing for $X \leq \overline{X}$,

(ii) $\alpha(\cdot)$ is continuous, and nonincreasing for $X \geq X^c$.

Then the competitive menu C is an RSW menu.

Proof. In appendix.

Using this RSW menu, we can express the function $\overline{\Pi}_D(x_{-D})$ used in Proposition 2 as

$$\overline{\Pi}_D(x_{-D}) = \max_{(x_i,t_i)\in C \text{ for } i\in D} \left[\sum_{i\in D} t_i - c(\sum_{i\in D} x_i + \sum_{i\notin D} \widehat{x}_i)\right].$$

This function expresses the profitability of the principal's multilateral deviations. In particular, for the principal's deviations to a single agent $i = \{D\}$, we have $\overline{\Pi}_i(x_{-i}) = \Pi^C(\widehat{X}_{-i})$, where $\Pi^C(\cdot)$ is given by (6). Thus, a necessary condition for $(\widehat{x}, \widehat{t})$ to be a weak PBE outcome of the offer game is

$$\widehat{t}_i - c(\widehat{X}) \geq \Pi^C(\widehat{X}_{-i}) \text{ for all } i \in N.$$

It turns out that when a strict competitive equilibrium exists, this condition also ensures that the principal's multilateral deviations considered in Proposition 2 are unprofitable as well. This yields the following characterization of equilibrium outcomes of the offer game:

Proposition 7 *Suppose that a strict competitive equilibrium* (p^c, X^c) *exists, and the assumptions of Lemma 3 hold. Then* $(\widehat{x}, \widehat{t})$ *is a weak PBE outcome of the offer game if and only if the agents' participation constraints hold:*

$$\alpha(X)x_i - t_i \geq 0 \text{ for all } i \in N, \tag{11}$$

$\widehat{X} \leq X^c$, *and*

$$\widehat{t}_i - c(\widehat{X}) \geq p^c(X^c - \widehat{X}_{-i}) - c(X^c) \text{ for all } i \in N. \tag{12}$$

Proof. The fact that $\widehat{X} \leq X^c$ is established in Proposition 4. This implies that $\Pi^C(\widehat{X}_{-i}) = p^c(X^c - \widehat{X}_{-i}) - c(X^c)$. The necessity of (11) and (12) follows immediately from the necessity part of Proposition 2.

For sufficiency, note first that when $\widehat{X} \leq X^c$, we have

$$\overline{\Pi}_D(\widehat{x}_{-D}) = \pi^c - p^c(\widehat{X} - \sum_{i \in D} \widehat{x}_i), \tag{13}$$

where $\pi^c = p^c X^c - c(X^c)$. Note that adding (12) over all agents in D gives

$$\sum_{i \in D} \widehat{t}_i - |D|\, c(\widehat{X}) \geq |D| \left[\pi^c - p^c \widehat{X}\right] + p^c \sum_{i \in D} \widehat{x}_i.$$

Since $p^c\widehat{X} - c(\widehat{X}) \leq \pi^c$, this implies

$$\sum_{i \in D} t_i - c(\widehat{X}) \geq \pi^c - p^c(\widehat{X} - \sum_{i \in D} \widehat{x}_i) = \overline{\Pi}_D(x_{-D}).$$

Therefore, condition (5) is satisfied, and Proposition 2 implies the result. ■

This result implies that two important kinds of outcomes are sustainable in a weak PBE of the offer game. One kind of outcome is competitive equilibrium allocations:

Corollary 2 *Under the assumptions of Proposition 7, any outcome* $(\widehat{x}, \widehat{t})$ *such that* $\widehat{t}_i = p^c \widehat{x}_i$ *for all* i *and* $\sum_i \widehat{x}_i = X^c$ *is a weak PBE outcome of the offer game.*

Proof. All such outcomes satisfy agents' participation constraints (11) and condition (12) with strict equality. Proposition 7 implies the result. ■

The other kind of sustainable outcome involves trades that maximize bilateral surplus for all principal-agent pairs as defined in Section 2:[20]

Corollary 3 *Under the assumptions of Proposition 7, any outcome* $(\widehat{x}, \widehat{t})$ *that satisfies agents' participation constraints (11) with strict equality, and that has*

$$\widehat{x}_i \in \arg\max_{x_i \in \mathcal{X}_i} x_i \alpha(x_i + \widehat{X}_{-i}) - c(x_i + \widehat{X}_{-i}) \text{ for } i \in N, \tag{14}$$

is a weak PBE outcome of the offer game.

Proof. Observe first that any outcome satisfying (14) must involve $\widehat{X} \leq X^c$. To see this, suppose not. Consider the agent i with the largest trade, $\widehat{x}_i > 0$. Then either (i) $\widehat{X} > X_{-i} \geq X^c$ or (ii) $\widehat{X} > X^c \geq X_{-i}$. The first possibility is ruled out by Lemma 6 while the second is ruled out by Lemma 7 (both lemmas are in the appendix).

Condition (14) means that for all $x_i \in \mathcal{X}_i$,

$$\widehat{x}_i \alpha(\widehat{X}) - c(\widehat{X}) \geq x_i \alpha(x_i + \widehat{X}_{-i}) - c(x_i + \widehat{X}_{-i}).$$

Substituting $x_i = X^c - \widehat{X}_{-i}$, and using agents' binding participation constraints (11), $\widehat{x}_i \alpha(\widehat{X}) = \widehat{t}_i$, we obtain condition (12). Proposition 7 implies the result. ■

When only a partial competitive equilibrium exists, we have the following characterization of equilibrium outcomes:

Proposition 8 *Suppose that the assumptions of Lemma 3 hold. Then* $(\widehat{x}, \widehat{t})$ *is a weak PBE outcome of the offer game if and only if agents' participation constraints (11) hold,* $\widehat{X} \leq X^c$*, (12) hold for all for all* $i \in N$ *such that* $\widehat{X}_{-i} \in \left[\widetilde{X}, X^c\right]$*, and*

$$\sum_{i \in D} \widehat{t}_i \geq c(\widehat{X}) - c(\widehat{X}_{-D}) \text{ for all sets } D \subset N \text{ such that } \widehat{X}_{-D} < \widetilde{X}. \tag{15}$$

[20] As we have seen in Section 2, any passive beliefs equilibrium in offer games in which agents must offer point contracts must maximize bilateral surplus for all principal-agent pairs.

Proof. Observe first that for all sets $D \subset N$ such that $\widehat{X}_{-D} \geq \widetilde{X}$, $\overline{\Pi}_D(\widehat{x}_{-D})$ is given by (**??**), while for sets $D \subset N$ such that $\widehat{X}_{-D} < \widetilde{X}$, we have $\overline{\Pi}_D(\widehat{x}_{-D}) = -c(\widehat{X}_{-i})$. The proof of the necessity part follows in a similar manner to the necessity proof of Proposition 7. For the sufficiency part, the proof that condition (5) in Proposition 2 is satisfied for sets $D \subset N$ such that $\widehat{X}_{-D} \geq \widetilde{X}$ are unprofitable is the same as in Proposition 7. For sets $D \subset N$ such that $\widehat{X}_{-D} < \widetilde{X}$, (15) ensures that condition (5) is satisfied for these sets as well. ■

Observe that (15) is exactly the condition that $(\widehat{x}, \widehat{t})$ is "subsidy-free" for all sets D having $\widehat{X}_{-D} < \widetilde{X}$ from the subsidy-free pricing literature. This condition in fact holds as well for sets D such that $\widehat{X}_{-D} \geq \widetilde{X}$ since for such sets it is implied by the facts that (12) hold for all $i \in D$ and that $p^c(X^c - X_{-i}) - c(X^c) > c(X_{-i})$ for all such i.[21]

For symmetric equilibria, Proposition 8 implies that X can be sustained as the aggregate trade in a PBE of an offer game if and only if either $(\frac{N-1}{N})X < \widetilde{X}$ and $\alpha(X)X - c(X) \geq 0$, or $(\frac{N-1}{N})X \geq \widetilde{X}$ and $\left(\frac{1}{N}\right)\alpha(X)X + \left(\frac{N-1}{N}\right)p^c X - c(X) \geq p^c X^c - c(X^c)$.

5.2 A Partial Competitive Equilibrium Does Not Exist: Decreasing Marginal Cost

We now consider a situation in which the marginal cost $c'(\cdot)$ is strictly decreasing, and thus a competitive equilibrium does not exist. In this case, we show that the null contract is an RSW menu:

Lemma 4 *Suppose that*

(i) $c(\cdot)$ is differentiable and $c'(\cdot)$ is decreasing,

(ii) there exists $\overline{X}$ such that for $X \geq \overline{X}$, $\alpha(X)$ is nonincreasing and $\alpha(X) \leq c'(X)$.

Then $N = \{(0,0)\}$ is an RSW menu.

Proof. In appendix. ■

Since the RSW menu is a point contract, allowing the principal to offer menus rather than point contracts does not reduce the set of outcomes of the

[21] This conclusion can alternatively be seen by observing that the null contract $(0,0)$ is also an acceptable menu and applying Proposition 2.

offer game in this case. Using Proposition 2, these outcomes are described as follows:

Proposition 9 *Suppose the assumptions of Lemma 4 hold. Then* $(\widehat{x}, \widehat{t})$ *is a weak PBE equilibrium outcome of the offer game if and only if agents' participation constraints (11) hold and*

$$\sum_{i \in D} \widehat{t}_i \geq c(\widehat{X}) - c(\widehat{X}_{-D}) \text{ for all sets } D \subset N. \tag{16}$$

For a given $\widehat{x} \geq 0$, a necessary and sufficient condition for conditions (16) and (2) to hold for some transfer profile $\widehat{t}$ is that

$$\alpha(\widehat{X})\widehat{X}_D \geq c(\widehat{X}) - c(\widehat{X}_{-D}) \text{ for all } D \subset N. \tag{17}$$

In particular, taking $D = N$, we see that the total surplus must be nonnegative:

$$\alpha(\widehat{X})\widehat{X} - c(\widehat{X}) \geq 0. \tag{18}$$

Furthermore, with decreasing marginal cost, (18) is sufficient for condition (17). Indeed, decreasing marginal cost implies that

$$\frac{c(\widehat{X})}{X} \leq \frac{c(\widehat{X}_{-D})}{X_{-D}},$$

which is equivalent to

$$\frac{c(\widehat{X}) - c(\widehat{X}_{-D})}{\widehat{X}_D} \leq \frac{c(\widehat{X})}{\widehat{X}}.$$

Together with (18) this implies (17).

6 Bidding Games

(THIS SECTION IS PRELIMINARY.)

In this section we study the subgame perfect Nash equilibria of bidding games in the linear settings considered in Sections 4 and 5. Our primary analysis in this section is limited to the case in which $c'(\cdot)$ is increasing. We begin with a lemma establishing an additional necessary condition for equilibria in bidding games whenever $c'(\cdot)$ is either increasing or decreasing.

Lemma 5 *Suppose that $\alpha(\cdot)$ is nonincreasing and that $c'(\cdot)$ is increasing (resp. decreasing). If $(\widehat{x}, \widehat{t})$ is an equilibrium outcome of the bidding game, then for any $i \in N$ and $x_i > \widehat{x}_i$ (resp. $x_i < \widehat{x}_i$) it must be that*

$$\alpha(\widehat{X})\widehat{x}_i - c(\widehat{X}) \geq \alpha(x_i + \widehat{X}_{-i})x_i - c(x_i + \widehat{X}_{-i}). \tag{19}$$

Proof. We establish the result for the case of $c'(\cdot)$ increasing. Suppose $(\widehat{x}, \widehat{t})$ is the outcome in an equilibrium in which the agents offer menus $(M_1, ..., M_N)$, but that there exists an i and x_i such that (19) is violated. Let (x_j, t_j) for $j \neq i$ be an optimal choice for the principal from menus M_j, $j \neq i$, given that she trades x_i with agent i. Observe that the fact that $c'(\cdot)$ is increasing (resp. decreasing) implies that $X_{-i} < \widehat{X}_{-i}$ (resp. $X_{-i} > \widehat{X}_{-i}$). Then

$$\begin{aligned} \alpha(\widehat{x}_i + \widehat{X}_{-i})\widehat{x}_i + \sum_{j\neq i} \widehat{t}_j - c(\widehat{x}_i + \widehat{X}_{-i}) \quad &< \quad \alpha(x_i + \widehat{X}_{-i})x_i + \sum_{j\neq i} \widehat{t}_j - c(x_i + \widehat{X}_{-i}) \\ &\leq \quad \alpha(x_i + X_{-i})x_i + \sum_{j\neq i} t_j - c(x_i + X_{-i}) \end{aligned}$$

where the second inequality follows from $X_{-i} \leq \widehat{X}_{-i}$ and $\sum_{j\neq i} t_j - c(x_i + X_{-i}) \geq \sum_{j\neq i} \widehat{t}_j - c(x_i + \widehat{X}_{-i})$. But this violates condition (iii) of Proposition 3 – a contradiction to $(\widehat{x}, \widehat{t})$ being an equilibrium. ■

When $c'(\cdot)$ is increasing, the necessary condition in Lemma 5 says that bilateral surplus cannot be raised for any principal-agent pair by increasing the principal's trade with that agent. This additional necessary condition follows from the fact that bidders know their opponents' menu offers in equilibrium. When $c'(\cdot)$ is increasing, an agent who offers a point contract with a trade level above his equilibrium trade level can be assured that the trades of all other agents will not increase if the principal accepts his offer. If bilateral trade were increased following such an increase in trade, then the agent could be assured of a higher payoff by deviating to a point contract that offered this trade level and gave the principal a payoff equal to her supposed equilibrium payoff if she did not alter her trade with other agents. As an example of the application of this result, observe that it implies that in the example of Section 2 every agent must have a trade of at least $\widehat{X}_N^p/N$.

Our main result of this section provides necessary and sufficient conditions for a trade profile x to be an equilibrium trade profile in the bidding game.

Proposition 10 *If $c'(\cdot)$ is increasing and the hypotheses of Lemma 5 hold then $(\widehat{x}_1, ..., \widehat{x}_N)$ is sustainable as the trade outcome of a subgame perfect Nash equilibrium of the bidding game if and only if, for all i,*

$$\alpha(\widehat{X})\widehat{x}_i - c(\widehat{X}) \geq \alpha(x_i + \widehat{x}_{-i})x_i - c(x_i + \widehat{x}_{-i}) \text{ for all } x_i > \widehat{x}_i, \tag{20}$$

$\widehat{X} \leq X^c$, and

$$\alpha(\widehat{X})\widehat{x}_i - c(\widehat{X}) \geq p^c(X^c - \widehat{X}_{-i}) - c(X^c). \tag{21}$$

Proof. In appendix. ■

Thus, when $c'(\cdot)$ is increasing, not only is condition (19) necessary, but when combined with the conditions that $\widehat{X} \leq X^c$ and our previous necessary condition (21), it is sufficient as well. Thus, in the example of Section 2, the set of aggregate trade levels $\widehat{X}$ arising in symmetric equilibria of the bidding game is precisely the set $[X_N^p, X^c]$.

7 Appendix

Define the function

$$B(X, X_{-i}) = \alpha(X)(X - X_{-i}) - c(X),$$

giving the bilateral surplus when the aggregate trade with agents $j \neq i$ is X_{-i} and the aggregate trade is X (and so the trade with agent i is $x_i = X - X_{-i}$). Note that if the principal of type X_{-i} offers agent i an acceptable menu M and chooses $x_i = X - X_{-i}$ from it, the agent's individual rationality implies that $\Pi^M(X_{-i}) \leq B(X, X_{-i})$. The proofs of the results in Section 4 rely on the following two lemmas concerning bilateral surplus:

Lemma 6 *Suppose that (p^c, X^c) is a competitive equilibrium, $X \geq X_{-i}$, and $\alpha(X) \leq p^c$. Then $B(X, X_{-i}) \leq B(X^c, X_{-i})$.*

Proof.

$$\begin{aligned} B(X, X_{-i}) &= \alpha(X)(X - X_{-i}) - c(X) \\ &= [p^c X - c(X)] + [\alpha(X) - p^c](X - X_{-i}) - p^c X_{-i} \\ &\leq [p^c X^c - c(X^c)] - p^c X_{-i} = B(X^c, X_{-i}), \end{aligned}$$

where the inequality uses the fact that (p^c, X^c) is a competitive equilibrium. ■

Lemma 7 *Suppose that* $\alpha(\cdot)$ *is nonincreasing and* $\alpha(X) < c'(X)$ *for all* $X > X_{-i}$*. Then* $B(X, X_{-i}) < B(X_{-i}, X_{-i})$ *for all* $X > X_{-i}$*.*

Proof.

$$\begin{aligned} B(X, X_{-i}) - B(X_{-i}, X_{-i}) &= \alpha(X)(X - X_{-i}) - c(X) + c(X_{-i}) \\ &= \int_{X_{-i}}^{X} [\alpha(X) - c'(Y)]\, dY. \end{aligned}$$

The result follows since $c'(Y) > \alpha(Y) \geq \alpha(X)$ for all $Y \in (X_{-i}, X)$. ■

Proof of Lemma 3: By Proposition 2, C is an acceptable menu, with the associated profit function $\Pi^C(\cdot)$ given by (6). Let $\widetilde{X}$ be as defined in (6). [Observe that when $c(\cdot)$ is differentiable, hence continuous, we have $p^c\widetilde{X} - c(\widetilde{X}) = p^c X^c - c(X^c)$.] We will show that for all X_{-i} and any acceptable menu M we must have $\Pi^M(X_{-i}) \leq \Pi^C(X_{-i})$.

For $X_{-i} \geq X^c$, this follows from the agent's individual rationality and Lemma 7:

$$\Pi^M(X_{-i}) \leq B(X, X_{-i}) \leq B(X_{-i}, X_{-i}) = \Pi^C(X_{-i}).$$

Now suppose in negation that there exists $X'_{-i} < X^c$ and an acceptable menu M such that $\Pi^M(X'_{-i}) > \Pi^C(X'_{-i})$. By Lemma 1, the union menu $U = M \cup C$ is also acceptable. Its associated profit function is $\Pi^U(X_{-i}) = \max\{\Pi^M(X_{-i}), \Pi^C(X_{-i})\})$. Let

$$X^0_{-i} = \min\left\{X_{-i} \in \left[X'_{-i}, X^c\right] : \Pi^U(X_{-i}) = \Pi^C(X_{-i})\right\}. \tag{22}$$

(The minimum is achieved because $\Pi^U(\cdot)$ and $\Pi^C(\cdot)$ are continuous as upper envelopes of continuous functions; see e.g. Milgrom and Segal [2000, Theorem 2].) By construction we have $X^0_{-i} > X'_{-i}$.

Take $\varepsilon = \min\left\{X^0_{-i} - X'_{-i}, X^c - \overline{X}\right\}$. Let $\left(x^M(X_{-i}), t^M(X_{-i})\right)$ be an optimal choice from a menu M for the principal of type X_{-i}, and let $X^M(X_{-i}) = x^M(X_{-i}) + X_{-i}$. Standard envelope theorem arguments of mechanism design imply that for any acceptable menu M,

$$\Pi^M(X^0_{-i}) = \Pi^M(X^0_{-i} - \varepsilon) - \int_{X^0_{-i}-\varepsilon}^{X^0_{-i}} c'(X^M(X_{-i}))dX_{-i}$$

Therefore,

$$\Pi^U(X^0_{-i}-\varepsilon)-\Pi^C(X^0_{-i}-\varepsilon)=\int_{X^0_{-i}-\varepsilon}^{X^0_{-i}}\left[c'(X^U(X_{-i}))-c'(X^C(X_{-i}))\right]dX_{-i}.$$

Since $\Pi^U(X^0_{-i}-\varepsilon)>\Pi^C(X^0_{-i}-\varepsilon)$, we must have $c'(X^U(X''_{-i}))>c'(X^C(X''_{-i}))$ for some $X''_{-i}\in\left(X^0_{-i}-\varepsilon,X^0_{-i}\right)$. At the same time, by (22), we must have $\Pi^U(X''_{-i})>\Pi^C(X''_{-i})$.

Note that we can rule out $X^U(X''_{-i})>X^c$, for in this case the agent's individual rationality and Lemma 6 imply

$$\Pi^U(X''_{-i})\le B(X^U(X''_{-i}),X''_{-i})\le B(X^c,X''_{-i})=\Pi^C(X''_{-i}).$$

This eliminates $X''_{-i}\ge\overline{X}$, for then $\max_{X\in[X''_{-i},X^c]}c'(X)=c'(X^c)=c'(X^C(X''_{-i}))$. This also eliminates $X''_{-i}\le\widetilde{X}$, for then $\max_{X\in[X''_{-i},X^c]}c'(X)=c'(X''_{-i})=c'(X^C(X''_{-i}))$.

Thus, we can focus on $X''_{-i}\in\left(\widetilde{X},\overline{X}\right)$. Then $X^C(X''_{-i})=X^c$, and $c'(X^U(X''_{-i}))>c'(X^C(X''_{-i}))$ implies $X^U(X''_{-i})<\overline{X}$. Let $(x'',t'')=\left(x^U(X''_{-i}),t^U(X''_{-i})\right)$. By (22) and the fact that $x''_i=X^U(X''_{-i})-X''_{-i}\le\overline{X}-X''_{-i}\le X^c$ is a feasible trade in menu C, we have

$$t''-c(X''_{-i}+x'')=\Pi^U(X''_{-i})>\Pi^C(X''_{-i})\ge p^cx''-c(X''_{-i}+x'').$$

Therefore, $t''>p^cx''$. But then, since $(x'',t'')\in U$, we have

$$\Pi^U(X^C-x'')\ge t''-c(X^C-x''+x'')>p^cx''-c(X^c)=\Pi^C(X^c-x'').$$

Since

$$X^c-x''=X^c-X^U(X''_{-i})+X''_{-i}>X^c-\overline{X}+X''_{-i}>X^c-\overline{X}+X^0_{-i}-\varepsilon>X^0_{-i},$$

this contradicts (22). ■

Proof of Lemma 4: N is trivially an acceptable menu, with the associated profit function $\Pi^N(X_{-i})=-c(X_{-i})$. To see that N is an RSW menu, take an arbitrary acceptable menu M. By Lemma 1, the union menu $U=M\cup N$ is also acceptable, with $\Pi^U(X_{-i})=\max\{\Pi^M(X_{-i}),\Pi^N(X_{-i})\}$. We will show that $\Pi^U(X_{-i})=\Pi^N(X_{-i})$, and therefore $\Pi^M(X_{-i})\le\Pi^N(X_{-i})$, for all X_{-i}.

First, note that if the principal with $X_{-i} \geq \overline{X}$ chooses $(x,t) \in U$, the agent's individual rationality and Lemma 7 imply that

$$\Pi^U(X_{-i}) \leq B(X_{-i}+x, X_{-i}) \leq B(X_{-i}, X_{-i}) = \Pi^N(X_{-i}).$$

Therefore, $(0,0)$ is an optimal choice for the principal with $X_{-i} \geq \overline{X}$ from U.

Note that with $c'(X)$ decreasing in X, the principal's payoff has strictly increasing differences in (x_i, X_{-i}). Therefore, by the Monotone Selection Theorem of Milgrom and Shannon [1994], $(0,0)$ must be an optimal choice from U for any $X_{-i} \leq \overline{X}$, which implies that $\Pi^U(X_{-i}) = \Pi^N(X_{-i})$ for all such X_{-i}. ■

Proof of Proposition 10: Necessity follows from Lemma 2, Proposition 4, and Lemma 5. Here we establish sufficiency. Let each agent i offer a menu in which the principal can choose any $x_i \leq \widehat{x}_i$ or any $x_i \geq X^c - \widehat{X}_{-i}$ with accompanying payments of:

$$t_i(x_i) = \left\{ \begin{array}{c} p^c x_i \text{ for } x_i \leq \widehat{x}_i \\ p^c x_i + \{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})\} \text{ if } x_i > \widehat{x}_i. \end{array} \right\}$$

Remark 1 *The idea of this schedule is that the principal is indifferent about expanding her trade by $X^c - \widehat{X}$ units. If any agent reduces his trade by ε, the principal strictly prefers to increase the total trade by $X^c - \widehat{X}$ units, and then for each additional unit that the agent further reduces his trade, the principal replaces it with an additional unit from another agent at the same price of p^c (hence, the principal's profit is unaffected by having a zero trade with any $N-1$ agents).*

Lemma 8 *Suppose we fix x_i. Any solution to*

$$\max_{x_{-i}} \sum_{j \neq i} t_j(x_j) - c(x_i + \sum_{j \neq i} x_j) \tag{23}$$

has at most 1 element k with $x_k > \widehat{x}_k$.

Proof. Suppose we have $(x_j, x_k) >> (\widehat{x}_j, \widehat{x}_k)$. Consider a change in which only j and k's trades are changed, and these are changed to $[\widehat{x}_j, x_k + (x_j - \widehat{x}_j)]$. The change in profit is

$$\begin{aligned} & p^c(x_j + x_k) + \{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})]\} \\ & -p^c(x_j + x_k) - 2\{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})]\} \\ = \ & -\{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})]\} \\ > \ & 0 \end{aligned}$$

since $\widehat{X} \leq X^c$ and $c(X^c) - c(\widehat{X}) \leq c'(X^c)(X^c - \widehat{X}) = p^c(X^c - \widehat{X})$. ■

Lemma 9 *Suppose we fix x_i. The solution and value of the solution [denoted $\pi_{-i}(x_i)$] to problem (23) take the following form:*

1. *If $x_i < \widehat{x}_i$: Any solution has $\sum_{j \neq i} x_j = X^c - x_i$ and has $\pi_{-i}(x_i) = p^c(\widehat{X} - x_i) - c(\widehat{X})$.*
2. *If $x_i > \widehat{x}_i$ but $x_i < X^c - \widehat{X}_{-i}$: Any optimal choice has $\sum_{j \neq i} x_j = \widehat{X}_{-i}$ and has $\pi_{-i}(x_i) = p^c \widehat{X}_{-i} - c(x_i + \widehat{X}_{-i})$.*
3. *If $x_i > \widehat{x}_i$ and $x_i \in [X^c - \widehat{X}_{-i}, X^c]$: Any optimal choice has $\sum_{j \neq i} x_j = X^c - x_i$ and has $\pi_{-i}(x_i) = p^c(X^c - x_i) - c(X^c)$.*
4. *If $x_i > \widehat{x}_i$ and $x_i \geq X^c$: Any optimal choice has $x_j = 0$ for all $j \neq i$ and has $\pi_{-i}(x_i) = -c(x_i)$.*

Proof. Given x_i, call a configuration with $x_j \leq \widehat{x}_j$ for all $j \neq i$ a type A configuration. Its value is $p^c \sum_{j \neq i} x_j - c(x_i + \sum_{j \neq i} x_j)$. When $x_i < X^c - \widehat{X}_{-i}$, the best solution within this class has $x_j = \widehat{x}_j$ for all j and has value $p^c \widehat{X}_{-i} - c(x_i + \widehat{X}_{-i})$. When $x_i \in [X^c - \widehat{X}_{-i}, X^c]$, the best solution within this class has $x_j \leq \widehat{x}_j$ for all j and $\sum_{j \neq i} x_j = X^c - x_i$, and has value $p^c(X^c - x_i) - c(X^c)$. When $x_i \geq X^c$, the best solution in this class has $x_j = 0$ for all $j \neq i$ and has value $-c(x_i)$.

Call a configuration with $x_k \geq \widehat{x}_k$ for some k a type B configuration. Its value is

$$p^c \sum_{j \neq i} x_j - c(x_i + \sum_{j \neq i} x_j) + \{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})]\}.$$

If $x_i \leq X^c - \widehat{x}_k$, this is maximized with $\sum_{j \neq i} x_j = X^c - x_i$ and yields a value of $p^c(\widehat{X} - x_i) - c(\widehat{X})$. When $x_i \geq X^c - \widehat{x}_k$, this is maximized by setting $x_k = \widehat{x}_k$ and $x_j = 0$ for all $j \neq i, k$ and yields a value of

$$p^c \widehat{x}_k - c(x_i + \widehat{x}_k) + \{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})\}.$$

Now consider the optimal choices in the four cases of the lemma:

1. If $x_i < \widehat{x}_i$ ($< X^c - \widehat{X}_{-i} \leq X^c - \widehat{x}_k$ for all k): The best type A configuration has value $p^c \widehat{X}_{-i} - c(x_i + \widehat{X}_{-i})$ while the best type B configuration has value $p^c(X^c - x_i) - c(X^c)$. So a type B configuration is better and any optimal solution to problem (23) has $\sum_{j \neq i} x_j = X^c - x_i$.

2. If $x_i > \widehat{x}_i$ but $x_i < X^c - \widehat{X}_{-i}$ ($\leq X^c - \widehat{x}_k$ for all k): The best type A configuration has value $p^c \widehat{X}_{-i} - c(x_i + \widehat{X}_{-i})$, while the best type B configuration has value $p^c(\widehat{X} - x_i) - c(\widehat{X})$. Since $\widehat{X} - x_i < \widehat{X}_{-i}$, a type A configuration is better and any optimal solution to problem (23) has $\sum_{j \neq i} x_j = \widehat{X}_{-i}$.

3. If $x_i > \widehat{x}_i$ and $x_i \in [X^c - \widehat{X}_{-i}, X^c]$: Then the best type A configuration has value $p^c(X^c - x_i) - c(X^c)$. If $x_i < X^c - \max_{k \neq i}\{\widehat{x}_k\}$, then the best type B configuration has value $p^c(\widehat{X} - x_i) - c(\widehat{X})$. If instead $x_i \geq X^c - \max_{k \neq i}\{\widehat{x}_k\}$, then the best type B configuration has value no greater than $\max_k \, p^c \widehat{x}_k - c(x_i + \widehat{x}_k)$. In either case, a type A configuration is optimal and any optimal solution to problem (23) has $\sum_{j \neq i} x_j = X^c - x_i$.

4. If $x_i > \widehat{x}_i$ and $x_i \geq X^c$: The optimal type A configuration has value $-c(x_i)$. Since $x_i \geq X^c - \max_{k \neq i}\{x_k^*\}$, the best type B configuration has value no greater than $\max_k \, p^c \widehat{x}_k - c(x_i + \widehat{x}_k)$. Since $x_i > X^c$, a type A configuration is optimal and any optimal solution to problem (23) has $x_j = 0$ for all $j \neq i$. ■

We now prove the result by verifying that conditions (i)-(iv) of Proposition 3 are satisfied:

To check condition (i) we show that the principal does not improve her payoff by deviating to some $x \neq \widehat{x}$. Suppose first that she deviates to an x with $X \leq \widehat{X}$. Then $x_i \leq \widehat{x}_i$ for some i. So the principal can earn at most $p^c x_i + p^c(\widehat{X} - x_i) - c(\widehat{X}) = p^c \widehat{X} - c(\widehat{X})$. Suppose that she deviates to an x with $X > \widehat{X}$. Then there is an i with $x_i > \widehat{x}_i$. If $\dot{x}_i < X^c - \widehat{X}_{-i}$, then the principal earns at most

$$p^c x_i + \{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})]\} + p^c \widehat{X}_{-i} - c(x_i + \widehat{X}_{-i})$$

which equals

$$p^c(x_i + \widehat{X}_{-i}) - c(x_i + \widehat{X}_{-i})] + \{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})]\}$$

which is bounded above by

$$p^c X^c - c(X^c) + \{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})]\}$$

which equals $p^c\widehat{X} - c(\widehat{X})$.

If instead $x_i \in [X^c - \widehat{X}_{-i}, X^c]$, then the principal earns no more than

$$\begin{aligned} & p^c x_i + \{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})] + p^c(X^c - x_{-i}) - c(X^c) \\ = \; & p^c\widehat{X} - c(\widehat{X}). \end{aligned}$$

Finally, if $x_i \geq X^c$, then the principal earns no more than

$$\begin{aligned} & p^c x_i + \{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})] - c(x_i) \\ \leq \; & p^c X^c - c(X^c) + \{[c(X^c) - c(\widehat{X})] - p^c(X^c - \widehat{X})] \\ = \; & p^c\widehat{X} - c(\widehat{X}). \end{aligned}$$

Condition (ii) is satisfied since agent i earns $\alpha(\widehat{X})\widehat{x}_i - p^c\widehat{x}_i \geq 0$.

To see that condition (iii) holds, note that

$$\alpha(\widehat{X})\widehat{x}_i + \sum_{j \neq i} t_j(\widehat{x}_j) - c(\widehat{X}) = \alpha(\widehat{X})\widehat{x}_i + p^c\widehat{X}_{-i} - c(\widehat{X}). \tag{24}$$

Consider four cases:

1. If $x_i < \widehat{x}_i$: For any optimal choice x_{-i} by the principal, we have

$$\begin{aligned} \alpha(x_i + \sum_{j \neq i} x_j)x_i + \pi_{-i}(x_i) &= \alpha(x_i + (X^c - x_i))x_i + p^c(X^c - x_i) - c(X^c) \\ &= \alpha(X^c)x_i + p^c(X^c - x_i) - c(X^c) \\ &= p^c X^c - c(X^c) \\ &\leq \alpha(\widehat{X})\widehat{x}_i + p^c\widehat{X}_{-i} - c(\widehat{X}) \end{aligned}$$

   by condition (21).

2. If $x_i > \widehat{x}_i$ but $x_i < X^c - \widehat{X}_{-i}$: For any optimal choice by the principal, we have

$$\alpha(x_i + \sum_{j \neq i} x_j)x_i + \pi_{-i}(x_i) = \alpha(x_i + \widehat{X}_{-i})x_i + p^c\widehat{X}_{-i} - c(x_i + \widehat{X}_{-i}) \leq \alpha(\widehat{x}_i + \widehat{X}_{-i})\widehat{x}_i + p^c\widehat{X}_{-i} - c(\widehat{x}_i + \widehat{X}_{-i})$$

   by condition (20).

3. If $x_i > \widehat{x}_i$ and $x_i \in [X^c - \widehat{X}_{-i}, X^c]$: For any optimal choice by the principal, we have

$$\begin{aligned}
\alpha(x_i + \sum_{j \neq i} x_j)x_i + \pi_{-i}(x_i) &= \alpha(x_i + (X^c - x_i))x_i + p^c(X^c - x_i) - c(X^c) \\
&= \alpha(X^c)x_i + p^c(X^c - x_i) - c(X^c) \\
&= p^c X^c - c(X^c) \\
&= [p^c(X^c - \widehat{X}_{-i}) - c(X^c)] + p^c \widehat{X}_{-i} \\
&\leq [\alpha(\widehat{X})\widehat{x}_i - c(\widehat{X})] + p^c \widehat{X}_{-i}
\end{aligned}$$

where the last inequality follows from condition (21).

4. If $x_i > \widehat{x}_i$ and $x_i \geq X^c$: Now

$$\begin{aligned}
\alpha(x_i + \sum_{j \neq i} x_j)x_i + \pi_{-i}(x_i) &= \alpha(x_i)x_i - c(x_i) \\
&\leq \alpha(X^c)x_i + p^c(X^c - x_i) - c(X^c) \\
&= p^c X^c - c(X^c) \\
&\leq [\alpha(\widehat{X})\widehat{x}_i - c(\widehat{X})] + p^c \widehat{X}_{-i}
\end{aligned}$$

by condition (21).

For condition (iv) note first that our analysis above tells us that the principal's optimal payoff when she does not trade with one agent i (i.e. when $x_i = 0$) is $p^c \widehat{X} - c(\widehat{X})$, which exactly equals her equilibrium payoff. ■

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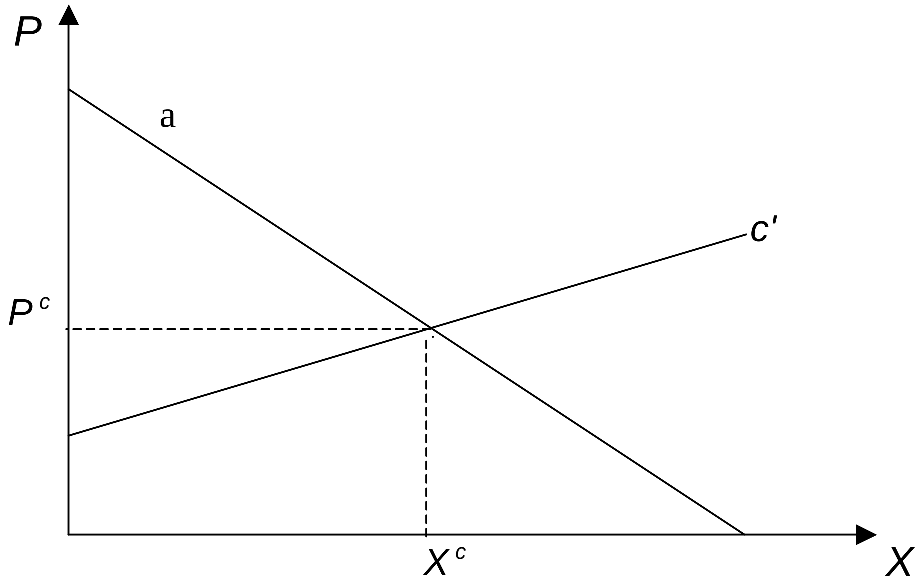

Figure 1.

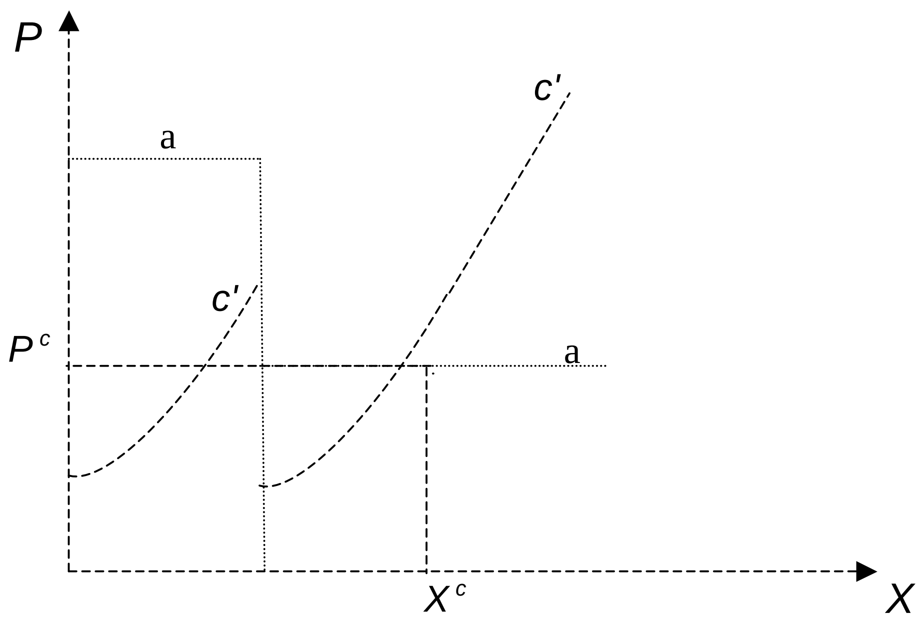

Figure 2.

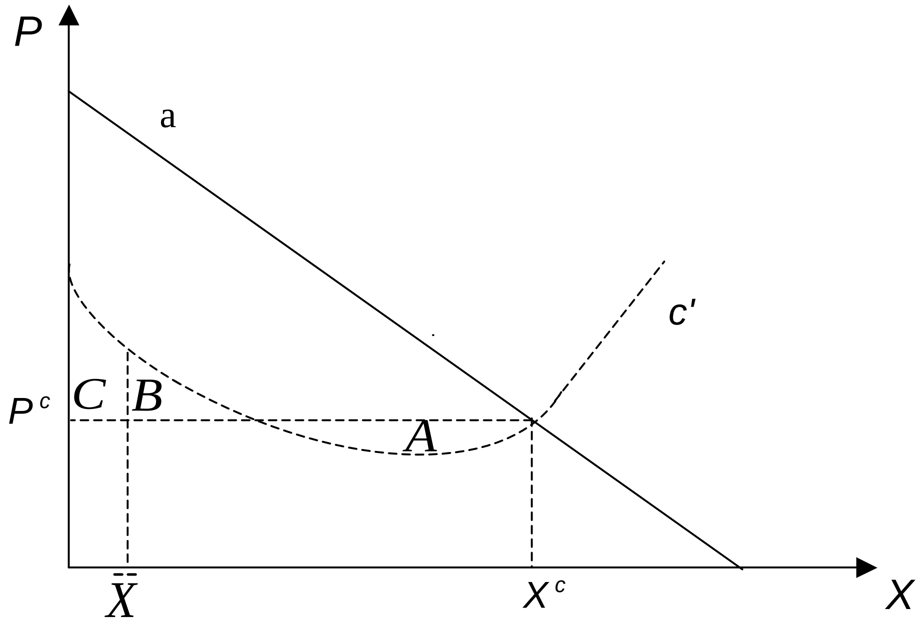

Figure 3(a).

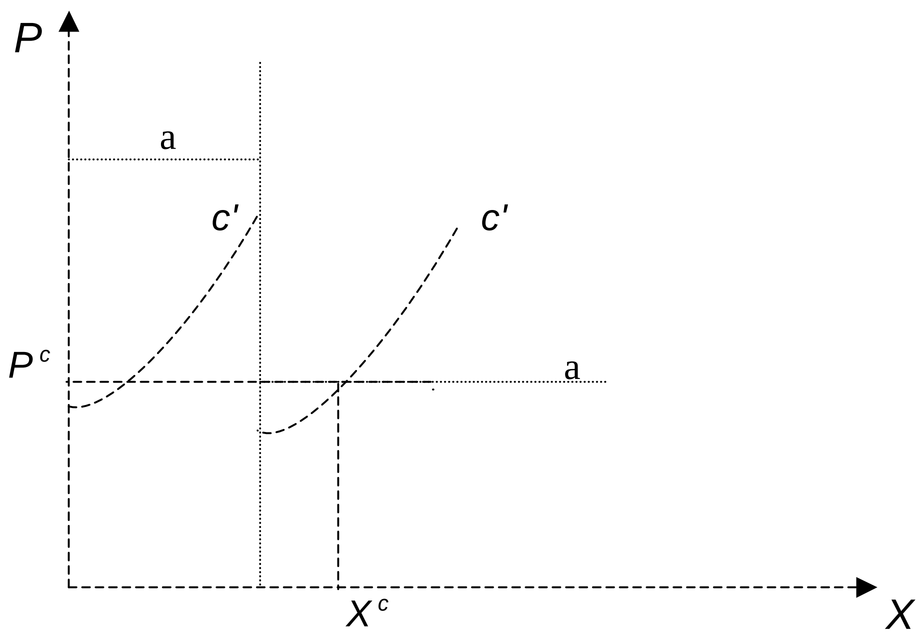

Figure 3(b).

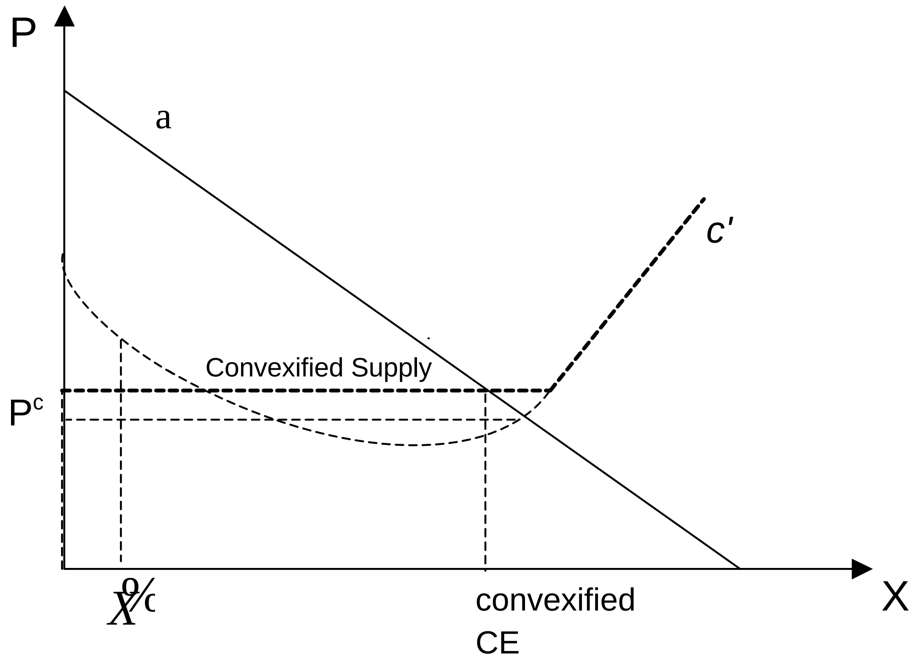

Figure 4.