Efficient Equilibria in Economies with Adverse Selection *

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Abstract

In economies with Adverse Selection the existing Competitive Equilibrium concept ([10]) imply either that the equilibrium does not exist, or that it is not necessarily efficient. We introduce and analyze, within the Rotschild and Stiglitz model of Adverse Selection, a competitive notion of equilibrium, *a constrained competitive equilibrium,* which yields a unique and constrained efficient allocation. At a constrained equilibrium, individuals of different types are allowed to make transfers and, as in ([1] and [2]), joint trades must satisfy a *market incentive*

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constraint. We also show that the Lindhal's equilibria of ([1] and [2]) may yield inefficient allocations (and are indeterminate).

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1 Introduction

In this paper we propose a competitive equilibrium notion for economies of Adverse Selection which yields constrained efficient outcomes. For sake of clarity we develop the analysis in the Adverse Selection model of Rotschild and Stiglitz ([12]). [1]

There is a large body of research on Adverse Selection economies and on the Rotschild and Stiglitz insurance model. Our work is in the line of research started by Prescott and Townsend ([10]). The problem is to model asymmetric information economies as standard competitive economies where equilibria exist under classical assumptions and equilibrium allocations are (constrained) efficient. This project has been successful in handling economies with private information and moral hazard ([10], [11], [7] and [14]), but the analysis of Adverse Selection economies has proved to be more problematic. Different competitive equilibrium concepts have been proposed for this class of economies. None of them solves the problem of reconciling existence and efficiency, as we have shown in [13] and [14]. Either the concept is too restrictive, and existence fails, or it is too weak, and while equilibria exist, some of

[1] We believe that the analysis of this paper can be extended to general economies of Adverse Selection and this is the goal of future research.

them may be inefficient.

Before explaining the details of our positive analysis, we need to clarify what makes Adverse Selection economies special. We start by quickly summarizing the approach initiated by Prescott and Townsend.

The Prescott and Townsend' s Model

Economies of asymmetric information are modeled as economies of individual risk as in Malinvaud ([8] and [9]) where, in addition, some important variables are private information. In economies of individual risk, feasibility is a restriction over average allocations (where the average is taken over types and personal states), and equilibrium prices are type dependent. In environment with asymmetric information, any sensible notion of efficiency or equilibrium narrows down the prescribed allocations to satisfy the incentive compatibility constraints. The key insight in [10] is to perform this restriction ex-ante by identifying the consumption set of the individuals with the set of incentive compatible consumption bundles. The lack of convexity generated by the incentive constraints is restored by the introduction of lotteries. Incentive constraints are restrictions over the set of individual allocations for economies with Moral Hazard and Private Information. Thus, for these economies, the individual allocation set, that is the set of individual incentive compatible lotteries, is convex and the individual utility function is linear. This takes us back to a standard individual risk economy.

There is, however, an additional complication. Since allocations are lotteries, the price domain coincide with its topological dual and, hence, it contains prices that are non-linear in basic commodities (although, by definition, they must be linear in lotteries). In absence of further restrictions, the richness of the price domain supports a plethora of equilibrium allocations potentially inefficient. The model is closed by the introduction of a price taking and profit maximizing firm (or intermediary). The production set of the firm coincide with the set of feasible signed measures. The constant return to scale property of the firm's technology forces prices to be linear in commodities, and selects efficient equilibria. Thus, an equilibrium exists and it is efficient. Different types will face different type-dependent prices as for the Malinvaud' s individual risk economy where each type faces fair personalized prices, i.e., linear prices for personal state contingent commodities adjusted by the appropriate probabilities. Whenever types are public information, this is not an aspect of concern for competitive markets.

Economies of Adverse Selection

Adverse Selection economies are very different than other asymmetric information environments. Since individual types are private information, incentive compatibility is a restriction over joint allocations and not over individual allocations. Indeed, for adverse selection economies, individuals can manipulate allocations by misreporting their types. Thus, the notion of a set of incentive compatible individual allocations is meaningless and must be replaced by the notion of an incentive compatible set of joint allocations.

The private information nature of the individual types collides with the personalized nature of the prices supporting efficient allocations. If prices are different for different types, any agent can trade in any of the markets he chooses. In absence of additional restrictions, a novel difficulty arises for the competitive model: personalized prices must be in an obvious sense "incentive compatible." The necessity of defining incentive compatibility over joint, rather then individual, allocations, is suggestive of the existence of a consumption externality. The consumption of a given type i is affected through the incentive constraint by the consumption chosen by a type j different than i. Every time consumption externality enters the picture of a competitive world one is naturally lead to use a Lindahl solution concept. Thus, there seems to be two fairly natural strategies of modeling a competitive economy with adverse selection. In the first, the adverse selection economy is modeled as a competitive economy. The consumption set of the individuals coincide with the set of individual lotteries, while the production set of the firm contains the intersection of the sets of the feasible and of the incentive compatible joint lotteries. In the second route, Adverse selection is modeled in the context of a Lindahl economy. The individual consumption set coincide with the set of joint allocations and individuals have access to markets for property rights over the consumption of any type. The firm produces feasible joint measures. For economies of Moral Hazard and Private Information, the introduction of the intermediary or, equivalently (see for instance [14]), restricting prices to be fair, pins down efficient competitive allocations. However, in Adverse Selection economies, this maneuver collides with the nature of the private information and it makes the equilibrium set potentially empty. This is robust to different specification of the production set and it is independent of whether the economy is modeled as a Lindahl economy with externalities, as it is shown in Rustichini and Siconolfi ([13] and [14]). Of course, the existence of an equilibrium can be restored, by either imposing additional restrictions on trade or suppressing both the firm and the price linearity, as in Gale ([6]) or Dubey and Geanakoplos ([4]). However the price to be paid is the loss of efficiency of the equilibrium allocations ([13]).

Restrictions on Joint Trade across Markets

A competitive or Lindahl equilibrium may fail to exists mainly because of the private information nature of the individual types. Bisin and Gottardi ([1] and [2]) get around this problem by introducing an incentive constraint across markets, for short, "a market incentive constraint." Identify a market with a (type dependent) price and attach to each markets a type tag. Thus, the $i{-}th$ market is the market where individuals of type i are meant to trade. In ([1] and [2]), a (non modeled) market institution prevents individuals that trade on the $i{-}th$ market to buy anything preferred by individuals of type j to what they can buy on the market with their own type tag. In different words, given a collection of type dependent prices, the market institution makes sure that each type i trade at the $i - th$ price vector by restricting the choice set of the individuals adding to the standard budget constraint the appropriate "market incentive constraint." In some sense, this maneuver is conceptually similar to the imposition of the incentive constraints in standard contract theory, e.g., [12]. There, individuals are given a (limited) set of incentive compatible contracts. Here, Bisin and Gottardi require that the market transactions are limited by the same constraints. We call "constrained" any equilibrium notion (Lindhal or competitive) of the Adverse Selection economy where the "market incentive constraints" and the Prescott and Townsend firm are present.

Bisin and Gottardi ([1] and [2]) propose a constrained Lindahl as an equilibrium notion for the study of equilibrium allocations of adverse selection economies. However, we show in the last section of this paper that Constrained Lindahl equilibria suffer two major drawbacks. The equilibrium set may contain inefficient allocations and it is indeterminate, that is it contains a continuum of (payoff relevant) equilibrium allocations. Our study of Constrained Lindhal equilibria is rather sketchy. A fully detailed analysis is available in the companion working paper ([15]).

Thus, the failure of the constrained Lindahl equilibrium motivates the search for a different equilibrium notion.

Constrained Competitive Equilibria

We propose a new equilibrium notion which is somewhat an hybrid between a Lindahl and a competitive one. The competitive element (rather than the Lindahl one) in our notion is captured by the fact that the consumption set of the individuals coincide with the set of individual lotteries. We also retain the market incentive constraints. Without them, as we have already argued, the search for an efficient competitive allocation is meant to fail. However, to overcome inefficiency individuals need to be able to affect the market constraints. Thus, we make the choice set of individuals richer than the set of individual lotteries, by opening a market for wealth transfers. An individual trading on market i can buy at a market price q^i (non negative amounts of) an "asset," which we call "wealth transfer", that pays off one unit of wealth or, equivalently, one unit of non contingent (personal) state consumption, to all the individuals trading on market j, $i \neq j$. By transferring wealth, each individual is able to modify the budget constraints of the individuals trading on the different market, thereby altering the market incentive constraint. This is the feature that produces efficient equilibrium allocations. The mechanism is simple. Because of the presence of the incentive market constraint, individuals of type i trade on market i, $i = g, b$. If an individual of type i find convenient to transfer wealth to type j it must be the case that the market constraint is binding. By transferring wealth, type i individuals increase the consumption and, hence, the welfare of type j individuals, as well as their own. As a result of the wealth transfer, everybody in the economy is better off.

Our equilibrium notion contains, as already stated, a Lindahl element. Individuals take as given the (average) transfer received by the individuals of the different type and "buy" wealth transfers at a type dependent price. Furthermore, a single individual can transfer wealth by affecting the market incentive constraint. At equilibrium, every individuals of a given type makes the same choice and, thus, at equilibrium, individual plans and aggregate outcome are consistent. Yet, conceptually, we are assuming that an individual (of negligible size) acts by thinking that his marginal impact on the wealth of the other type individuals is equal to the average impact of all the individuals of his type. This in the Lindahl spirit. In a Lindahl economy, when an individual buys (property rights over) the consumption of other individuals or buys public goods, her dollar contribution to total expenses is negligible. Nevertheless, Lindahl works because it allows individuals to decide the optimal consumption of everybody in the economy as well as the amount of public goods independently of the choices of everybody else. At a Lindahl equilibrium, plans are consistent because everybody makes the same choice. Out of equilibrium, there is a clear discrepancy between individual choices and market outcomes. The latter could not even be properly defined. Our notion is from this point of view identical. There is an externality generated via the market incentive constraint by the wealth of the other type. Each individual decide what that wealth is. At equilibrium every individual of the same type makes the same choice and the plans are consistent. Out of equilibrium individual plans and aggregate accounting are inconsistent.

The model is closed, following Prescott and Townsend, by the introduction of a competitive, price taking firm (or intermediary). The firm does two things. First, as in Prescott and Townsend, it produces feasible signed measures thereby pinning down the commodity prices to their fair values. Second, it supplies the assets that we have called wealth transfers and pay off their returns. This is going to pin down the market price of the wealth transfer activities.

We show that the Rotschild and Stiglitz economy of Adverse Selection has a unique and efficient constrained equilibrium. The equilibrium allocation is the one that maximizes the welfare of the good type individuals within the set of constrained efficient allocations that yields to the individuals of the bad type a welfare not inferior to that generated by the Rotschild and Stiglitz separating allocation. At the equilibrium allocation, the bad type individuals are fully insured and they have a binding incentive constraint. The good type individuals may choose to transfer wealth to the bad type ones. They will do that every time the separating allocation of Rotschild and

Stiglitz is inefficient. Thus, as the fraction of individuals of bad type becomes negligible, the constrained efficient allocation converges to the full Pareto efficient allocation that provides full insurance to the good type individuals. In different words, as the individuals of type b become negligible, it becomes negligible as well their influence on the equilibrium allocation.

The importance of the welfare implications of different institutions and equilibria has been emphasized in recent papers that focus on the consequences of predictive medicine (see for example, [3] and [5]). The paper [3] focuses on the classification risk: the risk for any individual of being classified (through testing) as being of the bad type. Chiappori estimates the welfare loss of increasingly precise testing, when the results are made publicly available to all agents in the economy (both consumers and insurance firms). Chiappori finds the loss very large even for realistic values of the risk aversion. He,then proceeds to discuss the welfare implications of two different institutional arrangements. One is based on exclusive contracts, which force linear prices. The other is based on non-exclusive contracts. Previous analysis concludes that the resulting equilibria are separating. In this situation, a small fraction of bad types sustains a heavy welfare loss.

An interesting contribution of the present paper is that in the equilibrium we describe a large fraction of good types can effectively subsidize the potentially large losses of the bad types, at a small cost, at the same time achieving efficiency in the outcomes and avoiding large inequalities in welfare.

1.1 The Economy

We consider the standard Rotschild-Stiglitz insurance economy.

The economy has a continuum of households. There is a set $I = \{g, b\}$ of two types (good and bad), with generic indexes i and j. There is set $S = \{L, H\}$ of two personal states, with generic term s. The probability of each state is type dependent, and it is private information of each household. On aggregate, there is a proportion $\lambda^i, i = b, g$ of households of type i. The probability $\pi^i = (\pi^i(H), \pi^i(L))$ denotes the probability over the two states for type i. The personal states, one for each household, are independent random variables. There is no aggregate state.

There is only one commodity. Endowments are type invariant and denoted by $e = (e(H), e(L))$. We assume that $\pi^g(H) > \pi^b(H)$ and $e(H) > e(L)$. The consumption set of basic commodities is X, a compact subset of $\Re^2_+$, the space of state dependent consumptions. It will be simpler to think of X as the product of two closed intervals $X' = [0, c]$, with c arbitrarily large. Preferences are represented by von-Neumann and Morgestern utility functions with type invariant cardinality indexes. Thus, preferences over state contingent consumption bundles $x = (x(H), x(L)) \in X$ for individuals of type i are

$$V^i(x) = \sum_{s \in S} \pi^i(s) U(x(s)). \tag{1}$$

We add the following restrictions on the economy:

Assumption 1 *1. The function U is continuous, strictly concave and strictly increasing,*

2. For every type i, the indifference curve through e, $\{x : V^i(x) = V^i(e)\}$, is contained in $R^2_{++} \cap X$.

3. The cardinality index U is smooth.

The first condition is standard. The second and third are innocuous regularity conditions.

We only consider allocations that give the same consumption to each household of the same type. In the following we consider the set of signed measures over X, endowed with the Borel σ-field, denoted by $M(X)$. We also consider the set of individual lotteries, that is of probability measures, denoted by $M_1(X)$. The generic individual lottery is denoted by a lower case Greek letter; for every such lottery γ^i we define:

$$V^i(\gamma^i) \equiv \int_X V^i(x)\gamma^i(dx),\ i = g, b.$$

A Rotschild and Stiglitz economy is denoted by E. It is a collection of cardinality indexes $U : R_+ \rightarrow R$, satisfying assumptions 1. - 3., of type sizes $\lambda^i > 0$, $i = g, b$, of a pair of probability vectors over personal states, $\pi^i = (\pi^i(H), \pi^i(L))$ with $\pi^g(H) > \pi^b(H)$, and of type independent endowments $e = (e(H), e(L))$, $e(H) > e(L)$.

An allocation is a pair of lotteries $\gamma = (\gamma^g, \gamma^b) \in (M_1(X))^2$. $\gamma \in (M_1(X))^2$ is feasible if

$$\sum_i \lambda^i \int_X \pi^i x \gamma^i(dx) \leq \bar{e} \equiv \sum_i \lambda^i \sum_s \pi^i(s)e(s).$$

The set of feasible allocations is denoted by Y'.

An allocation γ is incentive compatible if

$$V^i(\gamma^i) \leq V^i(\gamma^j),\ j \neq i,\ i = g, b.$$

The set of incentive compatible lotteries is denote by Σ.

2 Constrained Efficient Allocations

In this section, we study the properties of the set of constrained efficient allocations that will be instrumental in characterizing later on the equilibrium set. All the proofs of the various Lemmas are postponed to the appendix.

An allocation σ is constrained efficient if it is feasible, it is incentive compatible and it does not exists a feasible and incentive compatible allocation γ such that:

$$(V^g(\gamma^g), V^b(\gamma^b)) > (V^g(\sigma^g), V^b(\sigma^b)).$$

$P(E)$ denotes the set of constrained efficient allocation.

The feasible set Y' is convex and, by the linearity in individual lotteries of V^i, the set of incentive compatible allocation, Σ , is convex as well. Thus, the set of constrained efficient allocations can be parameterized by the elements of the simplex $\Delta = \{a = (a^g, a^b) > 0 : a^g + a^b = 1\}$ and by the following programming problem:

for $a \in \Delta$, a constrained efficient allocation $\gamma(a)$ is an optimal solution to

$$\max_{\beta \in Y' \cap \Sigma} \sum_i \alpha^i \lambda^i V^i(\beta) \tag{2}$$

As usual, every $\beta \in P(E)$ is a solution to the above programming problem for some $\alpha \in \Delta$. Viceversa, every solution to the programming problem 2, $\beta(a)$, for some $a \gg 0$, is an element of $P(E)$.

A first property of constrained efficient allocation pertains their stochastic nature. It is well known that in Rotschild and Stiglitz economies, constrained efficient allocations are deterministic, ([10], [1] , [13]). This is stated for sake of completeness in the next Lemma (where for $x \in X$, δ_x denotes the degenerate lottery that assigns probability 1 to x):

Lemma 2 *Let $\beta \in P(E)$, then $\beta = (\delta_{x^g}, \delta_{x^b})$, for some $x^i \in X$, $i = g, b$.*

Because of Lemma 2, we can identify elements of $P(E)$ with deterministic allocations in X^2 and, with some abuse of notation, denote them by $\hat{x} = (x^g, x^b)$.

An important role is played by the (efficient) pooling allocation $\hat{x}^p = (x^p, x^p)$, with $x^p = (\bar{e}, \bar{e})$. It is immediate to check that $\hat{x}^p$ is an optimal solution to the programming problem 2 for $a = (1/2, 1/2)$. Hence, $\hat{x}^p \in P(E)$.

The set of constrained efficient allocation can be partitioned in two disjoint sets defined by the type whose incentive constraint is binding. This is shown in the next lemma.

Lemma 3 *Let for $i = g, b$:*

$$\Theta^i = \{\hat{x} \in P(E) : \sum_i \lambda^i \pi^i x^i = \bar{e}, V^i(x^i) = V^i(x^j) \text{ and } V^j(x^i) < V^j(x^j), i \neq j\}$$

then $P(E) = \Theta^g \cup \Theta^b \cup \{\hat{x}^p\}$.

There are two important properties of the constrained efficient allocations contained in the set Θ^i. First, the elements in Θ^i assign state invariant (full insurance) allocations to individuals of type i . Second, individuals of type i strictly prefer allocations in Θ^j to allocations in Θ^i, $i \neq j$.

Lemma 4 *For $\hat{x} \in \Theta^i$, $x^i(H) = x^i(L)$ (and $x^j(H) \neq x^j(L)$). Furthermore, $V^i(x^i) > V^i(x^p) > V^i(x^{*i})$, for $(\hat{x}, \hat{x}^*) \in \Theta^j \times \Theta^i$, $i \neq j$.*

In our economy the set of strict constrained efficient allocation coincides with the set of constrained efficient allocations. This property will play a role in the characterization of the constrained competitive equilibria. The argument requires to identify individual allocations $\gamma^i \in M_1(X)$ with a pair of state contingent individual allocations $\gamma_s^i \in M_1(X')$, $s = H, L$. An individual lottery $\gamma^i \in M_1(X)$ and a pair of state contingent individual lotteries $\gamma_s^i \in M_1(X')$, $s = H, L$, are equivalent if and only if:

$$\begin{aligned} V^i(\gamma^i) &= \sum_s \pi^i(s) \int_{X'} U(x(s)) \gamma_s^i(dx(s)) \text{ and} \\ \int_X \pi^i x \gamma^i(dx) &= \sum_s \pi^i(s) \int_{X'} x(s) \gamma_s^i(dx(s)). \end{aligned}$$

Thus, for given γ^i the equivalent pair (γ_H^i, γ_L^i) is defined by setting $\gamma_s^i(O) = \int_{O \times X'} \gamma^i(dx(s), dx(s'))$, $s' \neq s$, for each Borel set $O \subset X'$. Again, with some abuse of notation, for $\gamma_s \in M_1(X')$, we set $U(\gamma_s) = \int_{X'} U(x(s)) \gamma_s(dx_s)$.

The next Lemma shows that "strict" constrained efficient allocations are constrained efficient. Recall that an allocation σ is strictly constrained efficient if $\sigma \in Y' \cap \Sigma$ and if it does not exist an allocation $\gamma \in Y' \cap \Sigma$ such that

$$(V^g(\gamma^g), V^b(\gamma^b)) \gg (V^g(\sigma^g), V^b(\sigma^b)).$$

Lemma 5 *If γ is a strict constrained efficient allocation, $\gamma \in P(E)$.*

3 Constrained Competitive Equilibria.

In this section, we define our competitive notion and we prove that, for the Rotschild and Stiglitz economy, this notion yields a unique and (constrained) efficient equilibrium allocation. Once again, all proof are in the appendix.

3.0.1 Prices and Budget Sets.

The consumption set of the individuals is $M_1(X)$. Prices are type dependent and they are pairs $p = (p^g, p^b)$ and $q = (q^g, q^b)$. p^i is the price on the i-th market for individual lotteries, a linear functionals over $M_1(X)$. Thus, the price of a lottery $\beta \in M_1(X)$ traded on the $i - th$ market is $p^i(\beta) = \int_X p^i(x)\beta(dx)$. $q^i \in \Re$ is the price of one unit of "wealth" transfer from an individual trading on the $i - th$ market to the individuals trading on the $j - th$ market, $i \neq j$. A unit of "wealth" is one unit of the only existing commodity in each realization of the personal state. If an individual buys one unit of wealth transfer, the endowments of the individuals trading on the other market are augmented by one unit of state uncontingent commodity. We can think of the wealth transfer activity as an asset. Thus, individuals of type i can buy at q^i non negative amounts of an asset which pays off one unit of wealth to the individuals of different type.

The constraint set faced by the individuals is defined by two elements. The first is the budget constraint. An individual trading on the $i-$market takes as given the price (p^i, q^i) and the (average) wealth transfer received by the individuals trading on $j - market$, $j \neq i$. The individual has to choose a lottery in $M_1(X)$ as well as a non negative transfer to the participants in the $j - th$ market, $t^i \geq 0$. For given pair of wealth transfer $t \equiv (t^g, t^b) \geq 0$, let $T^i(q,t) = p^i(\delta_{e^i(H)+t^j, e^i(L)+t^j}) - p^i(\delta_e) - q^i t^i$ denote the net transfer to individuals trading on the $i-$market and let $W^i(p,q,t) \equiv p^i(\delta_e) + T^i(q,t)$ denote their wealth.

For given (p^i, q, t), the budget set of an individual trading on market i is

$$B^i(p^i, q, t) = \{\beta \in M_1(X) : p^i(\beta) \leq W^i(p,q,t)\}.$$

3.0.2 Market Incentive Constraints and the Individual Problem.

The individual choices are restricted, in addition to the budget set, by a second element, the incentive constraint across markets. Nobody on the i-th market can buy anything that the j-th type prefers to what they can buy on the j-th market. For each given pair of wealth transfers t and prices p, the constraint sets faced by the individuals of type i, $C^i(\cdot)$, must be both budget feasible and incentive compatible in the precise sense stated in the next definition.

Definition 6 *A vector of sets $C = (C^i)_{i \in I}$ is:*

1. *incentive compatible at (p,t) if*

$$\max_{\sigma \in C^i} V^i(\sigma) \geq \max_{\tau \in C^j} V^i(\tau), \text{ for } i \neq j, \ j \text{ and } i = g, b \tag{3}$$

2. *budget feasible at (p,t) if*

$$C^i \subseteq B^i(p,t), \text{ for } i = g, b.$$

We assume that the institution governing the market incentive constraints guarantees that individual trade is restricted by the largest sets of choices that are both budget feasible and incentive compatible. Thus we look for the maximal (under set inclusion) pairs of sets that are incentive compatible and budget feasible. More precisely, C^* is maximal if for each pair of incentive compatible and (p,t) budget feasible sets C, $C^i \subseteq C^{*i}$, for all i.

We have to show that a maximal pair C^* exists. Its existence follows from the properties of an operator that we call IC operator defined next.

Definition 7 *The IC operator on vectors of sets is defined by:*

$$IC(C)^i \equiv \{\sigma \in C^i : \textit{there is } \tau \in C^j,\ j \neq i, \textit{ such that } V^j(\tau) \geq V^j(\sigma)\} \quad (4)$$

The operator is monotonic:

$$if\ D \subseteq C \text{ then } IC(D) \subseteq IC(C).$$

It is easy to show that a vector C is incentive compatible if and only if it is a fixed point of IC, namely if it solves

$$IC(C) = C. \quad (5)$$

In fact, (5) holds if and only if for no i and no j there is a $\sigma \in C^i$ with

$$V^j(\sigma) > \max_{\tau \in C^j} V^j(\tau),$$

and this is equivalent to C being incentive compatible. A simple way to prove that a largest fixed point exists and is incentive compatible is the following. Let

$$C_0^i(p,t) \equiv B^i(p,t) \quad (6)$$

and then for every n:

$$C_{n+1}^i(p,t) \equiv IC(C_n(p,t))^i \quad (7)$$

and finally let the vector C be defined by:

$$C^i(p,t) \equiv \cap_{n=0}^{\infty} C_n^i(p,t) \tag{8}$$

If for instance $\delta_0 \in B^i(p,t)$, for $i = g, b$, $C(\cdot)$ is clearly non empty, since δ_0, the vector of singletons equal to probability unit mass at the zero, is in every element $C_n(\cdot)$. In the next theorem we prove that any incentive and budget feasible pair of sets is contained in $C(\cdot)$. Evidently, this implies that $C(\cdot)$ is maximal.

Theorem 8 *If D is an incentive compatible and budget feasible (at (p,t)), then $D \subseteq C(p,t)$.*

Since individual trade is restricted by a market incentive constraint, there is no loss of generality in restricting individuals of type i to trade on the $i-th$ market, that is to transact at personalized prices (p^i, q^i).

In our economy, an individual of type i trades on the i-th market and takes as given both the prices (p,q) as well as the (average pro capita) wealth transfer received from the individuals of type j, t^j. Thus, the individual seeks an individual lottery and a wealth transfer in order to solve the following problem:

$$\max_{(\sigma,t^i)} V^i(\sigma), \ \textit{subject to } t^i \geq 0 \text{ and } \sigma \in C^i(p,q,(t^i,t^j)). \tag{9}$$

3.0.3 The Firm Problem, the Equilibrium Price Domain and the Definition of Constrained Competitive Equilibrium

Following Prescott and Townsend, we introduce a profit maximizer and price taker firm (or intermediary). The firm produces signed measure $\mu =$

$(\mu^g, \mu^b) \in (M(X))^2$ and wealth transfers $r = (r^g, r^b) \in \mathfrak{R}^2$. r^i is the (aggregate) supply to the individuals of type i of wealth transfers for individuals of type j, for short, it is the $i - type$ transfer supply. The revenue generated by the sale of r^i is obviously $q^i r^i$. Evidently the aggregate supply of r^i units of $i - type$ transfer corresponds to r^i/λ^i units of pro capita supply and, thus, the firm must guarantee that each individual of type j gets r^i/λ^i (personal) state invariant commodities. Since there are λ^j individuals of type j, the firm must, therefore, buy and deliver $(\lambda^j/\lambda^i)r^i$ units of state uncontingent commodity. Thus, to summarize, if the firm supplies the wealth transfer pair (r^g, r^b), it must deliver $(\lambda^g/\lambda^b)r^g + (\lambda^b/\lambda^g)r^b$ commodity units. Thus, by exploiting the law of the large numbers, the input requirement needed to produce an output of $(\lambda^g/\lambda^b)r^g + (\lambda^b/\lambda^g)r^b$ units is the set of signed measures $\rho = (\rho^g, \rho^b) \in [M(X)]^2$ satisfying the inequality:

$$\sum_i \int_X \pi^i x \rho^i(dx) \geq (\lambda^b/\lambda^g)r^g + (\lambda^g/\lambda^b)r^b.$$

If a firm supplies an allocation $\mu \in [M(X)]^2$, and wealth transfer $r \in \mathfrak{R}^2$ by using an input requirement $\rho \in [M(X)]^2$, the net amount of resources used by the firm is

$$\sum_i \int_X \pi^i x \mu^i(dx^i) + \sum_i \int_X \pi^i x \rho^i(dx) - (\lambda^b/\lambda^g)r^g - (\lambda^g/\lambda^b)r^b.$$

As usual, the production set Y restricts the firm to make feasible plans and, therefore, it is given:

$$\begin{gathered} Y = \{(\mu; r, \rho) \in [M(X)]^2 \times \mathfrak{R}^2 \times [M(X)]^2 : \\ \sum_i \int_X \pi^i x \mu^i(dx^i) + \sum_i \int_X \pi^i x \rho^i(dx) - (\lambda^b/\lambda^g)r^g - (\lambda^g/\lambda^b)r^b \leq \bar{e} \\ \text{and } \sum_i \int_X \pi^i x \rho^i(dx) \geq (\lambda^b/\lambda^g)r^g + (\lambda^g/\lambda^b)r^b\}, \end{gathered}$$

and the firm profit maximization problem is

$$\max_{(\mu,r,\rho)\in Y} \sum_i [p^i(\mu^i) + q^i r^i - p^i(\rho^i)]. \tag{10}$$

We are now ready to give a formal definition of equilibrium.

Definition 9 *An equilibrium of the economy E, is a collection of prices (p,q), of wealth transfers t and of an allocation γ such that: i) (γ^i, t^i) is an optimal solution to the individual programming problem 9 at (t^j, p, q), and ii) for some $(\rho^g, \rho^b) \in [M(X)]^2$, $(\lambda^g\gamma^g, \lambda^b\gamma^b)$, $(\lambda^g t^g, \lambda^b t^b)$ and (ρ^g, ρ^b) is an optimal solution to the profit maximization problem 10.*

Since $(\lambda^g\gamma^g, \lambda^b\gamma^b)$ is the optimal supply of lotteries, while (γ^g, γ^b) is the type dependent pair of pro capita demand, the lottery market clears. Similarly, by the same argument, the wealth transfer market clears. Finally, by the definition of Y, the equilibrium allocation γ, together t and the input requirement r are feasible.

The presence of the firm restricts the price domain. This is stated in the next Lemma.

Lemma 10 *The profit maximization problem has a solution if and only if $(p^g, p^b) = (\pi^g, \pi^b)$ and $(q^g, q^b) = ((\lambda^b/\lambda^g), (\lambda^g/\lambda^b))$.*

Furthermore, at $((\pi^g, \pi^b); (\lambda^b/\lambda^g), (\lambda^g/\lambda^b))$, the optimal profit is zero and the set of profit maximizing solutions is:

$$Y^* = \{(\theta, r, \rho) \in Y : \sum_i \lambda^i \int \pi^i x \theta^i(dx) = \bar{e},$$

$$\sum_i \int_X \pi^i x\rho^i(dx) = (\lambda^b/\lambda^g)r^g + (\lambda^g/\lambda^b)r^b\}.$$

Because of the last Lemma, we can drop (p, q) as an element of the various functions and sets previously defined. We can also ignore the firm and just think in terms of a pure exchange economy. Therefore, we give the following simple and direct definition of equilibrium, where prices are mentioned, but it is understood that $(p^g, p^b) = (\pi^g, \pi^b)$ and $(q^g, q^b) = ((\lambda^b/\lambda^g), (\lambda^g/\lambda^b))$:

Definition 11 *An equilibrium of the economy E, is a pair wealth transfers t and an allocation γ such that: i) (γ^i, t^i) is an optimal solution to the individual programming problem 9 at t^j, and ii) $\sum_i \lambda^i \int_X x\gamma^i(dx) = \bar{e}$.*

The last condition guarantees not only the feasibility of the equilibrium allocation, but also that $(\lambda^g\gamma^g, \lambda^b\gamma^b, \lambda^g t^g, \lambda^b t^b, \rho^b, \rho^g) \in Y^*$ for each ρ that satisfies $\sum_i \int_X \pi^i x\rho^i(dx) = (\lambda^g/\lambda^b)t^g + (\lambda^b/\lambda^g)t^b$ as for instance $\rho^i = \delta_{((\lambda^j/\lambda^i)t^i, (\lambda^j/\lambda^i)t^i)}$, $i = g, b$.

3.0.4 Equilibrium Properties of the Individual Optimal Solution and the First Fundamental Theorem of Welfare Economics.

The next Lemma by exploiting the equilibrium restrictions on the price domain provides an equivalent, but alternative definition of the constraint sets $(C^g(t), C^b(t))$ that will be handy for the analysis that follows. For any set $\Gamma \subset M_1(X)$, let $V^i(\Gamma) \equiv \max_{\gamma\in\Gamma} V^i(\gamma)$.

Lemma 12 *For all t such that $W^i(t) \geq W^j(t)$, define $\hat{C}^i(t) = \{\sigma^i \in B^i(t) : V^j(\sigma^i) \leq V^j(B^j(t)\}$, if $W^i(t) \geq W^j(t)$, while, $\hat{C}^i(t) = \{\sigma^i \in B^i(t) : V^j(\sigma^i) \leq V^j(\hat{C}^j(t)\}$, otherwise. Then, $(\hat{C}^g, \hat{C}^b)(.) = (C^g, C^b)(.)$.*

Lemma 12 allows for an immediate characterization of the equilibrium optimal solution to the programming problem 9. This is stated and shown in the next Lemma.

Lemma 13 *Let* (γ, t) *be an equilibrium, then,* $\gamma = \delta_x$ *with* $x^i = (W^i(t), W^i(t))$, *if* $W^i(t) \leq W^j(t)$, *while* x^i *is such that* $\pi^i x^i = W^i(t)$ *and* $V^j(x^j) = V^j(x^i)$, *otherwise.*

Thus by the last Lemma, equilibrium allocations are deterministic. We denote them as already done in the efficiency analysis by $\hat{x} = (x^g, x^b) \in X$.

An immediate consequence of Lemma 13 is the constrained efficiency of the equilibrium allocations. The constrained version of the First Fundamental Theorem of Welfare Economics is in the next Lemma.

Lemma 14 *If* $\hat{x}$ *is an equilibrium allocation,* $\hat{x} \in P(E)$

3.1 Existence and Uniqueness of the Incentive Market Equilibrium.

In order to characterize the equilibrium set, we show that Lemma 14 and simple rationality considerations pin down a unique allocation, $\hat{x}^*$, which is compatible with the optimizing behavior of the individuals. Later we show that $\hat{x}^*$ is indeed an equilibrium allocation and that it is unique.

A first observations shows that for the individuals of any type the utility levels associated with the equilibrium allocations are bounded below by those associated with the separating allocation of Rotschild and Stiglitz. The reason is simple. First, recall that the separating (deterministic) allocation, $\hat{x}_S = (x^g_S, x^b_S)$, is the solution for individuals of type i, $i = g, b$, to:

$$\max_{\sigma} V^i(\sigma) \textit{ subject to } \sigma \in C^i(0). \tag{11}$$

Since $W^b(0) < W^g(0)$, it follows from Lemma 13 that

$x_S^b = (W^b(0), W^b(0))$, $V^b(x_S^g) = V^b(x_S^b)$, $V^g(x_S^b) < V^g(x_S^g)$ and $\pi^H x_S^H = \pi^H e$.

Since $t^k \geq 0$, an individual of type i can always "undo" the given wealth transfer of individuals of type j, t^j, by setting $t^i = (\lambda^i/\lambda^j)t^j$. Observe that for t' satisfying $t'^i = (\lambda^i/\lambda^j)t'^j$, $B^k(t') = B^k(0)$ and $C^k(t') = C^k(0)$, for $k = g, b$. Thus, individuals of type i, by setting $t^i = (\lambda^i/\lambda^j)t^j$, can always reduce, if they find it convenient, the programming problem 9 to the programming problem 11.

The second observation shows that at equilibrium $T^b(t) = t^g - (\lambda^g/\lambda^b)t^b \geq 0$. First, as already observed, individuals of *type* b can always select values of the wealth transfer generating $T^b(t) \geq 0$. Second, $C^b(t) \subset B^b(t) \subset B^b(0)$, for $T^b(t) < 0$. Thus, since $V^b(x_S^b) = V^b(B^b(0)) = V^b(C^b(0))$, by strict monotonicity of individual preferences, $V^b(C^b(0)) \geq V^b(C^b(t))$, for $T^b(t) < 0$. Furthermore, since, $W^b(t) < W^b(0)$, for $T^b(t) < 0$, $x_S^b \notin C^b(t)$. Thus, the inequalities are strict, i.e., $V^b(C^b(0)) > V^b(C^b(t))$, for $T^b(t) < 0$.

The third observation establishes the existence of an upper bound to the value of wealth transfer $T^b(t)$ (or, a lower bound for $T^g(t)$). Consider the "pooling" efficient allocation $\hat{x}_P$. Recall that $x_P = (\bar{e}, \bar{e}) \in X$. Let $\bar{t}$ be a pair of wealth transfers such that $W^k(\bar{t}) = \bar{e}$, $k = g, b$. Evidently, for both types, $\delta_{x_P^i} \in \arg\max_{\sigma^i \in B^i(\bar{t})} V^i(\sigma^i)$. Hence, $V^g(B^g(\bar{t})) = V^g(C^g(\bar{t})) = V^g(x_P)$. Thus, since $C^g(t) \subset B^g(t) \subset B^g(\bar{t})$ and $\delta_{x_P} \notin B^g(t)$, for $T^g(t) < T^g(\bar{t})$, $V^g(x_P)(= V^g(C^g(\bar{t}))) > V^g(C^g(t))$. Thus, whatever is the transfer t^b

of the individuals of type b, the optimal choice of the individuals of type g yields a net transfer $T^g(t^g, t^b)$ bounded below by $\bar{T}^g \equiv T^g(\bar{t})$ and an allocation providing a utility value above $\max\{V^g(x_P), V^g(x_S^g)\}$.

We summarize the implications of the three observations under the form of a Lemma whose proof is omitted since the argument has already been explained.

Lemma 15 *Let* $(t^*, \hat{x}^*)$ *be an equilibrium, then* $T^g(t^*) \in [-T^g(\bar{t}), 0]$, $T^b(t^*) \in [0, T^b(\bar{t})])$, $V^b(x^{*b}) \geq V^b(x_S^{*b})$ *and* $V^g(x^{*g}) \geq \max\{V^g(x_P), V^g(x_S^g)\}$.

Lemma 15 pins down the equilibrium behavior of the individuals of type b. This is stated in the Lemma below. Let $x^b(t^g)$ be a deterministic and state invariant allocation for individuals of type b defined as $x^b(t^g) \equiv (W(t^g, 0), W(t^g, 0))$.

Lemma 16 *Let* $(t^*, \hat{x}^*)$ *be an equilibrium, then* $(t^{*b}, x^{*b}) = (0, x^b(t^{*g}))$.

To summarize, the analysis developed so far has the following three implications: if there exists an equilibrium, $(t^*, \hat{x}^*)$, 1) $\hat{x}^* \in P(E)$, 2) the wealth transfer of the individuals of type g is an element of the interval $[0, -T^g(\bar{t})]$, and 3) for each $t^g \in [-T^g(\bar{t}), 0]$, the optimal choice of the individuals of type b is to select the pair wealth transfer and allocation $(0, x^b(t^{*g}))$.

Thus, we are left with specifying (t^{*g}, x^{*g}), the (potential) equilibrium choices of the individuals of type g . By the three points previously stated, x^{*g} is found by solving the following programming problem:

$$\begin{aligned} &\max_{\hat{x} \in P(E)} V^g(x^g) \qquad (12) \\ \textit{subject to} \quad : \quad & V^b(x^b) \geq V^b(x_S^b), V^g(x^g) \geq \max\{V^g(x_S^g), V^g(x_P)\}. \end{aligned}$$

Denote by Φ^* the set of optimal solutions to 12 and, then, for $x^* \in \Phi^*$, define t^{*g} by setting $t^{*g} \equiv (\lambda^g/\lambda^b)(\pi^g e - \pi^g x^{*g})$.

Lemma 17 *Φ^* is a singleton $\{\hat{x}^*\}$, with $\hat{x}^* \in \Theta^b$.*

We conclude the argument by showing that the pair of wealth transfers $(t^{*g}, 0)$ and the allocation $\hat{x}^*$ are the unique competitive equilibrium.

Proposition 18 *The pair of wealth transfers $t^* = (t^{*g}, 0)$ and the allocation $\hat{x}^*$ constitute the unique competitive equilibrium.*

4 Comparative Statics

Many (competitive) equilibrium concepts used for Adverse Selection economy, e.g., Dubey and Geanakoplos [4], Gale [6] and Bisin and Gottardi [1] and [2], have the Rotschild and Stiglitz allocation either as an element of the equilibrium set or as the only possible equilibrium outcome. As known, the separating allocation is independent of λ^b an important parameter affecting its efficiency. Consider two economies E and E' identical in all aspects, but in the proportion of the individuals *type* b. Denote by $\hat{x}_S(\lambda^b)$ and $\hat{x}_S(\lambda'^b)$ the separating allocations for the two economies. By definition of the programming problems 11, $\hat{x}_S(\lambda^b) = \hat{x}_S(\lambda'^b)$. Thus, (and, as it is well known) the social cost of $\hat{x}_S(\lambda^b)$ increases as λ^b converges to zero. For the constrained competitive economy, the unique equilibrium allocation is constrained efficient. Thus, since our competitive notion does not suffer the drawbacks of the other competitive notions, as the proportion of the individuals of *type* b becomes small, it is small their influence on the equilibrium allocation of the $g - type$. More specifically, as $\lambda^b \to 0$, the equilibrium allocation of the individuals of type g (as well as of type b) converges to the full insurance allocation $x_0 = (\pi^g e, \pi^g e)$. This follows immediately by observing that, for $x_0 = (\pi^g e, \pi^g e)$, $V^g(x_0) \geq V^g(x^{*g}(\lambda^b)) \geq V^g(x^p(\lambda^b))$ and that for $\lambda^b \to 0$, the pooling allocation $\hat{x}_P(\lambda^b) \to \hat{x}_0$. Thus, the proof of the next Lemma is obvious and, therefore, it is omitted.

Lemma 19 *As $\lambda^b \to 0$, the equilibrium allocation $\hat{x}^*(\lambda^b)$ converges to $\hat{x}_P(0) = \hat{x}^0$.*

5 Constrained Lindahl Equilibria of Adverse Selection Economies

In this section, we analyze the equilibrium notion introduced by Bisin and Gottardi ([1] and [2]). Thus, the adverse selection economy is identified with an economy with consumption externalities and the equilibrium notion is the one of Lindahl. We simplify the analysis by limiting attention to deterministic allocations and linear prices. As shown in the companion working paper ([15]), all the results of this section survive the introduction of lotteries and non linear prices.

The individual consumption set is the set of incentive compatible pairs $\hat{x} = (x^g, x^b) \in X^2$ assigning a contingent consumption bundle to each type. $x^i = (x^i(L), x^i(H))$ is the consumption assigned to the i type, $i = g, b$.

5.1 Prices and Budget Sets

Prices are personalized, that is (potentially) type dependent, and they specify a price for each of the two markets. So the price p^i_j is the price on the market of type i for property rights over type j-th consumption vectors. A *price system* is a pair $p = (p^g, p^b)$, where the price in the i-th's type market is a pair $p^i = (p^i_g, p^i_b)$, for $i = g, b$. Again the convention that the first coordinate refers to the g type is followed.

Endowments have specific prices: the price of the endowment of an individual claiming of being of type i is q^i. Later, it will become clear what is the relation between q^i and p^i.

By denoting with Σ_d the set of incentive compatible and deterministic allocations, the budget set of an individual claiming of being of type i is:

$$B^i(p, q) = \{\hat{x} \in \Sigma_d : p^i \hat{x} \leq q^i e\}.$$

5.2 Price Dependent Incentive Constraint

Trade is further restricted. The trade on the i-th market is restricted so that type j, $j \neq i$, does not find convenient to misreport its type and trade on the i-th market. Let $\hat{x}_i \equiv (x^g_i, x^b_i)$ denote an allocation traded on the $i - th$ market. Specifically the trade on i-th market is restricted by the following price dependent constraint:

$$C^i(p, q) \equiv \{\hat{x} \in B^i(p, q) : for\ j \neq i,\ there\ is\ \hat{x} \in C^j(p, q),\ V^j(x^j) \geq V^j(x^i)\}. \tag{13}$$

The definition of the sets $C^i(.)$ for the Lindhal economy is basically identical to the one used for the same sets in the constrained competitive economy (see theorem 8 and Lemma 13). Thus, we do not need to add further explanations.

5.3 The Firm

There is a Prescott and Townsend firm, which buys and sells feasible bundles. The firm buys inputs $y^i \in \Re$, with $i = g, b$ from households claiming to be of type i. It produces outputs $z_i = (z_i^g, z_i^b) \in \Re^2$, with $i = g, b$.

The production set of the firm is:

$$Y = \{(z, y) \in \Re^{10} : \sum_{i \in I} \pi^i (z_i^i - y^i) \leq 0 \text{ and for all } i, j,\ z_i^i = (\lambda^i / \lambda^j) z_j^i\}.$$

The vector y^i is the net purchase of type i contingent commodity vectors. One may think of y as inputs for the production of (z_g^g, z_b^b). However, if the consumption vector z_i^i is supplied to individuals of type i, the pro capita consumption is z_i^i/λ^i. Whatever is the supply z_i^i, the production set requires the firm to provide the same supply to individuals of type j of property rights over the consumption vector of individuals of type i. Since, type j individuals are λ^j, consistency of supplies across types requires that $z_i^i = (\lambda^i/\lambda^j) z_j^i$. In different words, the condition $z_i^i = (\lambda^i/\lambda^j) z_j^i$ requires that pro-capita supplies across types coincide, as in any Lindahl equilibrium.

5.4 Lindahl Equilibrium

Definition 20 *A Constrained Lindahl equilibrium is a price system (p, q), a production plan (z^*, y^*) and allocations $\hat{x}^* \in \Sigma_d$ such that:*

1. *for every $i \in I$, $\hat{x}^*$ is an optimal solution to:*

$$\max_{\hat{x}} V^i(\hat{x}) \text{ subject to: } \hat{x} \in C^i(p, q),$$

2. (z^*, y^*) *is an optimal solution to*

$$\max_{(z,y)\in Y} \sum_{i\in I} (p^i z_i - q^i y^i) \tag{14}$$

3. for every $i \in I$, $\lambda^i x_i^{*i} = z_i^{*i}$ *and* $y^{*i} = \lambda^i e$ *(that is, markets clear).*

The feasibility of the equilibrium allocation is insured by the definition of the production set Y.

6 Lindahl Equilibria: Efficiency and Determinacy

We first show that the set of Lindahl equilibria has a simpler, equivalent, formulation. We then use this formulation to show that Lindahl equilibria of economies with Adverse Selection may be inefficient and that they are indeterminate.

6.1 Price Domain

The presence of the profit maximizing firm restricts the price domain in the precise sense stated in the following lemma whose proof is in the appendix.

Lemma 21 *The programming problem 14 has an optimal solution if and only if*

$$p_g^g + p_g^b(\lambda^b/\lambda^g) = q^g,\ \ p_b^b + p_b^g(\lambda^g/\lambda^b) = q^b,\ \ \textit{and } q^i = c\pi^i,\ i = g, b,$$

for some $c > 0$.

As usual, once the firm has restricted the price domain, we can safely get rid of it, and rewrite the economy as a simple pure exchange one. Every feasible allocation is profit maximizing. By taking into account the price restriction implied by the presence of the profit maximizing firm (and by normalizing, without loss of generality, $c = 1$), we can now write the budget set and price constrained set as simply functions of $p:\ D^i(p) \equiv C^i(p, (\pi^i)_{i\in I})$.

Definition 22 *A constrained Lindahl equilibrium of the incentive constrained pure exchange economy is a pair of personalized prices* (p^g, p^b) *and a feasible consumption* $\hat{x}^* = (x^{*g}, x^{*b}) \in X^2$ *such that:*

1. $\hat{x}^*$ *is, for* $i \in I$, *an optimal solution to*

$$\max_{\hat{x}} V^i(x^i) \text{ subject to: } \hat{x} \in D^i(p), \tag{15}$$

2. *The restriction on the price domain*

$$p_g^g + p_g^b(\lambda^b/\lambda^g) = \pi^g,\ p_b^b + p_b^g(\lambda^g/\lambda^b) = \pi^b. \tag{16}$$

is satisfied.

6.2 Lindahl Equilibria may be inefficient

Consider the standard Rotschild Stiglitz diagram, and take the case where the separating allocation is not efficient. Denote the separating allocation by $\hat{x}^* = (x^{*g}, x^{*b})$.

$x^{*b} = (\pi^b e, \pi^b e)$ maximizes the utility of the b type $\sum_{s\in S} \pi^b(s) U(x(s))$ subject to the constraint $\sum_{s\in S} \pi^b(s)(x(s) - e(s)) \leq 0$ and the incentive constraint $V^g(x^g) \geq V^g(x^{*g})$. x^{*g} is the optimal solution to the maximization of the utility of the g type, $\sum_{s \in S} \pi^g(s)U(x(s))$, subject to the constraint $\sum_s \pi^g(s)(x(s) - e(s)) \leq 0$, and the classical Rotschild Stiglitz incentive constraint $V^b(x^{*b}) \geq V^b(x^g)$.

From our initial assumption that the separating allocation is not efficient, this is not an efficient allocation. We claim that (x^{*g}, x^{*b}) is a Constrained Lindahl equilibrium allocation supported by the prices $p^{*g} = (\pi^g, 0)$, and $p^{*b} = (0, \pi^b)$.

First, $p^* \equiv (p^{*g}, p^{*b})$ satisfies the price domain restrictions (16). The set $D^g(p^*)$ is the set of allocations $x = (x^g, x^b)$ that are budget feasible for g at prices $(\pi^g, 0)$, and such that $\hat{x}^g$ is below the indifference curve of type b that passes through x^b. Similarly, the set $D^b(p^*)$ is the set of allocations x that are budget feasible for type$b$ individuals at prices $(0, \pi^b)$ and such that x^b is below the indifference curve of type g that passes through x^{*g}. Thus at prices p^*, the constraint sets of the individuals are:

$$D^g(p^*) = \{x \in \Sigma_d : \pi^g(x^g - e) \leq 0 \; and \; V^b(x^g) \leq V^b(x^{*b})\}$$

and

$$D^b(p^*) = \{x \in \Sigma_d : \pi^b(x^b - e) \leq 0 \; and \; V^g(x^b) \leq V^g(x^{*g})\}$$

Evidently, $\hat{x}^*_g = \hat{x}^*_b \equiv (x^{*g}, x^{*b})$ are the optimal solutions to the programming problem 15 for individuals of type g and b when the constraint sets $D^i(.)$ are as defined above and the prices are $p^* = ((\pi^g, 0), (0, \pi^b))$. Hence, the price system $p^{*g} = (\pi^g, 0)$, $p^b = (0, \pi^b)$, and the allocation (x^*_g, x^*_b) are a Lindahl equilibrium. Thus, the constrained Lindahl equilibrium set may contain constrained inefficient allocations.

The First Welfare Theorem

The Lindahl equilibrium set contains efficient allocations. The reason is simple. Consider first a Lindahl economy identical to the one we have just defined, but were types are observable, so that each household of type i may be forced by contract only in the i-th market.

In this economy the consumer is solving the problem

$$\max_{\hat{x}} V^i(x^i), \text{ subject to } \hat{x} \in B^i(p,q). \tag{17}$$

This economy, with the profit maximizing firm, has an equilibrium ([1]). Evidently, since for every price system (p,q), $B^i(p,q) \subset \Sigma_d$, the equilibrium of this economy is incentive compatible. In addition, this equilibrium is efficient, since this equilibrium satisfies the conditions of the First Welfare Theorem. Let $(x'^g, x'^b) \in \Sigma_d$ be this equilibrium allocation.

We now consider again the economy with private information. We want to sustain the described allocation as Lindahl equilibrium. To do this, we impose the additional incentive constraint. Rewrite $D^i(p) = \{\hat{x} \in B^i(p) : V^j(x^i) \leq V^j(x'^j), \text{ for } j \neq i\}$. This is now a constrained Lindahl equilibrium.

This argument proves that there are efficient equilibrium allocations in the constrained Lindahl economy. The converse however is not true. In particular the claim that the first welfare theorem holds for such economies is, as we have shown, false. It is easy to see why: the standard proof by contradiction assumes the existence of an allocation that is preferred by all types. From this one typically derives that the allocation is not in the budget set, and hence would give a higher profit to the firm. The conclusion that the allocation is not in the budget set however does not hold in the case of the incentive constrained economies, since this allocation might be in the budget set, but not chosen by the consumer because excluded by the incentive constraint.

6.3 Lindahl Equilibria are indeterminate

The Lindahl equilibrium notion analyzed so far suffers an additional drawback: there is a continuum of allocations. Furthermore, whenever the separating allocation of Rotschild and Stiglitz is constrained suboptimal, the constrained Lindahl equilibrium set contains a continuum of Pareto ranked allocations. This part is rather technical in nature and we state the result without a formal proof which is however available in the companion working ([15]).

Proposition 23 *The constrained Lindhal equilibrium set contains a continuum of payoff relevant allocations. Furthermore, if the separating allocation of Rothschild and Stiglitz is inefficient, the equilibrium set contains a continuum of inefficient allocations.*

7 Appendix

Proof of Lemma 3.

In order to prove the lemma, it suffices to show that the two following properties hold true:

i) if $\hat{x} \in P(E)$ and $V^i(x^i) = V^i(x^j)$, for all i *and* j, then $\hat{x} = \hat{x}^p$.

ii) if $\hat{x} = (x^g, x^b) \in Y'$ is such $V^i(x^i) > V^i(x^j)$ for all i and j, $i \neq j$, then $\hat{x} \notin P(E)$,

iii) if $\hat{x} \in P(E)$ then $\sum_i \lambda^i \pi^i x^i = \bar{e}$.

i) is an immediate consequence of the fact that if the two stated equalities hold true then $x^g = x^b$. Then, by the strict monotonicity and concavity of U, efficiency implies that $x^i = x^p$.

ii) is shown by contradiction. Thus, if an allocation in $P(E)$ satisfies the inequalities in ii), then $x^i(H) \neq x^i(L)$ for some i. Without loss of generality suppose that $x^i(H) > x^i(L)$. Pick an arbitrarily small $\varepsilon > 0$ and define the type i allocation $x^{\varepsilon i} = (x^i(H) - \pi^i(L)\varepsilon, x^i(L) + \pi^i(H)\varepsilon)$. Evidently, $(x^{\varepsilon i}, x^j)$ is feasible and, for ε small enough, incentive compatible. Furthermore,

$$D_\varepsilon V^i(x^{\varepsilon i})|_{\varepsilon=0} = \pi^i(H)\pi^i(L)(U'(x^i(L)) - U'(x^i(H)) > 0.$$

A contradiction.

In order to prove iii), suppose, by contradiction, that $\sum_i \lambda^i \pi^i x^i < \bar{e}$, for $\hat{x} = (x^g, x^b) \in P(E)$, pick a pooling deterministic allocation $\hat{z}^* = (z^*, z^*)$ with z^* arbitrarily large and such that $V^i(z^*) > V^i(x^i)$, $i = g, b$. Define, for $\varepsilon \in (0, 1)$, the lotteries $\sigma^i_\varepsilon = \varepsilon\delta_{z^{*i}} + (1-\varepsilon)\delta_{x^i}$. Since Σ is convex, $\sigma_\varepsilon = (\sigma^g_\varepsilon, \sigma^b_\varepsilon) \in \Sigma$. Furthermore, $V^i(\sigma^i_\varepsilon) = \varepsilon V^i(z^*) + (1-\varepsilon)V^i(x^i) > V^i(x^i)$, $i = g, b$. However,

since $\sum_i \lambda^i \pi^i x^i < \bar{e}$, there exists $\varepsilon \in (0,1)$, small enough, such that σ_ε is feasible. The latter contradicts the asserted constrained efficiency of x .
■

Proof of Lemma 4.

The proof of the first claim is virtually identical to the proof of point ii) and it is therefore omitted. In order to prove the remaining part of the Lemma, we show first that a) $V^i(x^p) > V^i(x^{*i})$, for $\hat{x}^* \in \Theta^i$, and then that b) $V^i(x^i) > V^i(x^p)$, for $\hat{x} \in \Theta^j$

If a) were not true, by lemma 4, $x^{*i}(H) = x^{*i}(L) \geq \bar{e}$. Furthermore, by the definition of constrained efficiency, $V^j(x^{*j}) \leq V^j(x^p)$, while by the definition of Θ^i , $V^j(x^{*j}) > V^j(x^{*i})$. However, the last two inequalities contradict $x^{*i}(H) = x^{*i}(L) \geq \bar{e}$.

If b) were not true, by definition of constrained efficiency, $V^j(x^j) > V^j(x^p)$. Since $\hat{x} \in \Theta^j$, b) is equivalent to a) once i is replaced by j. Thus, because i and j were arbitrary, the claim. ■

Proof of Lemma 5.

If by contradiction, $\gamma \notin P(E)$, $(V^g(x), V^b(x)) > (V^g(\gamma^g), V^b(\gamma^b))$, for some $\hat{x} \in P(E)$. Pick $\mu \in (0,1)$ and define the allocation $\sigma = (\sigma^g, \sigma^b)$ as $\sigma^i = \mu\delta_{x^i} + (1-\mu)\gamma^i$. By convexity, σ is incentive compatible and feasible. Furthermore, $V^i(\sigma^i) = \mu V^i(x^i) + (1-\mu)V^i(\gamma^i)$ and, thus, $V^i(\sigma^i) > V^i(\gamma^i)$, for some i, while $V^j(\sigma^j) \geq V^j(\gamma^j)$, for $j \neq i$. Identify γ^i and σ^i , $i = g, b$, with two pair of equivalent state contingent lotteries γ^i_s and σ^i_s, $s = \alpha, \beta$. Then, by definition, $\sigma^i_s = (1-\mu)\gamma^i_s + \mu\delta_{x^i(s)}$, $s = H, L$ and $i = g, b$. Since U is strictly concave and $\gamma \neq \delta_{\hat{x}}$, there exists a deterministic allocation $\hat{z} = (z^g, z^L)$ with $z^i(s) \leq \int_{X'} x(s)\sigma^i_s(dx^i(s) = \mu x^i(s) + (1-\mu)\int_{X'} x(s)\gamma^i_s(dx(s))$ and $U(z^i(s)) = U(\sigma^i_s) = \mu U(x^i(s)) + (1-\mu)U(\gamma^i(s))$, $i = g, b$ and $s = H, L$. Furthermore, since $\delta_{\hat{x}} \neq \gamma$, $z^i(s) < \mu x^i(s) + (1-\mu)\int_{X'} x(s)\gamma^i_s(dx(s))$ for at least a pair (i, s). Thus, since , $V^i(z^i) = V^i(\sigma^i)$, $i = g, b$, $\hat{z}$ is incentive compatible, but $\sum_{is} \lambda^i \pi^i(s)(z^i(s) - e(s)) < 0$. Exploiting the strict monotonicity of U, pick a pooling deterministic allocation $\hat{z}^* = (z^*, z^*)$ with z^* arbitrarily large and such that $V^i(z^{*i}) > V^i(z^i)$, $i = g, b$. Define, for $\varepsilon \in (0, 1)$, the lotteries $\beta^i_\varepsilon = \varepsilon\delta_{z^{*i}} + (1-\varepsilon)\delta_{z^i}$. By the convexity of Σ, $\beta_\varepsilon = (\beta^g_\varepsilon, \beta^b_\varepsilon) \in \Sigma$. Furthermore, $U^i(\beta^i_\varepsilon) = \varepsilon V^i(z^*) + (1-\varepsilon)V^i(z^i) > V^i(z^i) \geq V^i(\gamma^i)$, $i = g, b$. However, since $\sum_{is} \lambda^i \pi^i(s)(z(s) - \omega(s)) < 0$, there exists $\varepsilon \in (0, 1)$, small enough, such that β_ε is feasible. The latter contradicts the asserted strict constrained efficiency of γ. ■

Proof of Theorem 8.

We claim that for every n

$$D^i \subseteq C^i_n(p, t) \tag{18}$$

When $n = 0$ this is the condition that D is budget feasible.

Assume now the induction hypothesis that (18) holds for n, we claim it holds for $n + 1$. Let $\sigma \in D^i$. Since D is incentive compatible, there is a $\tau \in D^j$, $j \neq i$, such that

$$V^j(\tau) \geq V^j(\sigma). \tag{19}$$

By the induction hypothesis, $D^j \subseteq C_n^j(p,t)$, and so $\tau \in C_n^j(p,t)$. Therefore

$$\sigma \in C_{n+1}^i(p,t) \tag{20}$$

too, since for every $j \in I$ the τ insures (19) above. From (18) we now conclude that

$$D^i \subseteq C^i(p,t). \qquad \blacksquare$$

Proof of Lemma 10.

The claim $(p^g, p^b) = (\pi^g, \pi^b)$ is standard and it is an immediate consequence of Farkas' Lemma. Thus, $p^i(x) = \pi^i x$, for $x \in X$ and $i = g, b$. Hence, $\sum_i \int_X p^i(x)\rho^i(dx) = \sum_i \pi^i \int_X x\rho^i(dx)$ which implies that at an optimum

$$\sum_i \int_X p^i(x)\rho^i(dx) = \sum_i \pi^i \int_X x\rho^i(dx) = (\lambda^b/\lambda^g)r^g + (\lambda^g/\lambda^b)r^b.$$

The last equality implies, by the linearity in $(*, \rho)$ of the production set of the firm, that $q^i = \lambda^i$. $\blacksquare$

Proof of Lemma 12.

First, observe that, by definition, $IC(\hat{C}(.)) = \hat{C}(.)$. Moreover, by the strict concavity and monotonicity of U, $\delta_{(W^\kappa, W^\kappa)} = \arg\max_{\sigma^\kappa \in B^\kappa(t)} V^\kappa(\sigma^\kappa)$ and, hence, $V^k(\delta_{(W^\kappa, W^\kappa)}) = V^\kappa(B^\kappa)$. Suppose, without loss of generality, that, $W^i(t) \geq W^j(t)$, $i \neq j$. Then, since $\delta_{(W^j(t), W^j(t))} \in B^i(t)$, $IC(B^i(t), B^j(t))^i = \hat{C}^i(t)$, $IC(B^i(t), B^j(t))^j = B^j(t)$, while also $IC(\hat{C}^i(t), B^j(t))^i = \hat{C}^i(t)$ and $IC(\hat{C}^i(t), B^j(t))^j = \hat{C}^j(t)$. Thus, $IC(IC(B^i(t), B^j(t))) = \hat{C}(t)$. Hence, the claim. $\blacksquare$

Proof of Lemma 13.

If $W^i(t) \leq W^j(t)$, the claim is an immediate consequence of Lemma 12. Thus consider the case $W^i(t) > W^j(t)$. First we show that γ^i is degenerate and, then, that the budget set is satisfied with an equality. Argue by contradiction and identify γ^i with two pair of equivalent state contingent lotteries μ_s^i , $s = H, L$. Since U is strictly concave, there exists a deterministic consumption $z^i = (z^i(H), z^i(L))$ with $z^i(s) \leq \int_{X'} x(s)\mu_s^i(dx^i(s))$ and $U(z^i(s)) = U(\mu_s^i)$, $s = H, L$. Furthermore, since γ^i is a non trivial lottery, $z^i(s) < \int_{X'} x(s)\gamma_s^i(dx(s))$, for at least one state s, while by construction, $V^\kappa(z^i) = V^\kappa(\gamma^i)$, $\kappa = g, b$. Therefore, the allocation (z^i, x^j), is incentive compatible, but $\pi^i z^i < \pi^i(\gamma^i) \leq W^i(t)$. Let N_ε be an $\varepsilon - ball$ around z^i. Since $\pi^i \neq \pi^j$, the definition of V^k, $k = g, b$, implies that, for each $\varepsilon > 0$, there exists $z'^i \in B_\varepsilon$ such that $V^i(z'^i) > V^i(z^i) = V^i(\gamma^i)$, while $V^j(z'^i) \leq V^j(z^i) = V^j(\gamma^j)$. However, for ε appropriately small, $\pi^i z'^i \leq W^i(t)$. The latter shows that, if γ^i is a non trivial lottery, (γ^i, t^i) cannot be a solution to the individual programming problem 9, for any $t^i \geq 0$, and it show as well that at the optimal solution the budget constraint is satisfied with an equality. The last condition $V^j(x^j) = V^j(x^i)$ follows immediately from the fact that if it were $V^j(x^j) > V^j(x^i)$, then $x^i = (W^i(t), W^i(t))$. The latter cannot be incentive compatible since $W^i(t) > W^j(t)$. ■

Proof of Lemma 14.

Denote by t be the equilibrium pairs of wealth transfer and argue by contradiction. Then by Lemma 5, there is $\hat{x}' \in P(E)$ such that $V^k(x'^k) >$

$V^k(x^k)$, for all k . By Lemma 13, summing over types the budget constraints we get that $\sum_i \lambda^i \pi^i (x^i - e) = 0$. Similarly, by Lemma 3, $\sum_i \lambda^i \pi^i (x'^i - e^i) = 0$. Thus, there exists i such that $\pi^i x^i > \pi^i x'^i$, because otherwise $x'^k \in C^k(t)$, for $k = g, b$, and since $V^k(x'^k) > V^i(x^k)$, (x^k, t^k) cannot be an optimal choice. Set, $t' = (t'^i, t^j)$ with $(\lambda^j/\lambda^i)(t^i - t'^i) = \pi^i(x^i - x'^i)$. Then, it is immediate to verify that $x'^k \in B^k(t')$, $k = g, b$. However, since $\hat{x}' \in P(E) \subset \Sigma$, by Lemma 12, $\hat{x}'^\kappa \in C^\kappa(t')$, for $\kappa = g, b$. The latter together with the inequality $V^i(\hat{x}'^i) > V^i(x^i)$ imply that given t^j , (x^i, t^i) is not an optimal solution for type i to the programming problem 9 . A contradiction. ■.

Proof of Lemma 16.

By Lemma 15, $T^{*g} \in [-T^g(\bar{t}), 0]$ and, hence, $W^b(t^{*g}, 0) \leq W^g(t^{*g}, 0)$. Thus, $x^b(t^{*g}) \in C^b(t^{*g}, 0)$. Observe that $V^b(x^b(t^{*g})) = V^b(B^b(t^{*g}, 0))$ and that, for $t^b > 0$, $C^b(t^{*g}, t^b) \subset B^b(t^{*g}, t^b) \subset B^b(t^{*g}, 0)$. Since, for $t^b > 0$, $x^b(t^{*g}) \notin B^b(t^{*g}, t^b)$, $V^b(x^b(t^{*g})) > V^b(C^b(t^{*g}, t^b))$. The latter implies the thesis ■.

Proof of 17.

If Φ^* contains two distinct allocations $\hat{x}_1$ and

x$_2$, by the definition of the programming problem 12, $V^g(x_1^g) = V^g(x_2^g)$. Then, since $\Phi^* \subset P(E)$, $V^b(x_1^b) = V^g(x_2^b)$. Thus the non trivial lottery $\gamma_\mu = \mu\delta_{x_1} + (1-\mu)\delta_{x_2}$, $\mu \in (0, 1)$, is an element of $\Sigma \cap Y'$ and, by construction, $V^i(\gamma_\mu^i) = V^i(x_1^i) = V^i(x_2^i)$, $i = g, b$. Thus, $\gamma_\mu \in P(E)$. However, by Lemma 2, constrained efficient allocations are deterministic. Thus, $\Phi^* = \{\hat{x}^*\}$. Since,

$\hat{x}^* \in P(E)$ and since $V^g(x^{*g}) \geq \max\{V^g(x^g_S), V^g(x_P)\}$, by Lemma 4, $\hat{x}^* \in \Theta^b \cup \{\hat{x}_P\}$. Then, Lemma 4 implies that $t^{*g} \in [0, T^g(\bar{t})]$ and $x^{*b} = x^b(t^{*g})$. Thus we just have to show that $\hat{x}^*$ is not the pooling and efficient allocation $\hat{x}_P$. For $\varepsilon > 0$, let $\hat{x}_\varepsilon = (x^b_\varepsilon, x^g_\varepsilon)$ with $x^b_\varepsilon = (\bar{e} - \varepsilon, \bar{e} - \varepsilon)$ and x^g_ε satisfying the following two restrictions:

$$\lambda^g \pi^g x^g_\varepsilon = \lambda^g \bar{e} + \lambda^b \varepsilon \text{ and } V^b(x^b_\varepsilon) = V^b(x^g_\varepsilon)$$

The allocation $\hat{x}_\varepsilon$ is, for ε small enough, feasible. We are going to show that for ε arbitrarily small $V^g(x^g_\varepsilon) > V^g(x^p)$. The latter implies that $\hat{x}_\varepsilon$ is also incentive compatible and, thus, the claim.

Totally differentiating the two equations with respect to ε and evaluating the derivatives at $\varepsilon = 0$ we get:

$\lambda^g[\pi^g D_\varepsilon x^g_\varepsilon)]|_{\varepsilon=0} = \lambda^b$ and $-U'(\bar{e}) = U'(\bar{e})(\pi^b D_\varepsilon x^g_\varepsilon)$.

Hence:

$D_\varepsilon x^g_\varepsilon(H)|_{\varepsilon=0} = -\frac{\lambda^g \pi^g(L) + \lambda^b \pi^b(L)}{\lambda^g \pi^g(H) + \lambda^b \pi^b(H)} D_\varepsilon x^g_\varepsilon(L)|_{\varepsilon=0} > 0$ and

$$D_\varepsilon x^g_\varepsilon(L)|_{\varepsilon=0} = \frac{-(\pi^b(H)\lambda^b + \lambda^g)}{[\pi^b(L)\pi^g(H) - \pi^b(H)\pi^g(L)]\lambda^g} < 0$$

Then:

$$D_\varepsilon V^g(x^g_\varepsilon)|_{\varepsilon=0} = -U'(\bar{e})[\pi^g D_\varepsilon x^g_\varepsilon)|_{\varepsilon=0} =$$

$$-(U'(\bar{e}) D_\varepsilon x^g_\varepsilon(L)|_{\varepsilon=0})[\pi^g(H) \frac{\lambda^g \pi^g(L) + \lambda^b \pi^b(L)}{\lambda^g \pi^g(H) + \lambda^b \pi^b(H)} - \pi^g(L)] =$$

$$-(U'(\bar{e}) D_\varepsilon x^g_\varepsilon(L)|_{\varepsilon=0})[\frac{\lambda^b \pi^g(H)\pi^b(L) - \lambda^b \pi^b(H)\pi^g(L)}{\lambda^g \pi^g(H) + \lambda^b \pi^b(H)}] > 0. \qquad \blacksquare$$

Proof of Proposition 18.

First observe that, $\sum_i \lambda^i \pi^i(x^{*i} - e) = 0$ and, hence, the second requirement in the definition of equilibrium is satisfied. By Lemma 16, given t^{*g}, $(0, x^{*b})$ is the optimal solution of the individuals of *type* b to the programming problem 9.

Given $t^{*b} = 0$, (t^{*g}, x^{*g}) is a feasible solution to the type g' programming problem 9. Suppose, by contradiction, that $(t^g, \beta^g) \neq (t^{*g}, \delta_{x^{*g}})$ is the optimal solution. By Lemma 15 , $t^g \in [-T^g(\bar{t}), 0]$, thus, by lemma 16, $(0, x^b(t^g))$ is the optimal solution of the individuals of type b given t^g. However, by definition of the programming problem 9, the allocation $(\beta^g, \delta_{x^b(t^g)})$ is feasible. Thus, the pair $(0, t^g)$ and $(\beta^g, x^b(t^g))$ constitutes an equilibrium different than $(t^*, \hat{x}^*)$ and $(\beta^g, x^b(t^g))$ is, therefore, efficient. Furthermore, since $t^g \in [0, -T^g(\bar{t})]$, $(\beta^g, \delta_{x^b(t^g)}) \in \Theta^b \cup \{\hat{x}_P\}$, $V^g(\beta^g) < V^g(x^{*g})$. A contradiction. Thus, $(t^*, \hat{x}^*)$ is an equilibrium. It is also unique, since if there were a different equilibrium, (t', β'), by Lemma 15 and 16, $t'^b = 0$ and $t'^g \in [-T^g(\bar{t}), 0]$. Then, the same argument of before implies the thesis. ∎

Proof of Lemma 21.

First, substitute the constraints $z_i^i = (\lambda^i/\lambda^j) z_i^j$ in the programming problems and rewrite 14 as:

$$\max_{(\gamma_g^g, \gamma_b^b, \rho)} (p_g^g + (\lambda^b/\lambda^g) p_g^b) z_g^g - q^g y^g + (p_b^b + (\lambda^g/\lambda^b) p_b^g) z_b^b - q^b y^b$$

$$\textit{subject to}: \sum_{i \in I} \pi^i (z_i^i - y^i) \leq 0$$

Now just apply Farkas Lemma. ∎

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