Dynamic Stability and Reform of Political Institutions

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Abstract

When are political institutions stable? When do they tend toward reform? This paper examines a model of dynamic, endogenous institutional change. We introduce the class of *dynamic political games (DPGs)*, dynamic games in which the political aggregation rules used at date $t+1$ are chosen by the rules at date t, and the resulting institutional choices do not affect payoffs or technology directly. A political rule is *stable* if it selects itself for use in the following period. A *reform* occurs when an alternative rule is selected.

Absent an *essential* private sector, it is shown that institutional reform occurs if and only if the current political rule is *dynamically inconsistent*. Roughly, a rule is dynamically consistent if it is rationalized by a time separable, state invariant social welfare criterion. Simple majority rules are usually dynamically consistent. However, wealth-weighted voting rules are not. More generally, we identify sufficient conditions for stability and reform in terms of *recursive self-selection* constraints that treat the rules themselves as "players" who can strategically delegate future policy-making authority to different institutional types. These ideas are illustrated in a parametric model of dynamic public goods provision.

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1 Introduction

Reforms of political institutions are common throughout history. They come in many varieties. In some cases, the reforms correspond to changes in the voting franchise. Periodic expansions of voting rights occurred in governments of ancient Athens (700- 338BC), the Roman Republic (509BC-25AD), and most of Western Europe in the 19th and early 20th centuries, to name just a few examples.[1]

In other cases, modifications were made to the voting procedure itself. Medieval Venice (1032-1300), for instance, gradually lowered the required voting threshold from unanimity to a simple majority in its Citizens' Council. Nineteenth century Prussia, where votes were initially weighted by one's wealth, eventually equalized the weights across all citizens. The U.S. changed its rules under the 17th Amendment to require direct election of senators.

In still other instances, the scope of a government's authority changed. For example, England and France privatized common land during the 16th and 17th century enclosure movement thus reducing scope for rules governing the commons.[2] The U.S. government, on the other hand, increased its scope under the 16th Amendment by legalizing federal income tax in 1913.

Numerous historical examples suggest that institutional change is often gradual and incremental. Consider the progress of reforms in the Roman Republic. In 509BC, the Senate and Assembly were founded; in 494 BC the Patricians conceded the right of the plebs (the "commoners") to participate in the election of magistrates; in 336 BC one of the consulships became available for election by plebians; and in 287 BC Hortensian Law was introduced which gave resolutions in the plebian council the force of law. Gradual reform also characterized expansion of voting rights in 19th century England. In 1830, the voting franchise restricted to 2% of the population. In 1832, the First Reform Act extended the franchise to 3.5% of population. The Second Reform Act of 1867 extended it to some 7.7%. By 1884 it had been extended to 15% of population. Universal suffrage only passed in 1928 (see Finer (1997).

The objective in this paper is to understand how and why institutional reform occurs. Most basically, which environments tend toward institutional stability? Which environments admit change? What are the relevant forces that drive this change?

To address these questions, we posit a broad class of infinite horizon stochastic games in which political institutions are endogenous. The framework has three critical features. First, the process of change is recursive: the rules for choosing public decisions in period $t + 1$ are, themselves, objects of choice in period t. Second, the process is *instrumental*: institutions do not affect payoffs or technology directly. Third, institutional choice is wide and varied: the

[1] See Fine (1983), Finer (1997), and Fleck and Hanssen (2003).

[2] See MacFarlane (1978) and Dahlman (1980).

types of rules that can be chosen are not limited. We call games in this class *dynamic political games (DPGs).*

We study Markov Perfect equilibria of dynamic political games. These equilibria presume that neither government or individuals can commit to sequences of decisions or make non-credible threats. The restriction to Markov follows the general rule of thumb: in an economy-wide context, the participants are less apt to coordinate on non payoff-relevant history than they would if they were in a small group.

A companion paper (Lagunoff, (2005b)) establishes existence of Markov Perfect equilibria for a class dynamic political games. The present paper examines properties of these equilibria. In particular, we ask: when do institutional reforms occur, or alternatively, when are rules stable over time?

To make this question concrete, the model derives an endogenous law of motion for political institutions. Notationally, this law of motion is given by:

$$\mu(\omega_t, \theta_t) = \theta_{t+1}.$$

In this expression, ω_t is the "economic" state summarizing tangible features of the economy such as capital stocks and/or wealth distribution. By contrast, θ_t summarizes the *political rule* consisting of the formal procedures for governance, e.g., election laws, voting eligibility, fiscal constitutions, and so on. According to this expression, the date t economic state and political rule jointly determine the political rule to be used in date $t+1$. Fixing ω_t for the moment, a political rule θ_t *admits reform* whenever

$$\mu(\omega_t, \theta_t) \neq \theta_t$$

In other words, next period's political rule is chosen to be different than the present one. A rule is *stable* whenever $\mu(\omega_t, \theta_t) = \theta_t$, i.e., no reform occurs.

We develop the dynamic political game model in two stages. First, we examine a "pure public sector" model in which all decisions are made in the public/policy sector. Private sector decisions such as individual savings and investment choices are added later. The main result (Theorem 1) in this public sector model asserts that institutional reform occurs *if and only if* the political rule is *dynamically inconsistent*. Roughly, dynamically consistent (as distinct from time consistent) rules are those that are rationalized by a time separable social welfare function such that socially optimal choice of θ_{t+1} at the beginning of date t is the same as that which would be optimal at the beginning of $t+1$. The Condorcet (Majority Voting) Rule, for instance, is dynamically consistent whenever it is rationalized by the preferences of the same median voter over time. By contrast, wealth-weighted voting rules are usually dynamically *in*consistent since changes in the wealth distribution change the identity of a pivotal decision maker.

The intuition is straightforward. A dynamically consistent rule can be identified with a dynamically consistent objective function, hence may be interpreted as a dynamically consistent player (the "policy maker") in the game. Hence, dynamically optimal policy choices do no worse than full policy commitment from this player's point of view. In other words, there is no time consistency problem. Conversely, stable rules survive time consistent revision opportunities. Hence, such rules are dynamically consistent.

The logic is further illustrated in a parametric example of dynamic public goods provision. We show how show how two different rules can have very different stability properties. In the case of simple majority voting (the "Condorcet Rule"), reform need not occur since, in this example, the identity of the pivotal decision maker does not change over time. In other words, majority voting is dynamically consistent. However, in the case of a wealth-weighted voting rule (the "Wealth-is-Power Rule"), institutional reform does occur because changes in the wealth distribution changes the pivotal decision maker's identity.

In the full model with private, individual decisions, the sufficiency result is extended when the private sector is *inessential*. Formally, the private sector is inessential if individuals' dynamic payoffs satisfy a separability condition between one's own private decision and the policy choice. Theorem 2 asserts that if the political rule is dynamically consistent and if the private sector is inessential, then that rule is stable in every Markov equilibrium. That is, institutional reform *never* occurs if these two conditions are satisfied.

Although an essential private sector is not sufficient for reform, per se, it is easy to see how the policy maker's time consistency problem is mitigated by delegating authority to a different political institution when private sector decisions impact the policy sector. For instance, an influential argument of Acemoglu and Robinson (2000, 2001) asserts that the elites of 19th century Europe chose to extend the voting franchise because policy concessions alone could not "buy off" the threat of a peasant uprising. That is, franchise extension occurred because the threat (from the private sector) was *essential*. Jack and Lagunoff (2005a) display Acemoglu-Robinson logic as a special case in a recursive model in which gradual extension of the voting franchise is possible. The present result therefore helps to make sense of their logic in a larger context; it identifies some necessary features of an environment in order for institutional change to occur.

We develop this idea of "reifying" the rules further. One can interpret play of the dynamic political game in each period as a distinct normal form game. A rule is *recursively self-selected (RSS)* if, in this auxiliary game, the current rule never chooses to "delegate" decision authority to another rule for the subsequent period only. A rule is *recursively self-denied (RSD)* if it delegates authority to some other rule. Recursive self-selection and self-denial are institutional incentive constraints that treat each distinct rule as an institutional "player" in the induced normal form game with private individuals.

Recursive self-selection bears some relation to "self-selected rules" in the static social choice models of Koray (2000) and Barbera and Jackson (2000), and also to the infinite regress model of choice of rules by Lagunoff (1992). These all posit social orderings on the rules themselves based on the outcomes that these rules prescribe. Rules that "select themselves" do so on basis of selecting the same outcome as original rule. The present model has two differences. First, institutional choice occurs in real time — next period's rule is chosen by the present one. This makes possible an analysis of explicit dynamics of change. Second, the present model is more concrete; the trade-offs are explicitly derived from the interaction of economic fundamentals in the public and private sectors.

We show that recursively self-selected (RSS) rules are stable (Theorem 3). The converse, however, does not hold: there may be stable rules that are not RSS. In these instances, Rule A may choose not to delegate to Rule B, even though Rule A is recursively self-denied (RSD) by Rule B. Why? Because Rule B, with its decision authority, would subsequently delegate to Rule C which is unattractive from A's point of view. Given these intransitivities, Rule A remains stable. Consequently, we show in Theorem 3 that a rule admits institutional reform if either it is self-denied by another RSS rule, or, alternatively, if it is self-denied by every other rule.

Finally the parametric model with an added private sector is revisited. The public good is produced from both public taxation and private labor inputs. The RSS and RSD concepts are used to show that there exists an equilibrium that converges globally to a steady state political rule. This rule is uniquely recursively self selected. We show that every other political rule admits reform. In these cases, a time consistency problem arises when the current rule produces tax rates ill-adapted to private contributions. To alleviate this problem a reform occurs when the current rule commits future taxing authority to another rule.

Ideally, we eventually hope to obtain a full characterization of those environments that admit reform and those that do not. Notions of recursive self-selection and denial seem more representative of general ideas than their application so far suggests.

There are relatively few papers that model dynamic, endogenous political institutions. These include Roberts (1998, 1999), Justman and Gradstein (1999), Acemoglu and Robinson (2000, 2001), Lagunoff (2001), Barbera, Maschler, and Shalev (2001), Messner and Polborn (2004), Gradstein (2003), Greif and Laitin (2004), and Jack and Lagunoff (2005a,b). We review these in more detail in Section 5. For now, two main differences between the present paper and the literature are worth emphasizing. First, most dynamic models of institutional choice are not typically both recursive and instrumental.[3] Arguably both properties critical for modeling endogenous institutions. Recursivity "builds-in" an ongoing, incremental, and potentially reversible, process of change. Instrumental choice makes the rationale for reform nontrivial. Second, whereas the literature focuses on one particular type of reform (e.g. voting rights), DPGs admit a broader array of institutional changes. These include changes in the

[3]In prior work, an exception is Jack and Lagunoff (2005a,b) which is a special case of the present framework.

voting rule (majority vs supermajority rules), changes in voting rights (e.g. larger vs smaller voting franchise), and changes in the scope of the public sector (e.g., expansions vs contractions of regulatory authority).

The paper is organized as follows. In Section 2, the pure public sector model without private decisions is introduced. Section 3 illustrates the main trade-offs involved in keeping or reforming a political rule in a simple parametric model of public goods. Theorem 1 describes the trade off in terms of dynamically consistent decision making. Section 4 adds the private sector and defines a notion of equilibrium which combines standard Markov Perfection in private decisions with a political fixed point requirement for public decisions. There, the RSS and RSD concepts are introduced. Section 4 later revisits the parametric model. Section 5 summarizes the results, and examines the related literature in more detail. Section 6 is an Appendix with the proofs of the main results.

2 The Pure Public Sector Model

As a benchmark, we first examine issues of institutional reform in a pure public sector model. In this benchmark model, there are no individual private actions; all relevant decisions are carried out in the public sector. Though not realistic, this model isolates some of the relevant features of a political institution that determine its stability.

This model formally consists of a society $I = \{1, \dots, n\}$ of infinitely lived individuals, with n odd. At each date $t = 0, 1, 2, \dots$, this society must collectively choose policy p_t from a feasible set P. The stage payoffs of each individual depend on this policy and on a state variable ω_t drawn from a set, Ω. Both P and Ω are assumed to be Borel measurable subsets of Euclidian spaces, with P compact. Write $u_i(\omega_t, p_t)$ to denote the stage payoff of individual $i \in I$ at date t given state ω_t and policy p_t. The u_i, $i \in I$ are all assumed to be jointly continuous in the standard topology.

For now, omit (notationally) private sector behavior such as individuals' savings, investment, or labor decisions. Private sector behavior will be introduced in the next Section. Each individual's dynamic payoff is given by,

$$E\left[\sum_{t=0}^{\infty} \delta^t (1-\delta)\ u_i(\omega_t,\ \ p_t\ )\Big|\ \omega_0\right] \tag{1}$$

where δ is the common discount factor, ω_0 is the initial state, and the expectation is taken with respect to a stochastic transition technology that determines how current states and policies pin down distributions over future states. Formally, let $q(B|\ \omega_t,\ p_t)$ denote the probability that ω_{t+1} belongs to the (Borel measurable) subset $B \subseteq \Omega$, given the current state ω_t and policy p_t. The transition q is assumed to satisfy the standard measurability assumptions.

The payoffs and transition technology are assumed to yield a finite expectation in (1) for any initial state and any feasible sequence of policy choices.

This specification is sufficiently broad to fit a large variety of public decision problems. For example, ω_t could describe the distribution of assets $(\omega_{1t}, \ldots, \omega_{nt})$ across individuals and p_t a tax on those assets. Tax revenue is then used to augment future stocks of individual assets. Alternatively, ω_t might describe the stock of a common pool resource and p_t the publicly decided extraction rate. Since ω_t is directly relevant to payoffs and technology, we refer to it as the "economic state."

In each case, the public decision must be determined somehow, and a *political rule* describes the process that does it. Informally, a political rule at date t is a mapping from individuals' policy preferences at that date to a set of public decisions. Let θ_t denote a parameter that summarizes the political rule at date t. For example, if θ_t summarizes a simple majority voting rule, then policy p_t is a Condorcet Winner — an outcome that dominates any alternative in a pairwise majority vote. By contrast, if θ_t represents a dictatorship by, say, Citizen $i = 1$, then the only feasible decisions are those that maximize the payoff of the Dictator, $i = 1$. Let Θ, a compact subset of a Euclidian space, denote set of "admissible" political rules.

The formal definition of social choice under any political rule $\theta \in \Theta$ will be defined momentarily. The critical feature at this point, however, is that the prevailing political rule at date t determines both the policy p_t *and* the subsequent political rule, θ_{t+1}. Hence, the composite state at date t is denoted by $s_t = (\omega_t, \theta_t)$, consisting of the economic state and the political rule. The set of states is $S = \Omega \times \Theta$. A *public strategy* is a pair (ψ, μ) of (Markov) decision functions described as follows. Given $s_t = (\omega_t, \theta_t)$, ψ determines the policy $p_t = \psi(s_t)$ while μ determines next period's political rule $\theta_{t+1} = \mu(s_t)$. The institutional strategy μ is of particular interest since it describes a recursive process of institutional change. A public strategy (ψ, μ) produces a *public sector decision*, $(\psi(s_t), \mu(s_t)) = (p_t, \theta_{t+1})$ in each state and in each period.[4] Using a public strategy (ψ, μ), sequences of economic states, $\{\omega_t\}$ and political rules $\{\theta_t\}$ are built up recursively in the usual way from the transition q. An individual's recursive payoff under a public strategy (ψ, μ) in state $s_t = (\omega_t, \theta_t)$ is given by

$$V_i(s_t;\ \psi, \mu) \ = \ (1 - \delta) u_i(\omega_t, \psi(s_t)\ ) \ + \ \delta \int V_i(\omega_{t+1}, \mu(s_t); \psi, \mu) dq(\omega_{t+1} |\ \omega_t, \psi(s_t)\ ) \qquad (2)$$

Since Citizen i may have some say (e.g., voting) in determining a public sector decisions, his induced preferences over such decisions matter. These preferences are expressed by a *recursive payoff function*

$$v_i(p_t, \theta_{t+1} \mid s_t;\ \psi, \mu) \ \equiv \ (1 - \delta) u_i(\omega_t, p_t) \ + \ \delta \int V_i(\omega_{t+1}, \theta_{t+1};\ \psi, \mu) dq(\omega_{t+1} |\ \omega_t, p_t) \qquad (3)$$

[4] Strictly speaking ψ and μ need not be strategic choices. The could be determined by some mechanical rule as we will see. For most of the analysis, however, these will be explicit strategic choices of some "Player" and so I just label them as "strategies" from the start.

Each individual therefore has recursive preference $v_i(\cdot|\ s_t;\psi)$ in (3) defined over (p_t,θ_{t+1}) pairs of public decisions. By construction, $v_i(\psi(s_t),\mu(s_t)|\ s_t;\ \psi,\mu) = V_i(s_t;\ \psi,\mu)$. When the context is clear, we drop the conditioning variables, s_t, ψ, and μ from the notation and express the recursive payoff function simply as $v_i(p_t,\theta_{t+1})$. Simplifying even further, a *profile* of recursive payoff functions is expressed as $v=(v_i)_{i=1}^n$. Despite this notational simplification, it is worth noting that v_i is not an exogenous preference like a stage payoff but rather it is an endogenous object defined by a public strategy (ψ,μ) applied to all future periods.

The notation is now in place for a political rule to be defined formally. In much of what follows, we adopt the usual convention of using primes, e.g., θ' to denote subsequent period's variables, θ_{t+1}, with double primes, θ'' for θ_{t+2}, and so on.

Let $\mathcal{V}$ denote the set of recursive preference profiles obtained by varying s, ψ and μ. The collection of all political rules can now be described formally by a correspondence

$$C:\mathcal{V}\times S\rightarrow\rightarrow P\times\Theta$$

that associates to each preference profile $v=(v_i)$ and to each state s, a set $C(v,s)$ of public decisions. If $(p,\theta')\in C(v,s)$, then (p,θ') is a feasible public sector decision under C. The pair (C,Θ) intuitively describes the institutional constraints. We refer to a pair (C,Θ) as a *class of political rules* and the pair (ψ,μ) as *politically feasible* or simply *feasible* if it satisfies

$$(\psi(s),\mu(s))\in C(\ v(\cdot|\ s;\ \psi,\mu),\ s\ ),\quad \forall\ s=(\omega,\theta) \tag{4}$$

A companion paper of Lagunoff (2005b) focuses on finding solutions to the "political fixed point problem" in (4). If, indeed, satisfactory solutions may be found, the model endogenously determines the institutional rule.

Example 1. "Democracy" vs "Dictatorship" - revisited. *The bipolar case of "democracy" and "dictatorship" is described formally by:* $\Theta=\{\theta^C,\theta^D\}$ *where*

$$C(v,\ s)=\begin{cases}\textit{set of Condorcet Winners} & \textit{if}\ \ \theta=\theta^C\\[2ex] \textit{arg}\max\limits_{p,\theta'} v_1(p,\theta') & \textit{if}\ \ \theta=\theta^D\end{cases}$$

where, the "set of Condorcet Winners" is given by the set of pairs (p,θ') *such that for all* $(\hat{p},\hat{\theta})$*,*

$$|\{i\in I:\ v_i(\hat{p},\hat{\theta})>v_i(p,\theta')\}|\ \le\ \frac{n}{2}.$$

At this point, the model deserves further comment. By summarizing the political process in reduced form by (C,Θ), we adopt a "social choice" approach rather than a strategic approach to politics. Alternatively, one could make a case that political rules should be modeled as a noncooperative game. For instance, the political rule could define a noncooperative voting game whereby today's votes determine which voting game is to be used tomorrow. Consequently, endogenous institutions could arise as Nash equilibrium outcomes of a standard stochastic game. The issue becomes: which game? While there are agreed upon canonical social choice representations of voting, there are relatively few such games. There are many reasons for this. For one thing, strategic models of politics are notoriously sensitive to minor details.[5] For another, dynamic models of detailed politics are hard to model, which may explain why the literature is sparse. Ultimately, the trade-off is one of "explicitness" versus "representativeness" and tractability. The approach taken here favors the latter. The breadth and variety of institutions encompassed by this description is demonstrated in the examples given below.[6]

Example 2. Voting over the Voting Rule. *Here, the current super-majority voting rule determines which supermajority rule is used in the future.*[7] *In each state $s = (\omega, \theta)$, the political state θ identifies the fraction, $\theta \geq 1/2$ of individuals required to pass a public decision. Formally, let $(p, \theta') \in C(v, s)$ if for all $(\hat{p}, \hat{\theta}')$,*

$$|\{i \in I : \; v_i(\hat{p}, \hat{\theta}') > v_i(p, \theta')\}| \; \leq \theta n$$

Supermajority rules have been widely used, particularly to effect large or constitutional changes in policy. This example can be modified further so that the supermajority required for changing the policy is distinct from the supermajority for changing the current political rule (as in the U.S. constitution): let $\theta = (\theta^a, \theta^b)$, whereby θ^a is the supermajority required to determine policy, while θ^b is the supermajority required to determine the subsequent rule. Then $(p, \theta') \in C(v, s)$ if, for all $(\hat{p}, \hat{\theta}')$, EITHER

$$|\{i \in I : \; v_i(\hat{p}, \hat{\theta}') > v_i(p, \hat{\theta}')\}| \; \leq \theta^a n \;\; \text{OR} \;\; |\{i \in I : \; v_i(\hat{p}, \hat{\theta}') > v_i(\hat{p}, \theta')\}| \; \leq \theta^b n$$

Example 3. Voting over the Voting Franchise. *Consider the case of an endogenous voting franchise. The political state θ identifies the subset of individuals who currently possess the right to vote (the voting franchise). The chosen public decision is the one that is majority preferred within this restricted group. Each restricted voting franchise today uses a majority vote to determine what group of individuals have the right to vote tomorrow. Hence, the*

[5] Even in the more canonical endogenous candidate models of Besley and Coate (1997) and Osborne and Slivinsky (1996) there are reasonable and numerous, alternative specifications of how candidates could emerge.

[6] The Reader who prefers to skip the details of the Examples 2-4 can proceed, without losing the main ideas, to the next Section.

[7] Messner and Polborn (2004) examine a related model in an OLG rather than an infinitely-lived agent setting.

current voting franchise decides on a new voting franchise in the following period. Formally, $\Theta \supseteq 2^I$*, and let* $(p, \theta') \in C(v, s)$ *if for all* $(\hat{p}, \hat{\theta}')$*,*

$$|\{i \in \theta :\ v_i(\hat{p}, \hat{\theta}') > v_i(p, \theta')\}| \ \leq\ \frac{1}{2}|\theta|$$

The Example of endogenous voting franchise is an important special case. It the expansion of voting rights which occurred throughout Europe in the 19th century whereby new groups of voters were admitted. Typically, these expansions took the form of relaxed wealth or property qualifications for voting. Example 3 also conforms to the endogenous franchise models of Jack and Lagunoff (2005a,b) and generalizes the seminal models of endogenous enfranchisement by Justman and Gradstein (1999) and Acemoglu and Robinson (2000, 2001). In the latter two cases, the feasible institutions were $\Theta = \{M, I\},\ M \subset I$. The interpretation is that an elite group M chooses either to maintain the status quo, or to make an all-or-nothing extension of the vote to the entire population, I. In the Acemglu-Robinson (2000) model, the extension represents a commitment to lower taxes on the peasantry. The commitment is credible since the identity of the median voter is permanently altered. In the Justman-Gradstein OLG model, a franchise extension is assumed to lower production costs. The key differences between those models and this one is that here the process of change is gradual (i.e., partial extensions are possible).

Example 4. Voting over the Scope of Government. *Finally, consider an endogenous choice of public sector scope. Each period, society draws a line between private and public decisions. The political rule therefore identifies the domain of public decisions. Privatization of public land, for example, reduce the public sector while broadening surveillance capabilities of police increases it. Formally,* $\theta \subset P$ *so that* θ *denotes the set of feasible policies. Let* $(p, \theta') \in C(v, s)$ *if* $p \in \theta$ *and for all* $(\hat{p}, \hat{\theta}')$ *satisfying* $\hat{p} \in \theta$*,*

$$|\{i \in I :\ v_i(\hat{p}, \hat{\theta}') > v_i(p, \theta')\}| \ \leq \frac{1}{2}n$$

3 Stability versus Reform

Given a feasible public strategy (ψ, μ), a political rule θ *admits institutional reform in* μ if there exists a (set of positive measure) ω such that $\mu(\omega, \theta) \neq \theta$. Alternatively, *a rule* θ *is stable in* μ if it does not admit reform, i.e, if $\mu(\omega, \theta) = \theta$ for (almost) every ω.

When do political rules admit reform? When are they stable? This Section demonstrates that the answer depends on certain dynamic consistency properties of the rules themselves. We first develop the ideas in a simple parametric example.

3.1 A Parametric Example of Public Good Provision

Consider the following parametric model of durable public good provision. The economic state is $\omega_t = (G_t, y_t)$, where $y_t = (y_{1t}, \ldots, y_{nt})$ with y_{it} the wealth of Citizen i at date t (e.g, a parcel of land). Wealth y_{it} produces a one-to-one return each period of y_{it}. G_t is the stock of a durable public good at t. The public good in period $t+1$ depends on tax revenues in period t. p_t is a tax on the yield from one's wealth/land. There is no depreciation. The transition law for the public good is deterministic and is given by $G_{t+1} = G_t + (p_t \sum_j y_{jt})^\gamma, \quad 0 < \gamma < 1$. Finally, each citizen's stage payoff is a linear function of his after-tax returns and his use of the public good: $u_i = (1 - p_t)y_{it} + G_t$. (A concrete example is investment in public literacy whereby all individuals derive a benefit from the literacy rate G_t.)

Case A. Fixed Wealth Distribution. Suppose first that the wealth distribution $y_t = (y_{1t}, \ldots, y_{nt})$ is fixed for all time, so that $y_t = y$ for all t. Order the individuals' wealth holdings from poorest to richest:

$$0 = y_1 \leq y_2 \leq \cdots \leq y_n$$

Denote aggregate wealth by $Y = \sum_j y_j$. In this case, it is straightforward to show that the most preferred policy of a citizen is invariant to time and states: $p_i^* = A y_i^{-1/(1-\gamma)}$ where $A = (\frac{\gamma\delta}{1-\delta} Y^\gamma)^{1/(1-\gamma)}$ is a constant. Evidently, wealthier citizens prefer lower taxes.

Now add to this model an institutional choice over two political rules: $\Theta = \{\theta^C, \theta^W\}$, where θ^C describes the Condorcet (Majority) Rule (as defined before) while θ^W connotes the "Wealth-as-Power Rule" defined by: $(p, \theta') \in C(v, \omega, \theta^W)$ if for all $(\hat{p}, \hat{\theta}')$ with $M \equiv \{i \in I : v_i(\hat{p}, \hat{\theta}') > v_i(p, \theta')\}$, then $\sum_{i \in M} y_i < \sum_{i \notin M} y_i$. In other words, a public decision is feasible under the Wealth-as-Power Rule if it is undominated by any other rule where each individual is allocated a number of votes proportional to his wealth.[8] Generally speaking, wealth-weighted voting rules such as θ^W have been widely used historically,[9] Their formal properties are generally difficult to characterize.[10]

The differences between θ^C and θ^W are as follows. Let c_t denote the identity of the Condorcet-pivotal voter at date t. Voter c_t corresponds to the citizen with the median wealth endowment, i.e., $c_t = c = n/2 + 1$. The identity of this individual never changes over time.

[8] This is also referred to as the "dollar voting" rule.

[9] An interesting case is 19th century Prussia. In 1849, voting rights were extended to most citizens in Prussia, but not in an even-handed way. Right were accorded proportionately to the percentage of taxes paid. The electorate was divided into thirds, each third given equal weight in the voting. The wealthiest individuals who accounted for the first third of taxes paid accounted for 3.5% of the population. The next wealthiest group that accounted for the middle third accounted for 10-12% of the population. The remainder accounted for the remaining third.

[10] In a purely redistributive environment, Jordan (2005) shows that outcomes of the Wealth-as-Power rule correspond to the core of a cooperative game in which a blocking coalition's feasible allocations are determined endogenously by the status quo allocation.

Consequently, the resulting policy is $\psi(\omega, \theta^C) = Ay_c^{-1/(1-\gamma)}$. Under the Wealth-as-Power rule, the pivotal decision maker is also time invariant. A policy is feasible under θ^W if it is the ideal policy of a voter, denoted by w, with the "median wealth" in an economy with y_i identical voters of type $i \in I$.[11] The preferred policy for that individual is $p_w^* = Ay_w^{-1/(1-\gamma)}$. The resulting policy then is $\psi(\omega, \theta^W) = Ay_w^{-1/(1-\gamma)}$. In this case, it is easy to show that both θ^C and θ^W are stable. That is:

Proposition 1 *For all* ω, $\mu(\omega, \theta^C) = \theta^C$ *and* $\mu(\omega, \theta^W) = \theta^W$.

To see why Proposition 1 holds, observe that under a stable rule, the decision problem reduces to a single agent dynamic programming problem. By the well known Principle of Unimprovability (e.g., Howard (1960)), that agent's most desired policy each period is also dynamically optimal. Consequently, the pivotal citizen never has an incentive to delegate decision authority to a different individual whose preference over policy is different than his own.

Case B. Differential Wealth Accumulation. Suppose that at $t = 0$ the initial wealth distribution is $y_{i0} = y_i$ where y_i is the same as before. Recall that $y_1 \leq \cdots \leq y_n$. Rather than remaining fixed, suppose that for $t \geq 1$,

$$y_{i\,t+1} = y_{it} + \beta_i$$

with $\beta > 0$, $\sum_j \beta_j = 1$, and $\beta_1 < \cdots < \beta_n$. In this example, wealthier citizens accumulate wealth faster than poorer ones.

One can check that each citizen's most preferred policy is again of the form: $p_{it}^* = A_t y_{it}^{-1/(1-\gamma)}$ where $A_t = (\frac{\gamma\delta}{1-\delta} Y_t^{\gamma})^{1/(1-\gamma)}$ which now varies exogenously over time.

Under the differential accumulation rates, however, the two rules operate quite differently. Under the Condorcet Rule, the ordering of the wealth endowments never changes. Consequently, the identity of the Condorcet-pivotal voter c never changes. This voter's wealth at date t is given by y_{ct}. Hence, while the policy itself changes over time according to $\psi(\omega_t, \theta^C) = A_t y_{ct}^{-1/(1-\gamma)}$, the Condorcet-pivotal citizen has no incentive to delegate decision authority to another. Therefore, the Condorcet Rule remains stable.

By contrast, under the Wealth-as-Power rule, the identity of the pivotal voter may change over time. This gives a potential rationale for institutional reform: a current decision maker whose wealth is close to y_{ct} may prefer the Condorcet-pivotal voter as a "lesser of two evils." To illustrate this starkly, suppose $\beta_n > 1/2$. This inequality implies

$$\frac{y_{nt}}{Y_t} = \frac{y_n + t\beta_n}{Y + t\sum_j \beta_j} \rightarrow \beta_n > 1/2$$

[11] More precisely, this median may be found as follows: Citizen w satisfies $y_w = \max\{y_j : \sum_{k=j}^n y_k \geq Y/2\}$.

as $t \to \infty$, and the richest citizen eventually accumulates over half the aggregate wealth. Letting $\{w_t\}$ denote the sequence of pivotal decision makers under θ^W, the policy strategy is given by $\psi(\omega_t, \theta^W) = A_t y_{w_t t}^{-1/(1-\gamma)}$. It is clear that at some date t^*,

$$w_0 \leq w_2 \leq \cdots \leq w_{t^*} = n$$

Now consider an initial wealth distribution sufficiently egalitarian such that $\sum_{i=1}^{(n+1)/2} y_i > Y/2$. This implies that $w_0 = c$. That is, the initial pivotal decision maker under θ^W is precisely the Condorcet-pivotal (median) voter.

In this case, the institutional choice is made easy. Starting from θ^W, the Citizen $w_0 = c$ will choose θ^C. Clearly, by remaining in θ^W, decision making authority is eventually transfered from the median citizen to the richest one. By choosing θ^C, however, this median citizen retains power. In the more general case where $w_0 \neq c$, this Citizen w_0 may choose θ^C if his preferences more closely resemble c's over time than n's. To summarize:

Proposition 2 *Suppose an initial state $\omega_0 = (y, G_0)$ such that $\sum_{i=1}^{(n+1)/2} y_i > Y/2$. Then there exists some finite time τ such that $\mu(\omega_\tau, \theta^W) = \theta^C$.*

Since the preference of type c is strict, the Condorcet Rule is preferred for all ω in a neighborhood of ω_τ. Hence, θ^W admits a reform toward θ^C in μ.

The instability of the Wealth-is-Power Rule arises from a natural dynamic inconsistency. Namely, θ^W varies in the economic state ω_t. This leads to change in the "identity" of the policy maker. If the shift is substantial enough, the current decision maker implied by θ^W may select an alternative political rule for self protection. Because the Condorcet Rule retains the same pivotal decision maker over time, no such self-protection is necessary.

3.2 Dynamically Consistent Rules

Using the intuition of the Example, we identify a necessary and sufficient condition for institutional stability.

First, we make the identification of a rule with a policy authority explicit. Formally, (C, Θ) is *(partially) rationalized by* a measurable social welfare function $F : \mathbb{R}^n \times S \to \mathbb{R}$ if

$$C(v, s) = (\supseteq)\arg\max_{p,\theta'} F(v(p, \theta'), s)$$

A dictatorial rule is clearly rationalized by the preference of the dictator. When political rules are voting rules, as in Examples 2-3, then two well known conditions, either single peaked preferences or order restriction, imply that C is rationalized by the preferences of a Median Voter.[12] In what follows, we assume the class (C, Θ) is rationalized by a continuous social welfare function F.

Definition 1 A rule θ is said to be *dynamically consistent* if for every public strategy (ψ, μ) and every ω,

$$\begin{aligned} & \arg\max_{\theta'} \; F\left((1-\delta)u(\omega, \psi(\omega,\theta)\,) \;+\; \delta \int V(\omega', \theta';\; \psi, \mu) dq(\omega' |\, \omega, \psi(\omega,\theta)),\; \omega, \theta\right) \\ = \; & \arg\max_{\theta'} \delta \int F\left(V(\omega', \theta';\; \psi, \mu),\; \omega', \theta\right) dq(\omega' |\, \omega, \psi(\omega, \theta)) \end{aligned} \tag{5}$$

whenever the two sets of maximizers are nonempty.

Note that the left-hand side of (5) in nonempty if (ψ, μ) is politically feasible. Basically, a rule θ is dynamically consistent if in each state ω the political rules that maximize $F(\cdot, \omega, \theta)$ are those that maximize the expected continuation under F. Roughly, dynamic consistency presumes that future decision makers' points of view coincide with that of the present decision maker if the latter were to called upon to make the decision in the future. The rule therefore acts as if it were a dynamically consistent player whenever the individual participants are. A useful special case is that of a standard social planner with geometrically discounted payoffs:

$$\begin{aligned} & F\left((1-\delta)u(\omega, p) \;+\; \delta \int V(\omega', \theta';\; \psi, \mu) dq(\omega' |\, \omega, p),\; s\right) \\ = \; & (1-\delta) F\left(u(\omega, p),\; \theta\right) \;+\; \delta \int_{\omega'} F\left(V(\omega', \theta';\; \psi, \mu),\; \theta\right) dq(\omega' |\, \omega, p) \end{aligned} \tag{6}$$

The planner's criterion F is linear and does not vary in ω. We refer to Equation (6) as the *strong form* of dynamic consistency whereby F is linear and does not vary over economic states.

In much of the literature on dynamic policy choice, policies are chosen assuming the strong form of dynamic consistency. Examples include classic social planners (e.g., models of monetary authorities) and voting rules such as the Condorcet (Majority) Rule in certain environments. The present paper emphasizes that consistencies or inconsistencies can arise naturally from the structure of political aggregation. For example, the Wealth-is-Power Rule fails consistency due to the Rule's sensitivity to changes in the wealth distribution. Alternatively, a rule could fail due a present or future bias in the planner's criterion.[13] When these problems can be avoided, the following holds.

[12] The first conditions originated with Arrow (1951) and Black (1958). The second is due Rothstein (1990), although similar results can be found in applications of single crossing properties by Roberts (1977) and by Gans and Smart (1996).

[13] An example is a rule rationalized by a "Hyperbolic" social welfare function. See Krusell, Kuruscu, and Smith (2002) and Lagunoff (2005a)) for models of hyperbolic bias in government.

Theorem 1 *Suppose in the public sector model that (C,Θ) is single valued and rationalized by F. Let (ψ,μ) be any politically feasible pair. Then a political rule θ is stable in μ if and only if it is dynamically consistent.*

The proof is in the Appendix. The intuition is similar to that in parametric example. When rules are dynamically consistent, the implied decision maker remains the same over time. Hence, there is no reason to delegate authority to another (implied) decision maker. The more surprising part is the converse: stable rules must be dynamically consistent.

4 The Role of Private Sector Decisions

In the pure public sector model, there are no private decisions of individuals offset the pivotal decision maker's choices. Such decisions are considered here.

Let e_{it} denote i's private decision at date t, chosen from a compact feasible set E. A profile of private decisions is $e_t = (e_{1t},\ldots,e_{nt})$. These decisions may capture any number of activities, including labor effort, savings, or investment activities. They may also include "non-economic" activities such as religious worship or one's participation in a social revolt. The distinction between e_{it} and p_t is that while the latter is collectively determined, the former is chosen individually.

To express the dependence of payoffs and technology in private sector decisions, let $u_i(\omega_t,e_t,p_t)$ denote i's stage payoff and let $q(B|\ \omega_t,\ e_t,\ p_t)$ denote the probability that ω_{t+1} belongs to $B\subseteq\Omega$, both given the economic state ω_t, the private decision profile e_t, and the policy p_t. A *Dynamic Political Game (DPG)* is therefore summarized by the collection

$$G \equiv \langle\ \overbrace{(u_i)_{i\in I}, q, E, P, \Omega}^{\text{economic structure}}\ ,\ \overbrace{\Theta, C}^{\text{political structure}}\ ,\ \overbrace{s_0}^{\text{initial state}}\ \rangle$$

The class of dynamic political games (DPGs) constitutes a broad set of problems in which institutional changes occur endogenously and incrementally. The "economic structure," i.e., $(u_i)_{i\in I}, q, E, P$ and Ω is found in any standard stochastic game. The addition is the "political structure" given by Θ and C. The class of political rules is defined by the correspondence C as before. For tractability we restrict attention in the rest of the paper to dynamic political games G that satisfy: Θ, P, and E are all compact, convex subsets of Euclidian spaces, Ω is a convex set, and u_i and q are continuous in (ω,e,p).[14]

The individual strategies in DPGs are history-contingent choices. A history h^t at date t includes all past data up to that point of time, as well as the current state. In other words,

$$h^t = (\ (\omega_0,e_0,p_0),\ldots,(\omega_{t-1},e_{t-1},p_{t-1}),\omega_t)$$

[14]More precisely, for any Borel set $B\subset\Omega$, $q(B|\cdot)$ is continuous in (ω,e,p).

Given a public strategy (ψ, μ), a Subgame Perfect equilibrium in private strategies of the DPG would specify strategy profiles that map from histories to private actions such that each individual's strategy sequentially rational for him after every possible history. To make the theory tractable, we restrict attention to Markov Perfect equilibria. Markov Perfect equilibria in private strategies are Subgame perfect equilibria in which individuals' strategies are Markov - actions are conditioned only on the current state, s_t. By the well known One-shot Deviation Principle, it is clear that Markov strategies are best responses (among all strategies) to other Markov strategies.

A *private (Markov) strategy* for individual i is a function $\sigma_i : S \to E_i$ that prescribes action $e_{it} = \sigma_i(s_t)$ in state $s_t = (\omega_t, \theta_t)$. Let $\sigma = (\sigma_1, \ldots, \sigma_n)$. The strategy profile is therefore summarized by the triple

$$\pi \equiv (\underbrace{\sigma,}_{\text{private sector profile}} \underbrace{\psi,}_{\text{policy strategy}} \underbrace{\mu}_{\text{institutional strategy}})$$

A citizen's dynamic payoff, given by $V_i(s_t;\ \pi)$, generalizes the payoff in (2) in the obvious way. An individual's public payoff function also generalizes in the obvious way:

$$v_i(p_t, \theta_{t+1} |\ s_t; \pi) \equiv (1-\delta)u_i(\omega_t, \sigma(s_t), p_t) + \delta \int V_i(\omega_{t+1}, \theta_{t+1}; \pi) dq(\omega_{t+1} |\ \omega_t, \sigma(s_t), p_t\ ) \quad (7)$$

Definition 2 An *Equilibrium* of a dynamic political game, G, is a profile $\pi = (\sigma, \psi, \mu)$ of Markov strategies such that for all states $s = (\omega, \theta)$,

(a) *Private decision rationality*: For each citizen i, and each private strategy $\hat{\sigma}_i$,

$$V_i(s;\ \pi) \geq V_i(s;\ \hat{\sigma}_i, \sigma_{-i}, \psi, \mu\ ) \quad (8)$$

(b) *Political feasibility*: The public decision pair $(\psi(s), \mu(s)\ )$ satisfies

$$(\psi(s), \mu(s)\ ) \in C\left(v(\cdot |\ s; \pi),\ s\ \right) \quad (9)$$

In keeping with the standard definition of a stochastic game, both types of decisions are simultaneous.[15] Part (a) is the standard Markov Perfection property of a stochastic game. Private sector actions are individually optimal in each state. Part (b) asserts the feasibility of public sector strategies. As a part of that definition of feasibility, public strategies must also satisfy a "perfection" constraint in the sense that no current government can commit to future public decisions.

[15]There are sequential move alternatives, but none are clearly more compelling. In real time, private and public decisions are on-going. It seems natural then to define the dates as those intervals of time in which no agent is able to publicly pre-empt another.

4.1 An Inessential Private Sector

In the Pure Public Sector Model of Section 2, it was shown that dynamically consistent rules never admit institutional reform. In one sense, the result is robust to the addition of a private sector. Consider the following example.

Example 6. The "Libertarian Rule". *Consider the case in which all decisions are "private": let $P = E^n$ and let p^* denote an arbitrary Nash equilibrium of a game with payoff functions, $v_i(p_i, p_{-i}, \theta')$ for all i. Then define F by*

$$F(v(p, \theta'), s) \;=\; -||v(p, \theta') - v(p^*, \theta')||$$

In this example, there are no policy choices apart from the private decisions of individuals. θ' is fixed since it is not a choice variable. Formally, the rule prescribes "public sector decisions" that place decision authority for p_i in the hands of individual i who chooses his best response given choices of others. Because the rule simply labels private decisions as "public" the public sector can once again replace the effects of the private sector — this time by fiat.

Even when private decisions cannot be relabeled as public ones, they may still be replaceable or irrelevant. To develop this idea, we first maintain the assumption C is rationalized by some F (where F may be dynamically inconsistent). Next, for any initial economic state ω, a *feasible* continuation profile is a measurable function $x : \Omega \to \mathbb{R}^n$ such that for each $i = 1, \ldots, n$,

$$x_i(\omega) = E\left[\sum_{t=0}^{\infty}(1-\delta)\delta^t u_i(\omega_t, e_t, p_t) \;\Big|\; \omega = \omega_0\right],$$

where $\{e_t\}$ and $\{p_t\}$ are any pair of sequences of private sector and policy decisions, resp. Now given any feasible continuation x, define for each i,

$$H_i(\omega, e, p, x) \equiv (1-\delta)u_i(\omega, e, p) + \delta \int x_i(\omega')dq(\omega'|\omega, e, p).$$

Let $H = (H_i)_{i=1}^n$. Clearly, $H(\cdot, e, p, x)$ is itself a feasible continuation profile.

Definition 3 Fix the political state θ. *Private sector decisions are inessential* if for each feasible continuation x and each i, H_i may be decomposed according to

$$H_i(\omega, e, p, x) \;=\; H_i^1(\omega, e, x) + H_i^2(\omega, e_{-i}, p, x),\; \forall \omega, e, p \tag{10}$$

In words, private sector decisions are inessential if individuals' payoffs are separable in one's own private and public decisions. An extreme case of inessentiality is the complete absence of a private sector in Section 2. Example 6 is another example. Example 7 following the Theorem below is yet another example. It also illustrates the point that private sector inessentiality is not, by itself, that restrictive since the private sector can always be "defined away" by relabeling variables and pegging the planner's payoff to the private equilibrium.

Theorem 2 *Consider a dynamic political game in which* (C, Θ) *is single valued. Suppose that* θ *is dynamically consistent and private decisions are inessential. Then for any equilibrium* $\pi = (\sigma, \psi, \mu)$*, the political rule* θ *is stable in* μ*.*

The conclusion of the Theorem is that in any equilibrium, the type θ is stable, i.e., $\mu(\omega, \theta) = \theta$. According to the Theorem, a reform occurs only if either the rule is dynamically inconsistent or the private sector actions are *essential*.

The logic of the argument is straightforward. Interpret the parameter θ as a player — the social planner — whose payoff is $F(\cdot, \theta)$. We refer to this player as the "institutional type." In any state $s = (\omega, \theta)$, the institutional type θ chooses the public policy p and designates a subsequent decision maker, θ', the following period. A standard result of dynamic programming is that if type θ faces a single agent decision problem, it need never designate the decision authority over future policies to another player. Its own choice of policies $\{p_t\}$ would be optimal in each realized state. To put it another way, Type θ is willing to relinquish its authority over future decisions only if its designated choice can induce a more favorable response from actions of others. Consequently, inessentiality of the private sector collapses the model to the pure public sector model of Section 2. The "planner" of type θ can unilaterally reach any alternative social payoff using policies alone. Hence, this type need never relinquish decision making authority to another type $\hat{\theta} \neq \theta$. In such a case, the policy-path is optimal for type θ if that same type makes decisions each period. The institutional type θ is therefore stable.

Example 7. "Fish Wars" with User Taxes. *Let* ω *be a stock of a renewable resource (e.g., fish).* e_i *is* i*'s private extraction rate and* p *is a flat tax on each individual's extraction of the resource. Stage game preferences are* $u_i = \alpha_i \log(e_i \omega(1-p))$*. The transition is* $\omega' = (\omega[1 - \sum_j e_j])^{\gamma}(p \sum_j \omega e_j)^{1-\gamma}$ *where* $0 < \gamma < 1$*. This is a variant of the standard resource extraction or "fish war" model of Levhari and Mirman (1980) with an added user tax. Individuals have log preferences over after-tax consumption of the resource. Tomorrow's resource stock is a Cobb-Douglas function of two inputs — the public tax revenue and the left-over stock after private extraction. It is not hard to show in this case that the private sector is inessential. The key to this observation is the fact that a feasible continuation* x *is the discounted sum of logs which, in turn, is separable in private and public decisions. By Theorem 2, any dynamically consistent rule* θ *is stable.*

4.2 Recursively Self Selected Institutions

Even if the private sector is essential, change need not occur if the private sector response matters in a way that favors the current political institution. Moreover, it may be the case that change may or may not occur because the equilibrium selects "good equilibria" in certain states and "bad equilibria" in others.

This Section addresses some of these issues. For the remainder of the analysis, we assume that all feasible rules satisfy the strong form of dynamic consistency given by Equation (6). We proceed to characterize stability and reform in terms of a simple incentive compatibility condition, treating institutional types as potential players in the game.

For any fixed continuation profile x, and any state $s = (\omega, \theta)$, one can identify an $(n+1)$-player normal form game defined by the following payoffs,

$$(H_1(\omega, e, p, x), \ldots, H_n(\omega, e, p, x), F(H(\omega, e, p, x),\ \theta)\ )$$

Here, each Player $i = 1, \ldots, n$ is a participant in the original DPG and has payoff $H_i(\omega, e, p, x)$. With payoff $F(H(\omega, e, p, x), \theta)$, Player $n+1$ is the "Institutional Player" of type θ who maximizes welfare function $F(\cdot,\ \theta)$. Following a convention in the stochastic games literature, we refer to this game as an *auxiliary game*. Let $N(s, x)$ denote the set of Nash equilibria of this auxiliary game. A standard result establishes that under the given assumptions, $N(s, x)$ is a closed and nonempty set. Since feasible choice sets are compact, $N(s, x)$ is as well.

Recall that the choice of next period's institutional type θ_{t+1} is viewed as a strategic delegation decision. In any auxiliary game, one can view the delegation decision as a choice between equilibria from the sets $N(\omega, \theta, x)$ and $N(\omega, \hat{\theta}, x)$ for any pair θ and $\hat{\theta}$. However, multiplicity of equilibrium potentially muddles the comparison. Suppose, for instance, the current type θ_t chose θ simply because a "good" equilibrium was subsequently played in $N(\omega, \theta, x)$ while a "bad" equilibrium was subsequently played in $N(\omega, \hat{\theta}, x)$. Then θ_t's choice would arise largely come from a coordination failure and little else. To avoid this "modeler's fiat" as a basis for the analysis, we restrict attention in each auxiliary game to Nash equilibria that satisfy

$$\max_{(e,p)\in N(\omega,\hat{\theta},x)} F\left(H(\omega, e, p, x),\ \theta\right) \tag{11}$$

Notice that if $\theta \neq \hat{\theta}$, then type θ is not the institutional player in the auxiliary game.

Consider a continuation profile x^θ that defines a fixed point of the map in (11), i.e, a profile x^θ that satisfies:

$$F(x^\theta(\omega),\ \theta) \quad = \quad \max_{(e,p)\in N(\omega,\theta,x^\theta)} F\left(H(\omega, e, p, x^\theta),\ \theta\right), \qquad \forall\omega$$

Here, $F(x^\theta(\omega),\ \theta)$ is the largest social welfare for type θ over all possible Nash equilibria of the auxiliary game with x^θ as the continuation profile and with the same type θ as institutional Player. Existence of a fixed point in (11) is a nontrivial problem[16] Finding a fixed point x^θ is tantamount to finding a "restricted equilibrium" of the DPG in which μ is required to be

[16]Results on this can be found in the companion paper, Lagunoff (2005b).

stable, i.e, $\mu(\omega,\theta)=\theta$ for all ω. Clearly, if there exists an (unrestricted) equilibrium in the DPG then there exists a "restricted equilibrium," and, consequently, a fixed point x^{θ}.

If (11) admits a fixed point, then there exists a function F^* defined on arbitrary triples $(\omega, \hat{\theta},\ \theta)$ by

$$F^*(\omega,\hat{\theta},\ \theta) \quad = \quad \max_{(e,p)\in N(\omega,\hat{\theta},x^{\theta})} F\left(H(\omega,e,p,x^{\theta}),\ \theta\right), \qquad \forall\omega \tag{12}$$

The payoff $F^*(\omega,\hat{\theta},\theta)$ is the largest social welfare for type θ over all possible Nash equilibria of the auxiliary game in which $\hat{\theta}$ is the institutional player and x^{θ} is the continuation.

Definition 4 Type θ is *recursively self-selected (RSS)* if there exists a function F^* defined by (12) such that for all ω and all $\hat{\theta}$,

$$F^*(\omega,\theta,\ \theta)\ \geq\ F^*(\omega,\hat{\theta},\ \theta)$$

Type θ is *recursively self-denied (RSD)* by type $\hat{\theta}\neq\theta$ if for all ω,

$$F^*(\omega,\hat{\theta},\ \theta)\ >\ F^*(\omega,\theta,\ \theta)$$

Recursive self-selection describes an implicit incentive constraint on the institutional types. A recursively self-selected (RSS) θ is an institutional type that would never delegate decision authority in the public sector to another type, regardless of the realized state. A recursively self-denied (RSD) θ is a type that never delegates decision authority to itself in the subsequent period's game in any state.

Both RSS and RSD limit consideration only to one-shot delegation decisions, reverting back to auxiliary games in which type θ is the player thereafter. This restriction is sensible for checking for an institution's stability in light of the well-known One-Shot Deviation Principle. Consequently, it is not difficult to show that if the private sector is inessential in θ, then θ is RSS. The converse does not generally hold, and so RSS is a weaker condition.

Surprisingly, recursively self-denied types do not necessarily admit reform. Consider, for example, a dynamic political game with three types: $\theta_1,\theta_2,\theta_3$. Suppose that θ_1 is recursively self-denied by type θ_2, which, in turn, is recursively self-denied by type θ_3. Suppose that $F^*(\omega,\theta_1,\theta_1)\ >\ F^*(\omega,\theta_3,\theta_1),\ \ \forall\omega$, and $\mu(\omega,\theta_2)=\theta_3$ for all ω. If δ is close enough to one, then it can easily be shown that type θ_1 is stable. Basically, type θ_1 does not delegate to θ_2 because θ_2 is expected to delegate to θ_3 the following period. The failure to induce reform is due the fact that recursive self-denial is not transitive. Type θ_1 *would* delegate to θ_2 if it were certain that θ_2 were stable. Generally, θ_2 cannot commit *not* to delegate further. The lack of commitment leads to stability of θ_1.

Theorem 3 *Fix a DPG in which C is a single valued and dynamically consistent class of political rules. Suppose the DPG admits at least one equilibrium. Then:*

(i) If a rule θ is recursively self-selected, then there exists an equilibrium in which θ is stable;

(ii) If a rule θ is recursively self-denied by another rule $\hat{\theta}$ which is itself recursively-self selected, then there exists an equilibrium in which θ admits reform.

(iii) If a rule θ is recursively self-denied by every other rule, then there exists an equilibrium in which θ admits reform.

The result provides two scenarios in which reforms can occur. First, a type admits reform if it is self-denied by a stable type. In that case, the aforementioned commitment problem does not arise. Second, a type admits reform if it is self-denied by all other types. In this case, the public sector is completely ineffective. So much so, that any form of delegation is preferred by type θ.

In both scenarios, future private sector decisions compensate for the loss of control by type θ in the policy arena. The best outcome from type θ's point of view is one where it delegates decision authority to another type. In order to induce the preferred alternative, the social planner must "buy-off" the private sector by delegating future decision authority to another "type" $\hat{\theta} \neq \theta$. The delegation represents a strategic commitment in which a type cedes control over public decisions to gain more favorable treatment from decisions over which it is has no direct control. Hence, "Player" θ designates a new "player", $\hat{\theta}$, in order to elicit the desired response from the private sector.

4.3 Revisiting the Parametric Model: A Private Sector

This Section revisits the parametric example, adding a private sector to the production of a public good. Labor effort e_{it} at time t is now an input into the production of the durable public good. The transition is given by $G_{t+1} = G_t + (p_t \sum_j y_j)^\gamma \sum_j e_{jt}$ with $0 < \gamma < 1$. Stage game preferences are $u_i = y_i(1 - p_t) + G_t - e_{it}^2$, reflecting the cost or disutility of effort.[17]

As before, the wealth distribution is initially exogenous. Wealth is ordered from poorest to richest: $y_1 < \cdots < y_n$. For convenience, let $y_1 > 0$ and extend the class of rules as follows. Define C to behave *as if* the current political rule were a "dictator" with wealth anywhere in the interval $[y_1, y_n]$. Consequently, $\theta \in [y_1, y_n]$. By construction, C is dynamically consistent.

Proposition 3 *Institutional type $\theta = y_1$ is the unique, recursively self selected (hence, uniquely stable) political rule. Any other type θ with $y_1 < \theta \leq y_n$ is recursively self denied by some other type $\hat{\theta} < \theta$. Moreover, if $\gamma > 1/2$ then an equilibrium $\pi = (\sigma, \psi, \mu)$ exists, and it is given*

[17] A similar example with ideologically heterogeneous agents is developed in Jack and Lagunoff (JL) (2005b).

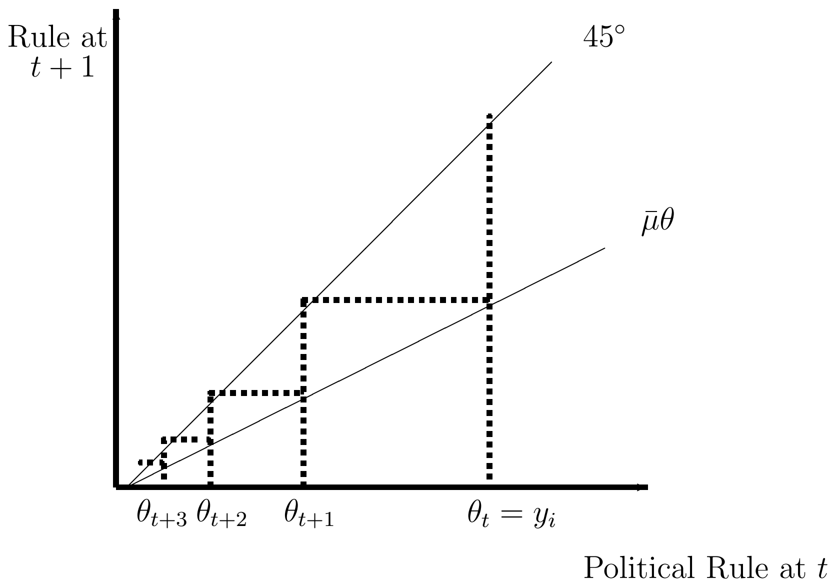

Figure 1: A Linear Rule in Wealth Holdings

by

$$
\begin{aligned}
\psi(\theta) &= {\frac{2(1-\delta)^2}{n\gamma\delta^2(\sum_j y_j)^{2\gamma}}}^{1/(2\gamma-1)} \theta^{1/(2\gamma-1)} \equiv B\ \theta^{1/(2\gamma-1)}, \\
\sigma_j(\theta) &= \frac{\delta(\sum_j y_j)^\gamma}{2(1-\delta)} B^\gamma\ \theta^{\gamma/(2\gamma-1)}, \quad \textit{and}
\end{aligned}
\tag{13}
$$

and $\mu(\theta) = \bar{\mu}\theta$ *where* $0 < \bar{\mu} < 1$ *is a constant given by the implicit solution to*

$$
\bar{\mu} = \frac{n}{2n-1} + \frac{\delta(n-1)}{2n-1}\bar{\mu}^{\frac{2\gamma}{2\gamma-1}} \tag{14}
$$

The Proposition asserts properties of RSS and RSD rules. Figure 1 illustrates the simple dynamics of the equilibrium institutional rule. In Figure 1 shows that every type except y_1 admits a reform downward. The unique stable political rule, $\theta = y_1$ (in the Figure, y_1 is small) is reached in finitely many periods.

The logic of the result is the following. If $\gamma > 1/2$ then tax rates and private effort increase in the wealth, θ, of the (implicit) dictator. Intuitively, from his own point of view, the rich dictator's chosen tax rate is too small to induce enough private investment in literacy. By delegating authority to a poorer dictator, the rich type commits to a higher tax rate, which, given the larger private response, is preferred from his point of view. Hence, rich policy makers are recursively self-denied by poorer ones. The proof is in the Appendix.

5 Summary

In general, dynamic, recursive models of political aggregation under *exogenous* political institutions are not new. One of the first is the pioneering work of Krusell, Quadrini, and Ríos-Rull (1997). More recent examples include Bernheim and Nataraj (2002), Klein, Krusell, and Ríos-Rull (2002), Banks and Duggan (2003), Hassler, et. al. (2003), Kalandrakis (2004), and Battaglini and Coate (2005).[18] In these paper, the always difficult "political fixed point" problem can be resolved in certain cases such as when the policy space is single dimensional, or when the voters are symmetric, or preferences satisfy a single crossing condition.

This paper introduces a dynamic recursive framework in which political institution are endogenous. I view the present results as, at best, a first step toward unifying a small but growing literature on dynamically endogenous institutions.[19] Informal discussions in North (1981) and Ostrom (1990) both hint at recursivity in the process of institutional change. In more formal work, Messner and Polborn (2004) examine an OLG model of endogenous changes to future voting rules under current ones. Lagunoff (2001) studies a dynamic recursive model of endogenously chosen civil liberties. Greif and Laitin (2004) model institutions as equilibrium outcomes in a repeated game.

The largest segment of this literature concerns progressive expansion of voting rights. Acemoglu and Robinson (2000, 2001) propose an endogenous voting rights model to explain the observed expansion of the voting franchise in Europe in the 19th century. Their claim is that the voting franchise historically was extended by an elite to head off social unrest. They posit a dynamic model in which an elite faces a choice each period between either: preserve the status quo by restricting voting rights to the elite, or: extend rights to everyone (full, universal manhood suffrage). Public decisions in the restricted franchise may be undercut by the threat of revolt. Consequently, the political state of the restricted franchise was — in the language of the present paper — self-denied by the stable state of the universal franchise.[20]

One virtue of Acemoglu and Robinson's approach is that political institutions are instru-

[18]See Persson and Tabellini (2001) for other references.

[19]In focusing attention on dynamic models, I neglect a larger literature on static models of endogenous political rules such as, for example, Lizzeri and Persico (2002) and Aghion, Alesina, and Trebbi (2002).

[20]Though neither are models of endogenous institutions per se, Powell (2003) and Egorov and Sonin (2005) both construct interesting dynamic games in which rulers can be overthrown. Powell constructs a dynamic game in which a temporarily weak government may lack credibility to induce another government to restrain its inefficient use of power such as launching a coup or attacking. Egorov and Sonin construct a game in which potential rivals may be killed if they constitute a likely threat to the ruler.

mental choices that do not enter preferences or technology directly. However, because the franchise choice in their model is binary, Acemoglu and Robinson do not address gradualism in expansions of voting rights. Gradualism, though not instrumental choice, is addressed in Justman and Gradstein (1999), Roberts (1998, 1999), Barbera, Maschler, and Shalev (2001), and Gradstein (2003). Justman and Gradstein examine endogenous voting rights under exogenous costs of disenfranchisement. Roberts and Barbera, Maschler, and Shalev examine club formation games in which players have exogenous preferences over the size or composition of the group. Gradstein (2003) examines choices over institutional quality.

A goal of the present framework is to combine both gradualism, as these papers do, with instrumental choice of institutions as in Acemoglu and Robinson. In this, the present work builds on some previous work (Jack and Lagunoff (2005a,b)) that examines gradual extensions of voting rights. In the present paper, however, an unrestricted array of institutional choices are examined, and reversibility in the dynamics is possible.

The paper is intended to illustrate how questions of institutional stability and/or reform can be addressed without many stylized assumptions. Without a private sector, political rules are stable if and only if they are dynamically consistent. When a private sector exists, but is inessential, dynamically consistent rules remain stable. A political rule is also stable if it is recursively self-selected. Alternatively, a political rule is shown to admit reform if the rule is recursively self-denied by a stable rule or by all rules. Clearly, the "intermediate" environments where the rules are sometimes self selected are obvious focal points for future work.

There are naturally many more cases in which political rules are neither recursively self-selected or self-denied. So far, I produce no general results on these intermediate cases. Nevertheless, the intuition above suggests that reform or stability depends on a clearer understanding of the incentive constraints on the institutional "types." Many possibilities exist for future research.

6 Appendix

Proof of Theorem 1 We omit the necessity part of the proof (i.e., dynamically consistent θ implies stability) since it is a special case of Theorem 2. It suffices to show: if θ is stable in μ then θ is dynamically consistent.

By stability, $\mu(\omega, \theta) = \theta$ a.e. ω. Because the rule is single valued, political feasibility of (ψ, μ) then implies

$$\theta = \arg\max_{\tilde{\theta}} F\left((1-\delta)u(\omega, \psi(\omega, \theta)\ ) + \delta \int V(\omega', \tilde{\theta}; \psi, \mu) dq(\omega' |\ \omega, \psi(\omega, \theta)\ ), \omega, \theta\right)$$

Suppose by way of contradiction, that θ is not dynamically consistent. Then

$$\theta \notin \arg\max_{\tilde{\theta}} \int F\left(V(\omega', \tilde{\theta}; \psi, \mu), \omega', \theta\right) dq(\omega' |\, \omega, \psi(\omega, \theta)\,) \tag{15}$$

where the set of maximizers on the right-hand side of (15) is nonempty (since otherwise, θ is trivially dynamically consistent). Let $\hat{\theta}$ denote a maximizer of (15). Since $\hat{\theta} \neq \theta$, there is some set of ω' with positive measure such that

$$F\left(V(\omega', \hat{\theta}; \psi, \mu), \omega', \theta\right) \;>\; F\left(V(\omega', \theta; \psi, \mu), \omega', \theta\right) \tag{16}$$

However, by the definition of political feasibility, for all ω',

$$\begin{aligned} & F\left(V(\omega', \theta; \psi, \mu), \omega', \theta\right) \\ = \;& \max_{\tilde{p}, \tilde{\theta}} \; F\left((1-\delta)u(\omega', \tilde{p})\,) + \delta \int V(\omega'', \tilde{\theta}; \psi, \mu) dq(\omega'' |\, \omega', \tilde{p}\,), \omega', \theta\right) \\ \geq \;& F\left((1-\delta)u(\omega', \psi(\omega', \hat{\theta})\,) + \delta \textstyle\int V(\omega'', \mu(\omega', \hat{\theta}); \psi, \mu) dq(\omega'' |\, \omega', \psi(\omega', \hat{\theta})\,), \omega', \theta\right) \\ = \;& F\left(V(\omega', \hat{\theta}; \psi, \mu), \omega', \theta\right) \end{aligned} \tag{17}$$

In other words, $F\left(V(\omega', \theta; \psi, \mu), \omega', \theta\right) \geq F\left(V(\omega', \hat{\theta}; \psi, \mu), \omega', \theta\right)$ which contradicts (16). We therefore conclude that θ is dynamically consistent. ∎

Proof of Theorem 2 Fix a dynamic political game and a political state θ. Suppose θ is dynamically consistent and private decisions are inessential in state θ. Fix an equilibrium $\pi = (\sigma, \psi, \mu)$. We proceed to show that θ is stable in μ, i.e., $\mu(\omega, \theta) = \theta$ a.e. ω.

By definition,

$$\{(\psi(\omega, \theta), \mu(\omega, \theta))\} = C(v(\cdot |\, \omega, \theta; \pi), s) = \arg\max_{p, \theta'} F(v(p, \theta' |\, \omega, \theta; \pi),\; \theta)$$

In other words,

$$F(V(\omega, \theta; \pi), \theta) \;\equiv\; F(v(\psi(\omega, \theta), \mu(\omega, \theta) |\, \omega, \theta; \pi),\; \theta) \;\geq\; F(v(p, \theta' |\, \omega, \theta; \pi), \theta),\; \forall (p, \theta') \tag{18}$$

By inessentiality, observe that for all i, and for all continuation profiles, x,

$$\arg\max_{e_i} H(\omega, e_i, \sigma_{-i}(\omega, \theta), \psi(\omega, \theta), x) \;=\; \arg\max_{e_i} H^1(\omega, e_i, \sigma_{-i}(\omega, \theta), x)$$

Moreover, the solution σ_i is independent of θ if $\sigma_{-i}(\omega, \theta) = \sigma_{-i}(\omega)$. Hence, θ is not a Markov state for σ_i, $i \in I$. It follows that for all i,

$$V_i(\omega, \theta; \pi) = V_i^1(\omega; \pi) + V_i^2(\omega, \theta; \pi)$$

where

$$V_i^1(\omega;\ \pi) \equiv H_i^1(\omega, \sigma(\omega),\ V_i^1(\cdot\ ;\ \pi)\ ),$$

and

$$V_i^2(\omega, \theta; \pi) \equiv H_i^2(\omega, \sigma_{-i}(\omega), \psi(\omega, \theta),\ V^2(\cdot, \mu(\omega, \theta);\ \pi)\ )$$

Now fix some ω' and $\hat{\theta} \neq \theta$. Then by definition of equilibrium,

$$\begin{aligned}
& F(V(\omega', \theta; \pi), \omega', \theta) \\
& F(V^1(\omega'; \pi), \theta) + V^2(\omega', \theta; \pi),\ \omega', \theta) \\
= \;& F\left(V^1(\omega'; \pi), \theta) + H^2(\omega', \sigma_{-i}(\omega), \psi(\omega', \theta), V^2(\cdot, \mu(\omega', \theta); \pi)\ ),\ \omega', \theta\right) \\
= \;& \max_{\tilde{p}, \tilde{\theta}}\ F\left(V^1(\omega'; \pi), \theta) + H^2(\omega', \sigma_{-i}(\omega), \tilde{p}, V^2(\cdot, \tilde{\theta}; \pi)\ ),\ \omega', \theta\right) \qquad (19) \\
\geq \;& F\left(V^1(\omega'; \pi), \theta) + H^2(\omega', \sigma_{-i}(\omega), \psi(\omega', \hat{\theta}), V^2(\cdot, \mu(\omega', \hat{\theta}); \pi)\ ),\ \omega', \theta\right) \\
= \;& F(V^1(\omega'; \pi), \theta) + V^2(\omega', \hat{\theta}; \pi),\ \omega', \theta) \\
= \;& F\left(V(\omega', \hat{\theta}; \pi), \theta\right)
\end{aligned}$$

We have therefore shown

$$F(V(\omega', \theta; \pi), \theta)\ \geq\ F\left(V(\omega', \hat{\theta}; \pi), \theta\right) \qquad (20)$$

However, since (20) holds for each economic state ω' and since $\hat{\theta}$ is arbitrary, it follows that for all $\hat{\theta} \neq \theta$,

$$\int F\left(V(\omega', \theta; \pi), \theta\right)\ dq(\omega'|\ \omega, \sigma(\omega, \theta), \psi(\omega, \theta))\ \geq\ \int F\left(V(\omega', \hat{\theta}; \pi), \theta\right)\ dq(\omega'|\ \omega, \sigma(\omega, \theta), \psi(\omega, \theta))\ . \qquad (21)$$

Hence, by dynamic consistency of θ,

$$\begin{aligned}
& F\left(v(\psi(\omega, \theta), \theta|\ \omega, \theta; \pi),\ \theta\right) \\
= \;& F\left((1-\delta)u(\omega, \sigma(\omega, \theta), \psi(\omega, \theta)) + \delta \int V(\omega', \theta; \pi) dq(\omega'|\ \omega, \sigma(\omega, \theta), \psi(\omega, \theta)\ ),\ \theta\right) \\
\geq \;& F\left((1-\delta)u(\omega, \sigma(\omega, \theta), \psi(\omega, \theta)) + \delta \int V(\omega', \hat{\theta}; \pi) dq(\omega'|\ \omega, \sigma(\omega, \theta), \psi(\omega, \theta)\ ),\ \theta\right) \qquad (22) \\
= \;& F\left(v(\psi(\omega, \theta), \hat{\theta}|\ \omega, \theta; \pi),\ \theta\right)
\end{aligned}$$

for all $\hat{\theta} \neq \theta$,

Hence, we have shown that $\mu(\omega, \theta) = \theta$ satisfies (22). But because C is single valued, it is the only such solution. Consequently, μ *must* satisfy $\mu(\omega, \theta) = \theta$. We therefore conclude that θ is stable in μ. ∎

Proof of Theorem 3, Part (i).

The following definition will prove useful for subsequent results.

Definition 5 An equilibrium π is *institutionally optimal for* θ^* if for any other equilibrium $\hat{\pi}$ and for each $s = (\omega, \theta)$,

$$F(V(\omega, \mu(s);\ \pi),\ \theta^*) \ \geq\ F(V(\omega, \hat{\mu}(s);\ \hat{\pi}),\ \theta^*) \tag{23}$$

The institutionally optimal equilibrium is the one that generates the most preferred social welfare for type θ^* in each state among all equilibria. The idea behind institutional optimality extends the best-case comparison to the full dynamic model. Clearly, if an equilibrium exists, then it is clear from the definition that an institutionally optimal one exists as well.

We show that if θ is recursively self selected, then there exists an institutionally optimal equilibrium in which θ is stable. Let $\pi = (\sigma, \psi, \mu)$ denote any institutionally optimal equilibrium. Now let μ^* denote an institutional strategy that satisfies:

$$\begin{aligned} \mu^*(\omega, \hat{\theta}) &= \mu(\omega, \hat{\theta}), \quad \forall\omega,\ \forall\hat{\theta} \neq \theta, \text{ and,} \\ \mu^*(\omega, \theta) &= \theta, \quad \forall\omega, \end{aligned}$$

Clearly, μ^* differs from μ in that it is stable in political state θ.

Choose some $\hat{\theta} \neq \theta$. Fix an arbitrary ω' in the full measure set on which recursive self selection holds. We will proceed to verify that the following string of equalities and inequalities hold.

$$
\begin{aligned}
& F(V(\omega,\theta;\pi),\ \theta) \\
= \ & (1-\delta)F\left(u(\omega,\sigma(\omega,\theta),\psi(\omega,\theta)),\ \theta\right) + \delta\int F\left(V(\omega',\mu(\omega,\theta);\ \pi),\ \theta\right) dq(\omega'|\,\omega,\sigma(\omega,\theta),\psi(\omega,\theta)) \\
\geq \ & (1-\delta)F\left(u(\omega,\sigma(\omega,\theta),\psi(\omega,\theta)),\ \theta\right) + \delta\int F\left(V(\omega',\theta;\ \pi),\ \theta\right) dq(\omega'|\,\omega,\sigma(\omega,\theta),\psi(\omega,\theta)) \\
= \ & (1-\delta)F\left(u(\omega,\sigma(\omega,\theta),\psi(\omega,\theta)),\ \theta\right) \\
& +(1-\delta)\delta\int F\left(u(\omega',\sigma(\omega',\theta),\psi(\omega',\theta)),\ \theta\right) dq(\omega'|\,\omega,\sigma(\omega,\theta),\psi(\omega,\theta)) \\
& +\delta^2\int\int F\left(V(\omega'',\mu(\omega',\theta);\ \pi),\ \theta\right) dq(\omega''|\,\omega',\sigma(\omega',\theta),\psi(\omega',\theta))dq(\omega'|\,\omega,\sigma(\omega,\theta),\psi(\omega,\theta)) \\
\geq \ & (1-\delta)F\left(u(\omega,\sigma(\omega,\theta),\psi(\omega,\theta)),\ \theta\right) \\
& +(1-\delta)\delta\int F\left(u(\omega',\sigma(\omega',\theta),\psi(\omega',\theta)),\ \theta\right) dq(\omega'|\,\omega,\sigma(\omega,\theta),\psi(\omega,\theta)) \\
& +\delta^2\int\int F\left(V(\omega'',\theta;\ \pi),\ \theta\right) dq(\omega''|\,\omega',\sigma(\omega',\theta),\psi(\omega',\theta))dq(\omega'|\,\omega,\sigma(\omega,\theta),\psi(\omega,\theta)) \\
\vdots \ & \ \vdots \\
\geq \ & F^*(\omega,\theta,\ \theta) \\
\geq \ & F^*(\omega,\hat{\theta},\ \theta) \\
= \ & (1-\delta)F\Big(u(\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta})),\ \theta\Big) + \delta\int F^*(\omega',\theta,\ \theta)dq(\omega'|\,\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta})) \\
\geq \ & (1-\delta)F\Big(u(\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta})),\ \theta\Big) + \delta\int F^*(\omega',\mu(\omega,\hat{\theta}),\ \theta)dq(\omega'|\,\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta})) \\
\geq \ & (1-\delta)F\Big(u(\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta})),\ \theta\Big) \\
& +(1-\delta)\delta\int F\Big(u(\omega',\mu(\omega,\hat{\theta})),\psi(\omega',\mu(\omega,\hat{\theta}))),\ \theta\Big) dq(\omega'|\,\omega,\sigma(\omega,\mu(\omega,\hat{\theta})),\psi(\omega,\mu(\omega,\hat{\theta}))) \\
& +\delta^2\int\int F^*(\omega'',\mu(\omega',\mu(\omega,\hat{\theta})),\ \theta)dq(\omega''|\,\omega',\sigma(\omega',\mu(\omega,\hat{\theta})),\psi(\omega',\mu(\omega,\hat{\theta})))dq(\omega'|\,\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta})) \\
\vdots \ & \ \vdots \\
\geq \ & F\Big(V(\omega,\hat{\theta};\pi),\ \theta\Big)
\end{aligned}
\tag{24}
$$

The string in equalities and inequalities in (24) are justified as follows. The first equality (second expression) in (24) follows by definition of $V(\omega, \theta; \pi)$ and the strong form of dynamic consistency of F. The first *in*equality (in the third expression) follows by definition of equilibrium. The second equality (fourth expression) follows again by the definition of $V(\omega, \theta; \pi)$ and dynamic consistency of F (applied recursively). The second inequality (fifth expression) follows by again by definition of equilibrium applied recursively. The third inequality follows by repeating the above substitutions recursively and iterating forward. The fourth inequality (seventh expression) follows by recursive self selection (RSS) of θ. The third equality (eighth expression) expands the expression using dynamic consistency. The fifth and sixth inequalities (nineth and tenth expressions, resp.) follows from the (recursive) application of the hypothesis that θ is RSS. The last inequality (and last expression) follows by iterating forward.

Using the same argument as in the Proof of Theorem 2 after Inequality (20 and to the end of Expression (22), we find that

$$F\left(v(\psi(\omega,\theta), \mu^*(\omega,\theta) \mid \omega, \theta; \pi),\ \theta\right) \ \geq\ F\left(v((\psi(\omega,\theta), \hat{\theta} | \ \omega, \theta; \pi),\ \theta\right)$$

or, equivalently, by dynamic consistency

$$\begin{aligned} & (1-\delta)F\left(u(\omega, \sigma(\omega,\theta), \psi(\omega,\theta)),\ \theta\right) + \delta \textstyle\int F\left(V(\omega', \mu^*(\omega,\theta);\ \pi),\ \theta\right) dq(\omega' | \ \omega, \sigma(\omega,\theta), \psi(\omega,\theta)) \\ \geq\ & (1-\delta)F\left(u(\omega, \sigma(\omega,\theta), \psi(\omega,\theta)),\ \theta\right) + \delta \int F\left(V(\omega', \hat{\theta};\ \pi),\ \theta\right) dq(\omega' | \ \omega, \sigma(\omega,\theta), \psi(\omega,\theta)) \end{aligned} \tag{25}$$

But since

$$V(\omega, \mu^*(\omega,\theta);\ \pi) \equiv (1-\delta)u(\omega, \sigma(\omega,\theta), \psi(\omega,\theta)) + \delta \int V(\omega', \mu(\omega,\theta);\ \pi) dq(\omega' | \ \omega, \sigma(\omega,\theta), \psi(\omega,\theta))$$

we can, by dynamic consistency, iteratively substitute μ^* in place of μ. Iterating forward in this way, we show $\pi = (\sigma, \psi, \mu^*)$ is, in fact, an equilibrium in which θ is stable. ∎

Parts (ii) and (iii) of Theorem 3.

Suppose that θ is recursively self denied (RSD) by either (a) another recursively self selected (RSS) type, or (b) every other type. Our proof works for both cases. Once again, let π be an institutionally optimal equilibrium. Suppose, by contradiction, that θ is stable in π, i.e., $\mu(\omega, \theta) = \theta$ for almost all ω. Since θ is RSD by either (a) a RSS type $\hat{\theta}$, or by (b) every type $\hat{\theta} \neq \theta$, we choose $\hat{\theta}$ in one or the other category such that

$$F^*\left(\omega, \hat{\theta},\ \theta\right) \ >\ F^*\left(\omega, \theta,\ \theta\right), \quad \forall \omega\ .$$

Since π is assumed stable, i.e., $\mu(\omega,\theta)=\theta$ on a set of ω with full measure, fix a state ω in that set. Then we verify that the following string of equalities and inequalities hold.

$$
\begin{aligned}
& F(V(\omega,\theta;\ \pi),\ \theta\ ) \\
= \ & (1-\delta)F\left(u(\omega,\sigma(\omega,\theta),\psi(\omega,\theta)),\ \theta\right)+\delta\int F\left(V(\omega',\mu(\omega,\theta);\pi),\ \theta\right)dq(\omega'|\,\omega,\sigma(\omega,\theta),\psi(\omega,\theta)) \\
= \ & (1-\delta)F\left(u(\omega,\sigma(\omega,\theta),\psi(\omega,\theta)),\ \theta\right)+\delta\int F\left(V(\omega',\theta;\pi),\ \theta\right)dq(\omega'|\,\omega,\sigma(\omega,\theta),\psi(\omega,\theta)) \\
= \ & (1-\delta)F\left(u(\omega,\sigma(\omega,\theta),\psi(\omega,\theta)),\ \theta\right) \\
& +(1-\delta)\delta\int F\left(u(\omega',\sigma(\omega',\theta),\psi(\omega',\theta)),\ \theta\right)dq(\omega'|\,\omega,\sigma(\omega,\theta),\psi(\omega,\theta)) \\
& +\delta^2\int\int F\left(V(\omega'',\theta;\ \pi),\ \theta\right)dq(\omega''|\,\omega',\sigma(\omega',\theta),\psi(\omega',\theta))dq(\omega'|\,\omega,\sigma(\omega,\theta),\psi(\omega,\theta)) \\
\vdots \ & \ \vdots \\
= \ & F^*(\omega,\theta,\ \theta\ ) \\
< \ & F^*(\omega,\hat{\theta},\ \theta\ ) \\
= \ & (1-\delta)F\Big(u(\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta})),\ \theta\Big)+\delta\int F^*(\omega',\theta,\ \theta)dq(\omega'|\,\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta})) \\
< \ & (1-\delta)F\Big(u(\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta})),\ \theta\Big)+\delta\int F^*(\omega',\mu(\omega,\hat{\theta}),\ \theta)dq(\omega'|\,\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta})) \\
< \ & (1-\delta)F\Big(u(\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta})),\ \theta\Big) \\
& +(1-\delta)\delta\int F\Big(u(\omega',\mu(\omega,\hat{\theta})),\psi(\omega',\mu(\omega,\hat{\theta}))),\ \theta\Big)\,dq(\omega'|\,\omega,\sigma(\omega,\mu(\omega,\hat{\theta})),\psi(\omega,\mu(\omega,\hat{\theta}))) \\
& +\delta^2\int\int F^*(\omega'',\mu(\omega',\mu(\omega,\hat{\theta})),\ \theta)dq(\omega''|\,\omega',\sigma(\omega',\mu(\omega,\hat{\theta})),\psi(\omega',\mu(\omega,\hat{\theta})))dq(\omega'|\,\omega,\sigma(\omega,\hat{\theta}),\psi(\omega,\hat{\theta})) \\
\vdots \ & \ \vdots \\
< \ & F\Big(V(\omega,\hat{\theta};\pi),\ \theta\Big)
\end{aligned}
\tag{26}
$$

The string in equalities and inequalities are justified as follows. The first equality (second expression) in (26) follows by definition and dynamic consistency of F. The second equality (in the third expression) follows from the hypothesis that the equilibrium π is stable in θ. The third equality (fourth expression) follows by recursive application of this hypothesis. The fourth equality (fifth expression) follows by iterative application of this hypothesis and the definition of F^*. The first *in*equality (sixth expression) follows by the hypothesis that θ is RSD by $\hat{\theta}$. The second inequality *in*equality (seventh expression) follows from the recursive application of the hypothesis that θ is RSD. Notice here that if $\hat{\theta}$ is itself RSS then the equilibrium is either stable in $\hat{\theta}$, i.e, $\mu(\omega', \hat{\theta}) = \hat{\theta}$ (such an equilibrium exists by the previous Theorem), OR $\mu(\omega', \hat{\theta}) \neq \hat{\theta}$ and $F^* \left(\omega', \mu(\omega, \hat{\theta}),\ \theta\right) > F^* (\omega', \theta,\ \theta)$ for all ω'. Otherwise, π could not have been institutionally optimal due to the availability of the $\hat{\theta}$-stable equilibrium. Of course, if θ is RSD against every other type $\hat{\theta}$, then clearly θ is RSS against $\mu(\omega', \hat{\theta})$. The third equality (eighth expression) applies the prior reasoning (that θ is RSD by either $\hat{\theta}$ which is itself RSS or by every other type) recursively. Finally, the last *in*equality (and last expression) follows by iterating forward.

From (26) we therefore conclude

$$F\left(V(\omega, \hat{\theta};\ \pi),\ \theta\right)\ >\ F(V(\omega, \theta;\ \pi),\ \theta\ )$$

And, since ω was chosen arbitrarily (from a full measure set), it follows that on a set of ω with positive measure, some set of

$$\int F\left(V(\omega', \hat{\theta}; \pi),\ \theta\right) dq(\omega' |\, \omega, \sigma(\omega, \theta), \psi(\omega, \theta))\ >\ \int F\left(V(\omega', \theta; \pi),\ \theta\right) dq(\omega' |\, \omega, \sigma(\omega, \theta), \psi(\omega, \theta))$$

But this clearly contradicts the supposition that $\mu(\omega, \theta) = \theta$ is an equilibrium institutional rule.

We conclude that $\mu(\omega, \theta) \neq \theta$ on a set ω with positive measure, and so θ admits institutional reform in $\pi = (\sigma, \psi, \mu)$. ∎

Proof of Proposition 3.

To verify the Proposition, consider any Markov strategy π and any state $s = (\omega, \theta)$, the recursive payoff function of both policy and effort is

$$(1 - \delta)[\omega + y_i(1 - p) - e_i^2] + \delta V_i(s'; \pi)$$

in which ω' is determined technologically and θ' is chosen by type $\theta = y_i \in [y_1, y_n]$. Observe that the state enters linearly in the stage payoff. This means that a more convenient expression of the recursive payoff function can be derived by grouping the subsequent period's state with the current stage payoff. Define $V_i^*(s;\ \pi) \equiv V_i(s; \pi) - \omega$. Notice, that the state variable in the right hand side of this equation cancels out. Consequently, V^* does not vary with ω and can be expressed a function of θ alone. The recursive payoff function then becomes

$(1-\delta)[y_i(1-p)-e_i^2+\delta(p\sum_j y_j)^\gamma \sum_j e_{jt}] + \delta V_i^*(\theta';\ \pi)$. Fixing $\theta = y_i$, the first order conditions in p and e_i imply the policy and private sector strategies of the form in (13).

With strategies in (13), the recursive payoff used by the current dictator is

$$V_i^*(\theta;\ \pi) \ = \ (1-\delta)\left[y_i(1-B\ \theta^{1/(2\gamma-1)})+A\theta^{2\gamma/(2\gamma-1)}\right] \ + \ \delta V_i^*(\theta';\ \pi) \tag{27}$$

where $A \equiv \frac{1}{4}(2n-1)\frac{\delta^2}{(1-\delta)^2}B^{2\gamma}(\sum_j y_j)^{2\gamma}$, a positive constant.

Consider now the special case where θ is stable, i.e., $\mu(\theta)=\theta$. Then V_i^* reduces to

$$V_i^*(\theta\ \pi) = x_i^\theta = \left[y_i(1-B\ \theta^{1/(2\gamma-1)})+A\theta^{2\gamma/(2\gamma-1)}\right]$$

It follows that when $\theta = y_i$ is the institutional type, then

$$F^*(\omega,\hat\theta,\theta) = (1-\delta)\left[\theta(1-B\ \hat\theta^{1/(2\gamma-1)})+A\hat\theta^{2\gamma/(2\gamma-1)}\right]+\delta\left[\theta(1-B\ \theta^{1/(2\gamma-1)})+A\theta^{2\gamma/(2\gamma-1)}\right]$$

Now evaluate $dF^*(\omega,\hat\theta,\theta)/d\hat\theta = 0$. A solution is given by linear function, $\hat\theta = \frac{n}{2n-1}\theta$. Hence, we have shown that institutional type θ is recursively self denied by any institutional type $\theta' \in [\frac{n}{2n-1}\theta,\theta)$. Since types are RSD only by lower types, it follows that the lowest type y_1 is recursively self selected, hence (by Theorem 3) stable. It also follows that all types $\theta \in (y_1, \frac{2n-1}{n}y_1]$ admit reform.

To find the equilibrium institutional rule, μ, we guess and verify a linear solution $\theta' = \bar\mu\theta$. Using the equation for V^* in (27), the recursive equilibrium payoff is of the form

$$V_i^*(\theta;\ \pi) = \sum_{t=0}^{\infty}(1-\delta)\delta^t\left[y_i(1-B\ (\bar\mu^t\theta)^{1/(2\gamma-1)})+A(\bar\mu^t\theta)^{2\gamma/(2\gamma-1)}\right] \tag{28}$$

Taking first order conditions and substituting for B and A, we verify that $\theta' = \bar\mu\theta$ is an equilibrium for $\bar\mu$ that satisfies (14). It remains to show that a solution to (14) exists. To verify this final step, observe that as $\bar\mu$ varies from 0 to 1, the left side (14) is continuously increasing from 0 to 1. Meanwhile, the right side of (14) is continously decreasing from $n/(2n-1)$ and approaches 0 asymptotically if $\gamma < 1/2$. If $\gamma > 1/2$, the right side of (14) is continously increasing from $n/(2n-1)$ but has value less than one at $\bar\mu = 1$. In either case, Brouwer's Theorem and the Intermediate Value Theorem imply a solution $\bar\mu \in (0,1)$ exists. ∎

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