Reputation and Sudden Collapse in Secondary Loan Markets *

V.V. Chari
UMN, FRB Minneapolis
chari@econ.umn.edu

Ali Shourideh
UMN, FRB Minneapolis
shour004@umn.edu

Ariel Zetlin-Jones
UMN, FRB Minneapolis
zetli001@umn.edu

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Abstract

Banks and intermediaries typically hold some of the loans they originate and sell others in the secondary market. This paper deals with the determinants of the decision of whether to hold or to sell loans. Secondary markets are often argued to suffer from adverse selection problems when originators of loans are better informed than potential purchasers regarding the quality of the loans. We analyze the role of reputation in mitigating such adverse selection problems. We argue that reputation can both be a blessing and curse, in the sense that reputational incentives lead to multiplicity of equilibria. In one of these equilibria, reputational forces help mitigate the adverse selection problem while in the other reputational forces actually worsen the adverse selection problem. We use a refinement adapted from the global games literature which leads to a unique equilibrium. This equilibrium is fragile in the sense that small fluctuations in fundamentals can lead to large changes in the volume of loans sold in the secondary market. Our model is consistent with the recent collapse in the volume of loans sold in the secondary market in the United States. We analyze a variety of policies that have been proposed to resolve adverse selection problems in the secondary loan market. We find that many such policies do not help resolve this problem and, indeed, worsen the allocative efficiency of the secondary loan market.

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1 Introduction

Following the sharp decline in the volume of new issuances in the U.S. secondary loan market in the fall of 2007, policymakers argued that the market was not functioning normally and proposed and carried out a variety of policy interventions intended to restore the normal functioning of this market. Here we construct a model in which new issuances in the secondary loan market abruptly collapse and this collapse is associated with an increase in inefficiency. We use this model to analyze proposed and actual policy interventions and argue that these interventions typically do not remedy the inefficiency associated with the market collapse.

Two features of our model, adverse selection and reputation, play central roles in generating abrupt collapses associated with increased inefficiency. In our model, loan originators have better information about the quality of the loans they originate than potential purchasers so that the market features adverse selection. We ask whether, in dynamic versions of our model, loan originators have incentives to acquire reputations for originating high quality loans. We show that reputational concerns lead to fragile outcomes in the sense that small changes in economic fundamentals lead to abrupt collapses. We also show that such abrupt collapses are associated with increased inefficiency.

We develop a finite horizon model in order to explore the role of adverse selection and reputation in the secondary market for loans. The model has two types of agents. Loan originators, or banks, differ in their ability to originate high quality loans. Those who always do that are called high quality banks, while those who never do are called low quality banks. Potential purchasers of loans, or buyers, cannot observe the quality of a bank. They receive noisy signals about the quality of a loan sold in the market and can use these signals to update their beliefs about the quality of the selling bank. If a bank chooses not to sell a loan, then buyers receive no information about the quality of the loans the bank holds or of the bank itself.

In our model, the secondary loan market has an allocative efficiency role. The secondary market has the potential to move each loan to a bank or a buyer that has the lowest holding cost for that loan. By holding cost, we mean the expenses of funding liquidity, servicing, negotiating a potential default, and any other costs of holding a loan that may be correlated with the rest of the agent's portfolio. An agent with the lowest holding cost we say has a *comparative advantage* over other agents. Under full information, in our model, when buyers know the quality of loans, the secondary loan market allocates loans based on comparative advantage.

When buyers are less well-informed about loan quality, however, the model features adverse selection. We illustrate this feature in a static model. We show that if the typical bank is likely to have low quality loans, then market prices are depressed, and high quality banks choose to hold their loans even when buyers have a comparative advantage in holding them. Our assumption that buyers have less information concerning the loan quality of a bank is in line with a body of literature that argues that secondary loan markets feature adverse selection (see, for example, the work of Dewatripont and Tirole (1994), Ashcraft and Schuermann (2008), and Arora et al. (2009)).

We then investigate whether adding reputational incentives can help mitigate the adverse selection problem, thereby increasing the allocative efficiency of the market. After a loan is sold, for example, a buyer will learn about its quality from the loan's repayment and default pattern. Potential buyers could thus use information about the quality of an originator's past loan sales to determine whether it has a *reputation* for selling high or low quality loans. Buyers may be willing to pay high prices for loans purchased from originators with a good reputation, those with a history of selling high quality loans. The originators of high quality loans thus may be willing to sacrifice short-term profits associated with holding their loans for long-term gains. With reputational incentives like this, which we label *positive reputational incentives*, even when the typical bank is likely to have low quality loans, the secondary loan market would include more than just low quality loans, and the adverse selection problem would be smaller.

The main finding of our analysis is that while reputational incentives can mitigate the adverse selection problem as described above, they can also worsen the adverse selection problem and lead to decreased allocative efficiency. To see that, suppose buyers believe that only low quality banks sell loans in the secondary market. Then buyers do not learn about the quality of the loan from its repayment and default pattern. Instead, buyers attribute the sale of any loan from a bank to a bank with low quality loans. The originators of high quality loans thus have strong incentives to hold their loans to avoid receiving a low reputation. With such *negative reputational incentives*, even when the typical bank is likely to have high quality loans, the secondary loan market includes only low quality loans. That is, our model with reputational incentives features multiple equilibria which are ranked by allocative efficiency. As a result, any sudden switch from the equilibrium associated with positive reputational incentives to the equilibrium with negative reputational incentives leads to an abrupt collapse in new issuances in the secondary loan market associated with an increased inefficiency.

Multiplicity of equilibria is generally undesirable in a model; it increases the difficulty of using the model to conduct policy experiments, for example. A refinement that produces a unique equilibrium is desirable. We adapt an equilibrium selection device from the literature on global games (Carlsson and Van Damme (1993) and Morris and Shin (2003)) that selects a unique equilibrium for games with multiple, Pareto ranked equilibria, like ours. Following that literature, we add to our model a perturbation, or shock, that introduces imperfect observability of the costs of holding loans. We show that the model has a unique equilibrium with this feature and that the equilibrium converges as the imperfections disappear. This unique equilibrium is characterized in the model as a cost threshold. Above this cost level, high quality banks choose to sell their loans, and below it, they choose to hold them.

This equilibrium is fragile in the sense that small changes in fundamentals, in our case the cost shock, can produce changes in outcomes for a bank with a given reputation. We also show that for a wide range of reputations, the associated cost thresholds are very close to each other. This finding implies that small fluctuations in costs around these thresholds can induce changes in outcomes for banks with a wide range of reputations. In this sense, small fluctuations in costs can induce large changes in aggregate outcomes.

The fragility of equilibrium in our model implies that it is consistent with the observed large fluctuations in the volume of new issuances in the market for asset backed securities. Figure 1 displays the volume of new issuances of asset-backed securities for various categories from the first quarter of 2000 to the first quarter of 2009. The figure shows that the total volume of new issuances of asset-backed securities rose from roughly \$50 billion in the first quarter of 2000 to roughly \$300 billion in the fourth quarter of 2006. The volume of new issuances fell abruptly to roughly \$100 billion in the third quarter of 2007 and then fell again to near zero in roughly the fourth quarter of 2008. The figure also shows similar large fluctuations in the volume of new issuances for each category. One interpretation of this data in light of our model is that in 2007 holding costs crossed the threshold for many issuers leading them to hold new issues rather than sell them in the marketplace.

Ivashina and Scharfstein (2008) document a similar pattern for new issues of syndicated loans.

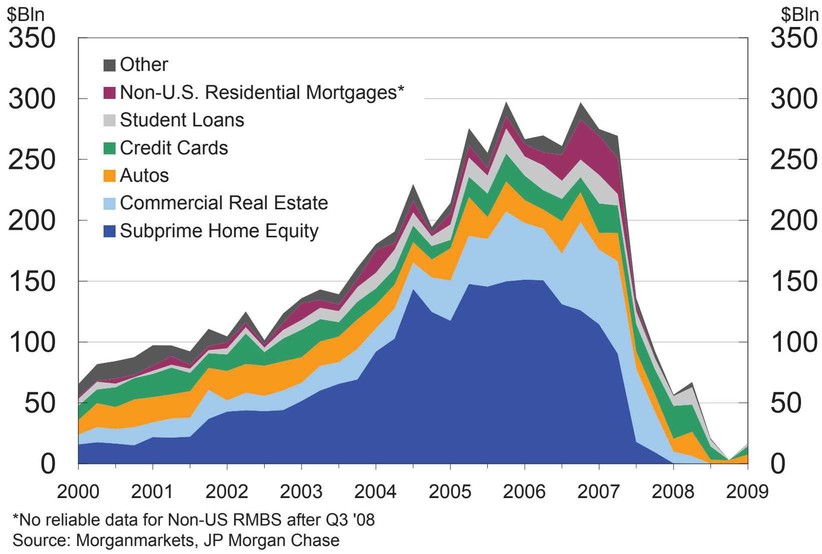

Figure 1: New Issuance of Asset Backed Securities (Source: JP Morgan Chase)

Figure 1, Panel-A of their paper shows that syndicated lending rose from roughly $300 billion in the first quarter of 2000 to roughly $700 billion in the second quarter of 2007. This lending declined sharply thereafter and fell to roughly $100 billion by the third quarter of 2008.

White (2009) has argued that the United States experienced a boom bust cycle in securitization of real estate assets in the 1920's similar to its recent experience. Figure 2 displays the change in the outstanding stock in real estate bonds in the 1920s based on data in Carter and Sutch (2006). Such bonds were issued against single large commercial mortgages or pools of commercial or real estate mortgages and were publicly traded. To make the data comparable to more recent data, we scale the data from the 1920s by nominal GDP in 2009. Specifically, we multiply the change in the nominal stock of outstanding debt in each year by ratio of the nominal GDP in 2009 to that in the relevant year. This figure shows that the changes in the stock rose dramatically from essentially 0 in 1919 to an average of 145 billion dollars in the period from 1925 to 1928. The market then collapsed sharply and changes in the stock fell to roughly 50 billion dollars in 1929. Such large changes in the stock are likely to have been associated with similar large changes in the volume of new issuances.

Figure 2: Change in Stock of Real Estate Bonds 1920-1930

We have argued that our model is consistent with abrupt collapses in secondary loan markets. Our model is also consistent with the widespread view among policymakers that such abrupt collapses were associated with sharp increases in the inefficiency of the operation of such markets. For example, the Treasury Department, in its Fact Sheet dated March 23, 2009 releasing details of a proposed Public-Private Investment Program for Legacy Assets asserts,

> "Secondary markets have become highly illiquid, and are trading at prices below where they would be in normally functioning markets." (Treasury Department 2009)

Similarly, the Federal Reserve Bank of New York, in a White Paper dated March 3, 2009 making the case for the Temporary Asset Loan Facility (TALF) asserts that

> "Nontraditional investors such as hedge funds, which may otherwise be willing to invest in these securities, have been unable to obtain funding from banks and dealers because of a general reluctance to lend." (TALF White Paper 2009)

In the wake of the 2007 collapse of secondary loan markets, policymakers proposed a variety of programs intended to remedy inefficiencies in the market for securitized assets. Some of these programs, such as the proposed Public-Private Partnership for purchasing assets held by distressed financial institutions, were not implemented. Others, such as TALF, were implemented. This program allows participants to purchase securitized assets by borrow from the Federal Reserve and using the assets as collateral. To the extent that the interest rate charged by the Federal Reserve is below market interest rates, this program is effectively a subsidy for the private purchase of assets in the secondary loan market. To the extent that the interest rate charged by the Federal Reserve is at market interest rates, it is not clear why this program would be effective.

We use our model to evaluate the effects of various policies. One such policy which resembles the Public-Private Partnership and the TALF program is that the government offers to purchase loans at prices above existing market values. Another policy, which is intended to capture the effects of the Federal Reserve's monetary policy actions, is to change the time path of interest rates. In terms of purchase policies, we show that if the price is set below that level that prevails in the positive reputational equilibrium, the policy by itself does not change equilibrium outcomes but it does involve transfers to banks and implies that the government makes negative profits. If the purchase price is set at a sufficiently high level, this policy can eliminate the fragility of equilibria. At this high level, the policy also involves transfers to banks and implies that the government makes negative profits.

In terms of policies that change the time path of interest rates, we show that temporary decreases in interest rates worsen the adverse selection problem. Interestingly, anticipated decreases in interest rates in the future can have beneficial current effects by reducing the range of reputations over which the economy has multiple equilibria.

1.1 Related Literature

Our work here is related to an extensive literature on adverse selection in asset markets, including the work of Myers and Majluf (1984), Glosten and Milgrom (1985), Kyle (1985), and Garleanu and Pedersen (2004) as well as to the related securitization literature, specifically, the work of DeMarzo and Duffie (1999) and DeMarzo (2005). We add to this literature by analyzing how reputational incentives affect adverse selection problems.

Additionally, a growing literature documents the presence of adverse selection in asset markets. For example, Downing et al. (2009) find that loans which banks held on their balance sheets yielded more on average relative to similar loans which they securitized and sold. Drucker and Mayer (2008) argue that underwriters of prime mortgage-backed securities are better informed than buyers and present evidence that these underwriters exploit their superior information when trading in the secondary market. Specifically, the tranches that such underwriters avoid bidding on exhibit much worse-than-average ex-post performance than the tranches that they do bid on.

Our work is also related to an extensive literature on reputation. Kreps and Wilson (1982) and Milgrom and Roberts (1982) argue that equilibrium outcomes are better in models with reputational incentives than in models without them. In the banking literature, Diamond (1989) develops this argument. More recently, Mailath and Samuelson (2001) analyze the role of reputational incentives in infinite horizon economies and provide conditions under which they can improve outcomes. In contrast, Ely and Välimäki (2003) and Ely et al. (2008) describe models in which reputational incentives can worsen outcomes. Our work here combines the results in this literature by showing that reputational models can have multiple equilibria. In some of these equilibria, reputational incentives can generate better outcomes; in others, worse. Furthermore, using techniques from the global games literature, we develop a refinement that produces a unique, fragile equilibrium. Perhaps the work most closely related to ours is that of Ordoñez (2008). An important difference between our work and his is that our model has equilibria that are worse than the static equilibrium, so that reputational incentives can lead to outcomes that are ex-post less efficient than in a model without these incentives.

Our analysis of policy is closely related to recent work by Philippon and Skreta (2009) who analyze a variety of policies in a model with adverse selection. The main difference with our work is that they focus on the incentives induced by reputation while they analyze a static model.

2 Reputation in a Secondary Loan Market Model

We begin our analysis by developing a finite horizon model of the secondary loan market.This allows us to demonstrate how adverse selection and reputation interact to yield abrupt collapses with increased inefficiency.

We begin with a static version of our benchmark model. We use this equilibrium to construct equilibria in a repeated finite horizon model. We show that reputational equilibria typically exhibit dynamic coordination problems in the sense that for a wide range of parameters, the repeated model has multiple equilibria. Although reputation is always valued, different equilibria require the informed player to choose different actions in order to signal her type.

2.1 Static Model: A Unique Equilibrium

We start with the static model. This model can also be interpreted as describing the last period of a finite horizon model. We show that the static version has a unique equilibrium, and that the unique equilibrium outcomes depend on the informed agent's reputation.

2.1.1 Agents and Timing

The model has three types of agents: a bank, a continuum of buyers, and a continuum of lenders. All agents are risk neutral.

The bank is endowed with a risky project indexed by π. We intend a project to represent an investment opportunity such as a loan, a mortgage, or an asset-backed security. Each project requires q units of inputs, which represents the project's size. A project of type π yields a return of $v = \bar{v}$ with probability π and no return with probability $1 - \pi$ at the end of the period. We assume that $\pi \in \{\underline{\pi}, \bar{\pi}\}$ with $\underline{\pi} < \bar{\pi}$. We refer to a bank which has a project of type $\bar{\pi}$ as a *high quality bank* and one with a project of type $\underline{\pi}$ as a *low quality bank*. We assume that $\underline{\pi}\bar{v} \geq q$ so that each project has positive net present value if sold.

At the beginning of the period, a bank has the opportunity to originate a loan. The bank can originate the loan and sells it immediately to buyers at a price p. This action generates a payoff of $p - q$. The purchaser of the asset is entitled to the resulting return. If the bank chooses to hold the loan to maturity, it must borrow q from lenders and repay $q(1 + r)$ at the end of the period, where r is the within-period interest rate paid to lenders. We allow r to be positive or negative in order to examine the effects of various policy experiments described below. The bank is entitled to the return from its projects; however, the bank then incurs a cost of holding the loan, c, in addition to the cost of repaying its debt, $q(1 + r)$.

Besides the quality of its loan, the bank is indexed by a cost type, which represents the costs, relative to the marketplace, that the bank incurs when it holds the loan to maturity. We intend the cost of the loan to represent funding liquidity costs, servicing costs, renegotiation costs in the event of a loan default, and costs associated with holding a loan that may be correlated in a particular way with the rest of the bank's portfolio, among other potential factors. We assume that $c \in \{\underline{c}, \bar{c}\}$ with $\underline{c} < -qr < 0 < \bar{c}$. We refer to a bank of type $\bar{c}$ as a *high cost bank* and a bank of type $\underline{c}$ as a *low cost bank*.

Hence, there are four types of banks in the model: $(\pi, c) \in \{\underline{\pi}, \bar{\pi}\} \times \{\underline{c}, \bar{c}\}$. We refer to the different types of banks, $(\bar{\pi}, \bar{c}), (\bar{\pi}, \underline{c}), (\underline{\pi}, \bar{c}), (\underline{\pi}, \underline{c})$, as, HH, HL, LH, LL banks, respectively.

Timing of the Static Game

We formalize the interactions in this economy as an extensive form game with the following timing.

1. Nature draws the quality and cost types of the bank.
2. The bank originates a loan of size q.
3. Buyers simultaneously offer a price to purchase a loan, p.
4. The bank sells the loan to one of the buyers or holds the loan to maturity.
5. If the bank sells the loan, then the loan's return v is realized publicly.
6. If the bank holds the loan, then the loan's return v is realized only by the bank.

We assume that, as perceived by buyers and lenders, the bank has quality type $\bar{\pi}$ with probability μ_2 and quality type $\underline{\pi}$ with probability $1 - \mu_2$. (The subscript 2 on the probability is meant to indicate that these are the beliefs of lenders associated with the second period of our two period model described below.) Following the work of Kreps and Wilson (1982) and Milgrom and Roberts (1982), we refer to μ_2 as the bank's reputation. Moreover, buyers believe that the bank has cost type $\underline{c}$ with probability α and cost type $\bar{c}$ with probability $1 - \alpha$.

2.1.2 Strategy and Equilibrium

A strategy for the bank consists of the choice of whether to sell or hold its loan, and which buyer to sell to if the bank chooses to sell. Clearly, the bank will choose the buyer offering the highest price if the bank decides to sell, so we suppress this aspect of the bank's strategy. Let a denote the loan decision of the bank whether to sell or hold the loan. If the bank chooses to sell, we denote the loan decision by $a = 1$, and if the bank chooses to hold the loan, we denote the loan decision by $a = 0$. A strategy for the bank is a function $a(\cdot)$ which maps prices into a loan decision. The payoffs to a type (π, c) bank are given by

$$w_2(a|p, \pi, c) = a(p - q) + (1 - a)\left[\pi\bar{v} - q(1 + r) - c\right]$$

A strategy for a buyer consists of the choice of a price to offer a bank for its loan. The payoffs to a buyer with an accepted price p and a strategy $a_2(\cdot|\pi, c)$ for each type of bank is

$$u_2(p|a_2) = E_{\pi,c}[v|a_2(p|\pi, c) = 1] - p.$$

Since buyers move simultaneously, they engage in a form of Bertrand competition, so that the price is equal to the expected return of the loan.

A (pure strategy) Perfect Bayesian Equilibrium is a price p_2 and a bank strategy for each bank type, $a_2(\cdot|\pi, c)$, such that for all p, each bank type chooses the optimal loan decision and buyers offer the highest price that yields a payoff of 0; i.e., $p_2 \in \max\{p|u_2(p|r, a_2) = 0\}$.

Before characterizing the equilibria of this game, we characterize the outcomes under full information, when the bank's type is known by buyers. When buyers and lenders are informed of the bank's type, (π, c), Bertrand competition among buyers implies that the price in the secondary loan market is $p = \pi\bar{v}$. Consider the decision of whether to sell or hold a loan by a bank of type (π, c). Facing a price p, the bank chooses to sell the loan in the secondary market if and only if

$$p - q \geq \pi\bar{v} - q(1 + r) - c.$$

Since Bertrand competition implies that the price $p = \pi\bar{v}$, the bank sells if and only if

$$qr + c \geq 0$$

which can also be written as $c \geq -qr$. Since $\underline{c} < -qr < 0 < \bar{c}$, in equilibrium if the bank has a high cost, it sells its loan while if it has a low cost it holds its loan.

Notice that the equilibrium allocation under full information is ex-post efficient. Low cost banks have a comparative advantage (over the market) in holding loans to maturity while the market has a comparative advantage over high cost banks. The full information equilibrium allocates loans to agents with a comparative advantage, i.e., if the bank has a low cost it holds its loan, and if the bank has a high cost it sells its loan.

Next, we characterize the equilibria of the game with private information. It is straightforward to show that the game has a unique equilibrium in which independent of the reputation μ_2, low cost banks (HL,LL) always hold their loans, and the LH bank always sells its loan. Moreover, the equilibrium outcomes for the HH bank can be characterized by a threshold level of μ_2, which we denote by μ_2^*, such that below μ_2^*, the high quality, high cost type bank holds its loan, and above μ_2^*, this type sells its loan.

To simplify the exposition in the text, we restrict the strategy sets of the low cost type banks as well as the low quality, high cost bank. Specifically, we assume that the low cost type banks must hold their loans while the LH bank must sell its loan. In the Appendix, we show that our results are still valid without this restriction.

Consider now the loan decision of the high quality, high cost (HH) bank. The HH bank sells if and only if

$$p - q \geq \bar{\pi}\bar{v} - q(1+r) - \bar{c}.$$

We now turn to the problem faced by buyers. Since buyers must break even in equilibrium, the equilibrium price offered by buyers must satisfy one of the following two conditions. Suppose first that buyers believe the HH bank will sell. Then, with probability μ_2, the selling bank is a high quality bank. Since we have assumed that a low quality high cost bank always sells, with probability $(1 - \mu_2)$, the selling bank is low quality. Thus, Bertrand competition among buyers implies that the equilibrium price satisfies the following equality:

$$\hat{p}(\mu_2) := \left[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}\right]\bar{v}. \tag{1}$$

If buyers believe that the HH bank holds, buyers believe that only the low quality bank sells, so that the price satisfies

$$p = \underline{\pi}\bar{v}. \tag{2}$$

When facing the highest possible price, $\hat{p}(\mu_2)$, the HH bank will sell if and only if

$$\hat{p}(\mu_2) - q \geq \bar{\pi}\bar{v} - q(1+r) - \bar{c}.$$

Let μ_2^* be the value of reputation such that the HH bank is indifferent between selling and holding at $\hat{p}(\mu_2)$. Then, μ_2^* must satisfy

$$\left[\mu_2^*\bar{\pi} + (1-\mu_2^*)\underline{\pi}\right]\bar{v} - q = \bar{\pi}\bar{v} - q(1+r) - \bar{c}$$

or

$$\mu_2^* = 1 - \frac{qr + \bar{c}}{(\bar{\pi} - \underline{\pi})\bar{v}}. \tag{3}$$

Clearly for $\mu_2 \geq \mu_2^*$, equilibrium involves the HH banks choosing to sell its loan and buyers offering $\hat{p}(\mu_2)$. When $\mu_2 < \mu_2^*$, equilibrium involves the HH bank holding its loan and buyers offering $p = \underline{\pi}\bar{v}$.

We use this characterization of the static equilibrium to calculate the payoffs associated with a given level of reputation μ_2 at the beginning of the period before a bank's cost type is realized. These payoff calculations play a crucial role in our dynamic game. They are given by

$$V_2(\mu_2) = \begin{cases} \bar{\pi}\bar{v} - q(1+r) - Ec, & \mu_2 < \mu_2^* \\ (1-\alpha)\left\{[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\bar{v} - q\right\} + \alpha[\bar{\pi}\bar{v} - q(1+r) - \underline{c}], & \mu_2 \geq \mu_2^*. \end{cases} \tag{4}$$

It is clear that V_2 is weakly increasing and convex in μ_2. We have proven the following proposition.

Proposition 1 *If $\underline{\pi}\bar{v} > q$ and $qr + \bar{c} > 0$, then for any $\mu \in [0,1]$, the game has a unique equilibrium. Let μ_2^* be defined by (3). For $\mu_2 < \mu_2^*$, the equilibrium price is $\underline{\pi}\bar{v}$ and the HH bank holds its loan. For $\mu_2 \geq \mu_2^*$, the equilibrium price is $[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\,\bar{v}$ and the HH bank sells its loan. Furthermore, the payoffs to the HH bank given in (4) is weakly increasing and convex in μ_2.*

Note that we have modeled buyers as behaving strategically. This modeling choice plays an important role in ensuring that the static game has a unique equilibrium. Suppose that rather than modeling buyers as behaving strategically, we had instead simply required that market prices satisfy a zero profit condition. One rationale for this requirement is that buyers take prices as given and choose how many loans to buy as in a competitive equilibrium. It is easy to show that with this requirement the economy has multiple equilibria in the static game if $\mu_2 \geq \mu_2^*$. One of these equilibria corresponds to the unique equilibrium of our game. In the other equilibrium, the buyers offer a price of $\underline{\pi}\bar{v}$. At this offered price, the HH bank holds its loan and only the low quality, high cost banks sells its loan. With strategic behavior by the buyers, this low price outcome cannot be an equilibrium. The reason is that each of the buyers now has an incentive to offer a price slightly below a price of $[\mu_2\bar{\pi} + (1 - \mu_2)\underline{\pi}]\,\bar{v}$. At this offered price, the HH bank strictly prefers to sell, and the deviating buyer makes strictly positive profits.

While we prefer our strategic formulation, we emphasize that our results that reputational incentives induce multiplicity do not rely on the static game having a unique equilibrium. We chose a formulation in which the static game has a unique equilibrium in order to argue that reputational incentives by themselves can induce multiplicity.

2.2 Two Period Version of the Benchmark Model

Consider now a two-period repetition of our static game. We assume that the bank's second period payoffs are discounted at rate β. In period 1, a continuum of buyers who are present in the market for only one period choose to offer one time prices. In period 2, a new set of buyers offer new one time prices. This new set of buyers observes whether the bank sold or held its loan in the previous period, and, if the bank sold its loan, buyers observe the realized value of the loan.

The timing of the game is an extension of that described in the static game. As in that game, at the beginning of period 1, nature draws the bank's quality and cost type. We assume that the bank's quality type is fixed for both periods. At the beginning of period 2, nature draws a new cost type for the bank. In any period, the bank's quality and cost types are unknown to buyers and lenders. The timing within each period is the same as in the static game. We also assume that the returns to successful loans, $v = \bar{v}$, and to unsuccessful loans, $v = 0$, are the same in both periods.

In order to define an equilibrium in this repeated game, we must develop language that will allow us to describe how second period buyers update their beliefs about the bank's type based on observations from period 1. To do so, we let the public history at the beginning of period 2 be denoted by θ_1 where $\theta_1 \in \{h, s0, s\bar{v}\}$ with $\theta_1 = h$ representing a decision by the bank to hold its loan in period 1, $\theta_1 = s0$ representing a decision to sell and the loan paying $v = 0$, and $\theta_1 = s\bar{v}$ representing a sale and the loan paying $v = \bar{v}$.

As in the static game, we restrict the strategy sets of the low cost type banks as well as the low quality, high cost bank. Specifically, we assume that the low cost type banks must hold their loans while the LH bank must sell its loan. A strategy for the high cost, high quality bank is now given by a pair of functions, $a_1(p_1)$ representing the decision in period 1 and $a_2(p_2, \theta_1)$ representing the loan decision in period 2, if the bank realizes a high cost in period 2, as a function of offered prices.

Consider next how the buyers in the last period update their beliefs about the bank's type. This update depends through Bayes rule on the prior belief of the buyers, the loan decision of the bank and the loan return realization if the bank sold, as well as on the first period strategies chosen by the HH bank and period 1 buyers. From Bayes rule, these posterior probabilities are given by

$$\mu_2(\mu_1, \theta_1 = h, a_1(\cdot), p_1) = \frac{\mu_1\left(\alpha + (1-\alpha)(1 - a_1(p_1))\right)}{\mu_1\left(\alpha + (1-\alpha)(1 - a_1(p_1))\right) + (1-\mu_1)\alpha} \tag{5}$$

$$\mu_2(\mu_1, \theta_1 = s\bar{v}, a_1(\cdot), p_1) = \frac{\mu_1 a_1(p_1)(1-\alpha)\bar{\pi}}{\mu_1 a_1(p_1)(1-\alpha)\bar{\pi} + (1-\mu_1)(1-\alpha)\underline{\pi}} \tag{6}$$

$$\mu_2(\mu_1, \theta_1 = s0, a_1(\cdot), p_1) = \frac{\mu_1 a_1(p_1)(1-\alpha)(1-\bar{\pi})}{\mu_1 a_1(p_1)(1-\alpha)(1-\bar{\pi}) + (1-\mu_1)(1-\alpha)(1-\underline{\pi})} \tag{7}$$

For notational convenience, we suppress the dependence on strategies and priors and let μ_h denote the posterior associated with the bank holding its loan, and $\mu_{s\bar{v}}$ and μ_{s0} denote the posteriors associated with selling and yielding a high or low return.

Given the updating rules, period 1 payoffs for the HH bank are given by

$$\begin{aligned} w_1(a|p) = & a\left[p - q + \beta\left(\bar{\pi} V_2(\mu_{s\bar{v}}) + (1-\bar{\pi}) V_2(\mu_{s0})\right)\right] \\ & + (1-a)\left[(\bar{\pi}\bar{v} - q(1+r) - \bar{c}) + \beta V_2(\mu_h)\right] \end{aligned}$$

where $\mu_h, \mu_{s\bar{v}}$, and μ_{s0} are given by equations (5), (6), and (7). Buyers' payoffs associated with an accepted price, p, in period t are given by

$$u_t(p|r, a_t, \mu_t) = \frac{\mu_t(1-\alpha)a_t(p)\bar{\pi} + (1-\mu_t)(1-\alpha)\underline{\pi}}{\mu_t(1-\alpha)a_t(p) + (1-\mu_t)(1-\alpha)}\bar{v} - p.$$

A Perfect Bayesian Equilibrium is a first period price, p_1, a first period loan decision for the good bank $a_1(\cdot)$ which maps accepted prices into loan decisions, updating rules $\mu_h, \mu_{s\bar{v}}, \mu_{s0}$ which map observations on loan decisions into posterior beliefs, a second period price, p_2, which maps second period beliefs into prices, and a second period loan decision $a_2(\cdot)$ which maps accepted prices and histories into loan decisions such that

1. for all p, the HH bank chooses the optimal action in period 1 so that $w_1(a_1(p)|p) \geq \max_{a'} w_1(a|p)$,
2. for all p,the HH bank chooses the optimal action in period 2 so that $w_2(a_1(p)|p) \geq \max_{a'} w_2(a|p)$,
3. the first period price, p_1 satisfies $p_1 \in \max\{p|u_1(p|a_1) = 0\}$,
4. the second period price, p_2 satisfies $p_2 \in \max\{p|u_2(p|a_2) = 0\}$,
5. the updating rules, $\mu_h, \mu_{s\bar{v}}, \mu_{s0}$ satisfy (5), (6), and (7).

Next, we characterize the set of equilibria in the two period game under the following assumption:

Assumption 2 *α and β satisfy $\beta(1-\alpha) \leq 1$.*

Later we provide a partial characterization of the set of equilibria when this assumption is relaxed.

We show that the game has two equilibria for a range of period 1 reputations, μ_1 around the static threshold, μ_2^*. In one type of equilibrium, the HH bank chooses to sell its loan in period 1. The posteriors associated with selling now depend non-trivially on returns of the loan. In particular, when the loan pays a high return, the bank is rewarded with a higher posterior, and when the loan pays a low return, the bank's posterior is lower than its prior. The posterior associated with holding the loan is exactly equal to the bank's period 1 reputation. These posteriors provide reputational incentives for the bank to sell the loan in order to signal its type and receive a higher period 2 reputation. Notice, for an HH bank with initial reputation above the static threshold, μ_2^*, the bank's equilibrium strategy coincides with repetition of the static perfect Bayesian equilibrium, but for HH banks with reputations below the static threshold, reputational incentives dominate their static incentives.

In the second type of equilibrium, the HH bank chooses to hold its loan. In this equilibrium, uninformed agents believe that the only type of bank that sells its loan is the LH bank. Hence, regardless of the return of the loan, if the bank that sells it receives a posterior reputation of 0. Because uninformed agents believe that high quality banks hold their loans, the posterior associated with holding the loan is higher than the prior reputation. These posteriors provide reputational incentives for the bank to hold its loan in order to signal its type. In this equilibrium, the action of HH banks with reputations below the static threshold coincides with repetition of the static perfect Bayesian equilibrium. High quality, high cost banks with reputations above the static threshold now hold their loan because of reputational concerns. In this sense, reputation is harmful to the market place as it induces high quality, high cost banks to hold their loans while in a static setting the market place can offer a sufficiently high price to induce these banks to sell their loans.

To see these results, consider first supporting equilibria in which the HH bank chooses to sell its loan in period 1. In this case, the period 1 price is given by equation $\hat{p}(\mu_1)$. Given this price, a sale decision is optimal if the difference in payoffs between choosing to sell or hold the loan is non-negative, or if the following incentive constraint is satisfied:

$$(\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi})\,\bar{v} - q + \beta\left(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0})\right) \geq \bar{\pi}\bar{v} - q(1+r) - \bar{c} + \beta V_2(\mu_h) \tag{8}$$

or if

$$\mu_1(\bar{\pi}-\underline{\pi})\bar{v} + \beta\left(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0}) - V_2(\mu_h)\right) \geq (\bar{\pi}-\underline{\pi})\bar{v} - (qr+\bar{c})$$

where $\mu_h = \mu_1, \mu_{s\bar{v}} = \frac{\mu_1\bar{\pi}}{\mu_1\bar{\pi}+(1-\mu_1)\underline{\pi}}$, and $\mu_{s0} = \frac{\mu_1(1-\bar{\pi})}{\mu_1(1-\bar{\pi})+(1-\mu_1)(1-\underline{\pi})}$. Let the reputational gain be defined as

$$\Delta^g(\mu_1) = \beta\left(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0}) - V_2(\mu_h)\right)$$

We characterize the reputational gain $\Delta^g(\mu_1)$ in the following lemma:

Lemma 3 *For all $\mu_1 \in (\mu_g, 1)$, the reputational gain $\Delta^g(\mu_1) > 0$, where μ_g satisfies*

$$\mu_2^* = \frac{\mu_g\bar{\pi}}{\mu_g\bar{\pi} + (1-\mu_g)\underline{\pi}}.$$

and $\Delta^g(\mu_1) = 0$ when $\mu_1\,[0, \mu_g]$. When $\mu_2^* = 1$, $\Delta^g(\mu_1) = 0$ for all $\mu_1 \in [0,1]$.

Proof. *In the Appendix* ■

This lemma shows that when $\mu_1 = \mu_2^* \in (0,1)$, the reputational gain is strictly positive. The reason is that the second period payoffs are convex in reputation. This result implies that when μ_1 is less than but close to μ_2^*, the dynamic gains from reputation exceed the static cost of selling. Therefore, there must exist $\underline{\mu} < \mu_2^*$ such that the HH bank sells when $\mu_1 \in [\underline{\mu}, 1]$ in the first period and holds the loan when $\mu_1 \in [0, \mu_1)$.

Now consider the equilibria in which the HH bank holds its loan in period 1. In this case the equilibrium price is given $\underline{\pi}\bar{v}$. A bank holds its loan if and only if

$$(\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi})\,\bar{v} - q + \beta\,(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0})) \leq \bar{\pi}\bar{v} - q(1+r) - \bar{c} + \beta V_2(\mu_h) \qquad (9)$$

where $\mu_h = \frac{\mu_1}{\mu_1 + (1-\mu_1)\alpha}$, and $\mu_{s\bar{v}} = \mu_{s0} = 0$. If the above inequality is reversed, there is a deviation by buyer to price $\hat{p}(\mu_1)$ that would break down the equilibrium. We show that there exists $\mu_2^* < \underline{\mu}$ such that when $\mu_1 \in [0, \underline{\mu}]$ (9) holds.

Assumption 1 implies that (9) holds when $\mu_1 = 1$. Again, we can define the reputational gain as

$$\Delta^b(\mu_1) = \beta(V_2(0) - V_2(\mu_h))$$

In the following lemma we show that the reputational gain is negative in this equilibrium.

Lemma 4 *For all $\mu_1 > \mu_b$, the reputational gain $\Delta^b(\mu_1) < 0$ where μ_b is given by*

$$\mu_2^* = \frac{\mu_b}{\mu_b + (1-\mu_b)\alpha}.$$

When $\mu_1 \leq \mu_b$, $\Delta^b(\mu_1) = 0$. Moreover, $\Delta^b(\mu_1)$ is a strictly convex function when $\mu_1 \in [\mu_b, 1]$.

Proof. In the Appendix. ■

Given the actions of various types, when future buyers observe a selling decision, they are certain that the selling bank is of low quality. Rationally, they ignore the realized return. This aspect of the way buyers update their beliefs means that there are losses from reputation.

When $\mu_1 \in [0, \mu_2^*]$, selling has a static cost, i.e. $\hat{p}(\mu_2) - q \leq \bar{\pi}\bar{v} - q(1+r) - \bar{c}$ as well as a loss from reputation, i.e. $\Delta^b(\mu_1) < 0$ so that the HH bank prefers to hold the asset. When

$\mu_1 \in (\mu_2^*, 1]$, there are benefits from selling the asset, i.e. $\hat{p}(\mu_2) - q \geq \bar{\pi}\bar{v} - q(1+r) - \bar{c}$, while there is a loss from reputation $\Delta^b(\mu_1) < 0$. Assumption 1 ensures that when $\mu_1 = 1$, the static benefit outweighs the loss from reputation, i.e. (9) is reversed at $\mu_1 = 1$. Moreover, by Lemma 4, $(\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi})\,\bar{v} - q + \Delta^b(\mu_1)$ is a strictly convex function of μ_1 for $\mu_1 \in [\mu_2^*, 1]$. Since the value of this function is strictly less than $\bar{\pi}\bar{v} - q(1+r) - \bar{c}$ at $\mu_1 = \mu_2^*$ and weakly higher when $\mu_1 = 1$, there exist a unique $\bar{\mu} \in (\mu_2^*, 1]$, such that (9) holds if and only if $\mu_1 \leq \bar{\mu}$.

We have proved the following proposition.

Proposition 5 *(Multiplicity of Equilibria) Suppose Assumption 1 is satisfied. Then, there exists $\underline{\mu}$ and $\bar{\mu}$ with $\underline{\mu} < \mu_2^* < \bar{\mu}$ such that*

1. *if $\mu_1 < \underline{\mu}$, the game has a unique equilibrium in which the HH bank holds its loan in period 1,*
2. *if $\mu_1 \geq \bar{\mu}$, the game has a unique equilibrium in which the HH bank sells its loan in period 1,*
3. *if $\mu_1 \in [\underline{\mu}, \bar{\mu})$, the game has two equilibria: in one the HH bank sells its loan, and in the other the HH bank holds its loan.*

Next we provide a partial characterization of the set of equilibria when we relax Assumption 1. We show that even when this assumption is relaxed, the game has a region of multiplicity near μ_2^*. We have also shown that multiplicity can arise for values of μ close to 1. Details are available upon request.

Proposition 6 *(Region of Multiplicity). There exist $\underline{\mu}$ and $\bar{\mu}$ with $\underline{\mu} < \mu_2^* < \bar{\mu}$ such that if $\mu_1 \in [\underline{\mu}, \bar{\mu})$, the game has two equilibria: in one the HH bank sells its loan, and in the other the HH bank holds its loan.*

Therefore, we have shown that introducing reputation as a device for mitigating lemons problems results in equilibrium multiplicity, that is, reputation can both be a blessing and a curse. The game has a positive reputational equilibrium in which, encouraged by reputational incentives, banks with high quality asset sell their asset. In other words, reputation helps sustain market activity in an otherwise illiquid market. The game also has a negative reputational equilibrium in which reputation discourages selling banks with high quality asset hold their asset.

In the positive reputational equilibrium, period 2 buyers believe that the high quality-high cost bank sells its asset. This belief implies that, on average future buyers increase the bank's reputation upon observing a sale while reputation is unchanged after a hold decision. Therefore, selling a high quality asset in the first period implies a future benefit for the bank. Given that the static costs of selling the asset are not high enough, reputation can induce high trade volume in the secondary loan market.

In the negative reputational equilibrium, future buyers believe that the high quality-high cost bank holds on to its asset. Therefore, after observing a sale, they will be certain that the bank is of low quality, while a hold decision increases the bank's reputation. This way of forming beliefs implies that the bank has a future loss from selling the asset. Hence, reputational incentives exacerbate the lemons problem and cause market failure in an otherwise liquid market.

As we have noted in Chari et al. (2009), we think of this multiplicity as arising from a coordination problem between future buyers and current banks, and hence, we refer to it as a *Dynamic Coordination Problem.* To see the sense in which lack of coordination leads to the multiplicity, suppose that in period 1, period 2 buyers could commit to buy the asset in period 2 at pre-specified prices contingent on observed realizations of asset quality. Then a bank whose quality type is just below the quality threshold μ_2^* has an incentive to sell the asset in period 1. Such commitment would eliminate the multiplicity of equilibria. The unique equilibrium with commitment is the positive reputational equilibrium. This argument suggests that coordination failure is at the root of the multiplicity result. This interpretation helps us develop a refinement concept similar to that in the Global Games literature.

To draw an analogy to models of reputation as incomplete information, our model nests features of the model in Mailath and Samuelson (2001) and Ordoñez (2008) as well as that of Ely and Välimäki (2003). In Mailath and Samuelson (2001) and Ordoñez (2008), strategic types are good and want to separate from non-strategic types - though in Mailath and Samuelson (2001) reputation generally fails to deliver this type of equilibria. Nevertheless, in their environments, there is no long run reputational loss from good behavior. Ely and Välimäki (2003), share the property that strategic types are good and want to separate, however, structure of learning is such that good behavior never implies long-run positive reputational gains and therefore reputation exacerbates bad behavior in equilibrium.

3 Achieving Uniqueness with Imperfect Observability

In this section, we use a perturbation of the model described above in order to select a unique equilibrium. The method used is in the spirit of the global games literature as in Carlsson and Van Damme (1993), Morris and Shin (2003).

Our motive for performing this exercise is twofold. First we would like to understand how outcomes in the model respond to various kinds of policy interventions. As is typically he case in models with multiple equilibria, comparative static exercises are not meaningful. Therefore, we seek a refinement that allows us to select a unique equilibrium. Second, we want to establish a well defined notion of fragility. In many macroeconomic environments with multiple equilibria, small shocks to the environment can cause sudden changes in behavior. Without a proper selection device, multiplicity leads to a lack of discipline on how equilibrium behavior changes in response to shocks. Global games techniques, however, enable us to impose such discipline. We demonstrate the precise nature of fragility in our environment using the unique equilibrium selected by our perturbation described below.

Consider the following change in the environment described above: In each period $t = 1, 2$, an aggregate shock $\bar{c}_t \sim F(\bar{c}_t)$ with finite mean is drawn. These shocks are drawn independently across periods. The cost of financing the asset for high cost banks is given by $\bar{c}_t$. Agents, however, observe a noisy signal of $\bar{c}_t$ given by $c_t = \bar{c}_t + \sigma\varepsilon_t$ where $\varepsilon_t \sim G(\varepsilon_t)$ with $E[\varepsilon] = 0$ is i.i.d. across periods and $\sigma > 0$. We assume that F and G have full support over $\mathbb{R}$.

The timing of the game is as follows:

1. At the beginning of each period t, agents observe $\bar{c}_{t-1}$. Buyers do not observe previous period signals c_{t-1} or the market price p_{t-1}. (We believe that our uniqueness result goes through if future buyers receive a noisy signal about previous prices.)

2. The new aggregate state $\bar{c}_t$ is drawn, banks and current period buyers do not observe the current state, $\bar{c}_t$, but they do observe the noisy signal, c_t.

3. Buyer offer prices.

4. Banks decide whether to sell or hold and choose the best price offered.

5. Holding banks pay their interest cost and buyer pay the price.

. The ex-post profit of financing to a high cost high quality bank is

$$\bar{\pi}\bar{v} - q(1+r) - \bar{c}_t$$

Hence, when trade occurs, the payoff from financing to the bank when the signal agents observe c_t is

$$\bar{\pi}\bar{v} - q(1+r) - E[\bar{c}_t|c_t]$$

Notice that when $\sigma = 0$, agents learn the exact value of the aggregate shock in the each period. As a result, the bank's state in period 1 is common knowledge to all players in both periods. However, when $\sigma > 0$, the bank has a private state that is not known to period 2 players. This aspect of the perturbation gives us uniqueness of equilibrium. Indeed, when $\sigma = 0$, the model is very similar to the model introduced in the previous section. The only difference is that the high cost value is a given number in the previous section. Here, the high cost value is a random variable. The same argument as before can be used to show that the game always has multiple equilibria if $0 < \mu_2^*(\bar{c}_1) < 1$. Thus in all but exceptional cases, the game has multiple equilibria.

Proposition 7 *Suppose that $\sigma = 0$. For every $\bar{c}_1$ such that $0 < \mu_2^*(\bar{c}_1) < 1$, there exists functions $\underline{\mu}(\bar{c}_1)$ and $\bar{\mu}(\bar{c}_1)$ satisfying $\underline{\mu}(\bar{c}_1) < \mu_2^*(\bar{c}_1) < \bar{\mu}(\bar{c}_1)$ such that for all $\mu_1 \in [\underline{\mu}(\bar{c}_1), \bar{\mu}(\bar{c}_1)]$, there are two equilibria. In one equilibrium, the HH bank sells its asset in period 1 and equilibrium price is $p_1 = [\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\,\bar{v}(= \hat{p}(\mu_2))$. The other equilibrium involves the HH bank using financing in period 1 and equilibrium price is given by $p_1 = \underline{\pi}\bar{v}$.*

Proof. In the Appendix. ■

This proposition can be proved exactly the same way as in the previous section. In fact, when bank's signal in period 1 is common knowledge, agents are able to coordinate on both types of equilibria. Therefore, the model with $\sigma = 0$ suffers from the same Dynamic Coordination Problem as the model in the previous section.

When $\sigma > 0$, the updating rules for the signal are given by

$$\Pr(c_1 \leq \hat{c}|\bar{c}_1) = \Pr(\bar{c}_1 + \sigma\varepsilon_1 \leq \hat{c}) = G\left(\frac{\hat{c} - \bar{c}_1}{\sigma}\right)$$
$$\Pr(\bar{c}_1 \leq \hat{c}_1|c_1) = \frac{\int_{-\infty}^{\hat{c}_1} f(\bar{c}_1) g\left(\frac{c_1 - \bar{c}_1}{\sigma}\right) d\bar{c}_1}{\int_{-\infty}^{\infty} f(\bar{c}_1) g\left(\frac{c_1 - \bar{c}_1}{\sigma}\right) d\bar{c}_1} = H(\hat{c}_1|c_1)$$

Assumption 8 *(Monotone Likelihood Ratio) For any* $c_1 > c_1'$, $\frac{g(c_1 - \bar{c}_1)}{g(c_1' - \bar{c}_1)}$ *is increasing in* $\bar{c}_1$*.*

This assumption ensures us that $H(\bar{c}_1|c_1)$ is a decreasing function of c_1. We prove this in Lemma 13 in the Appendix. The assumption implies that when the signal about the liquidity shock, c_1, is high, aggregate liquidity shock, $\bar{c}_1$, is more likely to be high.

We will show that when $\sigma > 0$, the equilibrium is unique. We begin with the second period.

Proposition 9 *In the second period, given a reputation level* μ_2 *and cost signal* c_2*, there is a unique equilibrium outcome in which bank's decision is the following:*

$$a_2 = \begin{cases} 1 & \text{if } \mu_2 \geq \mu_2^*(c_2) \\ 0 & \text{if } \mu_2 < \mu_2^*(c_2) \end{cases}$$

and market price is given by

$$p_2 = \begin{cases} \left[\mu_2\bar{\pi} + (1 - \mu_2)\underline{\pi}\right]\bar{v} & \text{if } \mu_2 \geq \mu_2^*(c_2) \\ \underline{\pi}\bar{v} & \text{if } \mu_2 < \mu_2^*(c_2) \end{cases}$$

where

$$\mu_2^*(c_2) = \max\left\{\min\left\{1 - \frac{qr + c_2}{(\bar{\pi} - \underline{\pi})\bar{v}}, 1\right\}, 0\right\}$$

The equilibrium in the subgame in the second period is similar to the previous section. The payoff from financing to a HH bank is

$$\bar{\pi}\bar{v} - q(1 + r) - c_2$$

since $E\left[\bar{c}_2|c_2\right] = c_2$ and the payoff from selling is $\left[\mu_2\bar{\pi} + (1 - \mu_2)\underline{\pi}\right]\bar{v} - q$. $\mu_2^*(c_2)$ is defined as the value of reputation that makes the bank indifferent between selling and financing. The equilibrium in the sub-game implies the following value function for the bank:

$$\hat{V}_2(\mu_2, c_2) = \alpha\left[\bar{\pi}\bar{v} - q(1+r) - \underline{c}\right] + (1-\alpha)\max\{\left[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}\right]\bar{v} - q, \bar{\pi}\bar{v} - q(1+r) - c_2\}$$

and the ex-ante value of period 2 reputation is

$$V_2(\mu_2) = \int\int \hat{V}_2(\mu_2, c_2) dG\left(\frac{c_2 - \bar{c}_2}{\sigma}\right) dF(\bar{c}_2) \tag{10}$$

Proving that the perturbed game has a unique equilibrium, is easiest when F is an improper uniform distribution, $U[-\infty, \infty]$. However, an improper uniform implies that the ex-ante value function, $V_2(\mu_2)$, is not well-defined. To ease the exposition, in the next proposition we assume that $\bar{c}_t$ is distributed independently but not identically across periods. In particular, we assume $\bar{c}_1$ is drawn from an improper uniform distribution while $\bar{c}_2$ is drawn from a proper distribution F. In section 3.2, we state our result for the case where $\bar{c}_t$ is i.i.d across periods and F is a proper distribution. Notice that when, $\bar{c}_1$ is drawn from an improper uniform distribution,

$$H(\bar{c}_1|c_1) = G\left(\frac{c_1 - \bar{c}_1}{\sigma}\right)$$

The next proposition states our uniqueness result.

Proposition 10 *For each $\sigma > 0$ and $V_2(\mu_2)$ given by (10), the game with uniform improper priors has a unique equilibrium in which in period 1, HH bank's action is characterized by a cutoff $c_1^*(\sigma) \in \mathbb{R}$ and is given by*

$$a_1(c_1) = \begin{cases} 1 & \text{if } c_1 \geq c_1^*(\sigma) \\ 0 & \text{if } c_1 < c_1^*(\sigma) \end{cases}$$

while the market price is given by

$$p_1^*(c_1) = \begin{cases} \left[\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi}\right]\bar{v} & \text{if } c_1 \geq c_1^*(\sigma) \\ \underline{\pi}\bar{v} & \text{if } c_1 < c_1^*(\sigma) \end{cases}$$

We prove this proposition using a similar method to the original paper of Carlsson and Van Damme (1993). We begin by restricting attention to switching strategies in which the bank sells for all costs above a threshold and holds for all costs below that threshold. We show that the game has a unique equilibrium in switching strategies. We then prove that the equilibrium switching strategy is the only strategy that survives iterated elimination of strictly dominant strategies so that we have a unique equilibrium.

The intuition for the iterated elimination argument is as follows. Note that we can define equilibrium as a strategy for period 1 bank and a belief - about banks' action in period 1 - by period 2 buyer used for Bayesian updating. In equilibrium beliefs have to coincide with strategies. Obviously reputational incentives depend on future buyers' belief. When c_1 is very large, independent of future buyers' belief, an HH bank sells. Similarly, when c_1 is very low, an HH bank holds onto the asset, independent of future beliefs. This establishes two bounds $\hat{c}^1 > \tilde{c}^1$, such that any equilibrium strategy must prescribe a sale for c_1 higher than $\hat{c}^1$ and financing for c_1 lower than $\tilde{c}^1$. This means that the set of beliefs by future buyers have to satisfy the same property. Limiting the set of beliefs puts tighter upper and lower bounds on reputational incentives, which in turn implies new bounds $\hat{c}^2 > \tilde{c}^2$. We will show that iterating in this manner implies that the bounds $\hat{c}^n$ and $\tilde{c}^n$ converge to a common limit.

Here we sketch the steps of the proof and leave the details to the Appendix.

3.1 Outline of Proof with Improper Priors

1. **Unique Equilibrium in Switching Strategies**

   (a) **Switching Strategies**

   Restrict attention to Bank strategies of the following form:

   $$d_k(c_1) = \begin{cases} 1 & c_1 \geq k \\ 0 & c_1 < k \end{cases}$$

   where k represents the switching point. We want to characterize the best response of the HH bank when future buyers use d_k to form their posterior over the banks type. To do so, we must define Bayesian updating.

   (b) **Bayesian Updating**

   Consider an arbitrary belief $\hat{a}_1(\cdot)$ by period 2 buyers about the HH bank's period 1 action. Based on the observed history and signal $\bar{c}_1$, Bayesian updating implies the following rules:

$$\begin{aligned}
\mu_{s\bar{v}}(\bar{c}_1;\hat{a}_1) &= \frac{\mu_1\bar{\pi}\int \hat{a}_1(c_1)dG\left(\frac{c_1-\bar{c}_1}{\sigma}\right)}{\mu_1\bar{\pi}\int \hat{a}_1(c_1)dG\left(\frac{c_1-\bar{c}_1}{\sigma}\right)+(1-\mu_1)\underline{\pi}} \\
\mu_{s0}(\bar{c}_1;\hat{a}_1) &= \frac{\mu_1(1-\bar{\pi})\int \hat{a}_1(c_1)dG\left(\frac{c_1-\bar{c}_1}{\sigma}\right)}{\mu_1(1-\bar{\pi})\int \hat{a}_1(c_1)dG\left(\frac{c_1-\bar{c}_1}{\sigma}\right)+(1-\mu_1)(1-\underline{\pi})} \\
\mu_h(\bar{c}_1;\hat{a}_1) &= \frac{\mu_1\left[(1-\alpha)\int\left[1-\hat{a}_1(c_1)\right]dG\left(\frac{c_1-\bar{c}_1}{\sigma}\right)+\alpha\right]}{\mu_1\left[(1-\alpha)\int\left[1-\hat{a}_1(c_1)\right]dG\left(\frac{c_1-\bar{c}_1}{\sigma}\right)+\alpha\right]+(1-\mu_1)\alpha}
\end{aligned}$$

For switching strategies, these formulas simplify to

$$\begin{aligned}
\mu_{s\bar{v}}(\bar{c}_1;d_k) &= \frac{\mu_1\bar{\pi}\left[1-G\left(\frac{k-\bar{c}_1}{\sigma}\right)\right]}{\mu_1\bar{\pi}\left[1-G\left(\frac{k-\bar{c}_1}{\sigma}\right)\right]+(1-\mu_1)\underline{\pi}} \\
\mu_{s0}(\bar{c}_1;d_k) &= \frac{\mu_1(1-\bar{\pi})\left[1-G\left(\frac{k-\bar{c}_1}{\sigma}\right)\right]}{\mu_1(1-\bar{\pi})\left[1-G\left(\frac{k-\bar{c}_1}{\sigma}\right)\right]+(1-\mu_1)(1-\underline{\pi})} \\
\mu_h(\bar{c}_1;d_k) &= \frac{\mu_1\left[(1-\alpha)G\left(\frac{k-\bar{c}_1}{\sigma}\right)+\alpha\right]}{\mu_1\left[(1-\alpha)G\left(\frac{k-\bar{c}_1}{\sigma}\right)+\alpha\right]+(1-\mu_1)\alpha}
\end{aligned}$$

(c) **Gain from Reputation**

Given any belief $\hat{a}_1$, we define gain from reputation as

$$\Delta(c_1;\hat{a}_1)=\beta\int\left[\bar{\pi}V_2(\mu_{s\bar{v}}(\bar{c}_1;\hat{a}_1))+(1-\bar{\pi})V_2(\mu_{s0}(\bar{c}_1;\hat{a}_1))-V_2(\mu_h(\bar{c}_1;\hat{a}_1))\right]dG\left(\frac{c_1-\bar{c}_1}{\sigma}\right)$$

In the appendix we show that $\Delta(c_1;\hat{a}_1)$ is bounded and strictly increasing in $\hat{a}_1$ according to a point-wise ordering on beliefs. Moreover, when $\hat{a}_1$ is a switching strategy, $\Delta(c_1;\hat{a}_1)$ is strictly increasing in c_1. In particular, we show that if $\hat{a}_1$ is a switching strategy, d_k, then $\Delta(c_1;d_k)$ is strictly decreasing in k.

Facing a switching strategy belief of future buyers, d_k, clearly, the HH bank sells if and only if

$$\hat{p}(\mu_1)-q+\Delta(c_1;d_k)\geq\bar{\pi}\bar{v}-q(1+r)-c_1.$$

(d) **Equilibrium in Switching Strategies**

Since the value of selling, $\hat{p}(\mu_1) - q + \Delta(c_1; d_k)$, is increasing in c_1, while the value of financing, $\bar{\pi}\bar{v} - q(1+r) - c_1$, is decreasing in c_1, there exists a unique solution, $b(k)$, that solves the equation

$$\hat{p}(\mu_1) - q + \Delta(b(k); d_k) = \bar{\pi}\bar{v} - q(1+r) - b(k).$$

Hence, the best response of the HH bank to a switching strategy belief of future buyers, d_k, is a switching strategy, $d_{b(k)}$ in which the bank sells for all costs above $b(k)$ and holds for all costs below $b(k)$. An equilibrium in switching strategies must be a fixed point of the above equation, so an equilibrium switching point, k^* satisfies

$$\hat{p}(\mu_1) - q + \Delta(k^*; d_{k^*}) = \bar{\pi}\bar{v} - q(1+r) - k^*.$$

In the Appendix, we prove that $b(k)$ has a unique fixed point c_1^* which is globally stable. Hence, the game with switching strategies has a unique equilibrium.

2. **Restriction to Switching Strategies is Without Loss of Generality**

(a) **Limit Dominance Regions**

We show that regardless of future buyers belief functions, the bank has a dominant strategy for extreme values of costs. Consider two numbers $\hat{c} < \tilde{c}$. We define an *extreme monotone strategy* to be a strategy that calls for selling when $c_1 \geq \tilde{c}$ and financing for $c_1 \leq \hat{c}$. We define $A_{\hat{c},\tilde{c}}$ to be the set of such strategies. Notice that $A_{-\infty,\infty}$ is the set of all strategies Moreover, for future reference, we define the Best Response set operator on a subset of beliefs, A, as

$$BR(A) = \{a_1 | \exists \hat{a}_1 \in A; a_1(c_1) = 1 \Leftrightarrow \hat{p}(\mu_1) - q + \Delta(c_1; \hat{a}_1) \geq \bar{\pi}\bar{v} - q(1+r) - c_1\}$$

Given that the gain from reputation is bounded above and below, and since support of c_1 is $\mathbb{R}$, we show that there exist bounds $\hat{c}^0 < \tilde{c}^0$ such that the HH bank uses financing for $c_1 \leq \hat{c}^0$ and it sells the asset for $c_1 \geq \tilde{c}^0$, independent of future buyers' belief function

$\hat{a}_1$. That is

$$\begin{aligned} \forall \hat{a}_1, c_1 &\geq \tilde{c}^0; \left[\mu_1 \bar{\pi} + (1-\mu_1)\underline{\pi}\right] \bar{v} - q + \Delta(c_1; \hat{a}_1) \geq \bar{\pi}\bar{v} - q(1+r) - c_1 \\ \forall \hat{a}_1, c_1 &\leq \hat{c}^0; \left[\mu_1 \bar{\pi} + (1-\mu_1)\underline{\pi}\right] \bar{v} - q + \Delta(c_1; \hat{a}_1) \leq \bar{\pi}\bar{v} - q(1+r) - c_1 \end{aligned}$$

We have established that any equilibrium strategy must be an extreme monotone strategy with cutoffs $\hat{c}^0 < \tilde{c}^0$. That is,

$$BR(A_{-\infty,\infty}) \subseteq A_{\hat{c}^0,\tilde{c}^0}.$$

Thus, we can restrict attention to extreme monotone strategies without loss of generality.

(b) **Bounding Best Response Sets.**

We show that the best response set operator is decreasing in the sense that it induces a best response set which is a strict subset of any arbitrary set of extreme monotone beliefs. Repeatedly applying this operator induces a decreasing sequence of sets which converges to a unique equilibrium.

To show that the best response set operator is decreasing, we show that for any $\hat{c} < \tilde{c}$, $BR(A_{\hat{c},\tilde{c}}) \subseteq A_{b(\hat{c}),b(\tilde{c})} \subset A_{\hat{c},\tilde{c}}$. Since $\Delta(c_1; \hat{a}_1)$ is increasing in $\hat{a}_1$, for all $\hat{a}_1 \in A_{\hat{c},\tilde{c}}$ we have

$$\hat{p}(\mu_1) - q + \Delta(c_1; d_{\tilde{c}}) \leq \hat{p}(\mu_1) - q + \Delta(c_1; \hat{a}_1) \leq \hat{p}(\mu_1) - q + \Delta(c_1; d_{\hat{c}})$$

because $\hat{a}_1$ first order stochastically dominates $d_{\tilde{c}}$ is dominated by $d_{\hat{c}}$. This implies that

$$\bar{\pi}\bar{v} - q(1+r) - c_1 \geq \hat{p}(\mu_1) - q + \Delta(c_1; \hat{a}_1)$$

if

$$\bar{\pi}\bar{v} - q(1+r) - c_1 \geq \hat{p}(\mu_1) - q + \Delta(c_1; d_{\hat{c}})$$

This means that the best response to $\hat{a}_1$ must call for financing whenever best response to $d_{\hat{c}}$ or $d_{b(\hat{c})}$ calls for financing. That is if a_1 is the best response to $\hat{a}_1$, we must have

$$\forall c_1 < b(\hat{c}), \quad a_1(c_1) = 0$$

Similarly, we can show that the best response to $\hat{a}_1$ must satisfy $a_1(c_1) = 1$ for all $c_1 \geq b(\tilde{c})$. This proves $BR(A_{\hat{c},\tilde{c}}) \subseteq A_{b(\hat{c}),b(\tilde{c})}$. Furthermore, $A_{b(\hat{c}),b(\tilde{c})} \subset A_{\hat{c},\tilde{c}}$ follows from global stability of the fixed point of $b(k)$. Finally, $BR^n(A_{-\infty,\infty}) \to A_{k^*,k^*} = \{d_{k^*}\}$ because $b(k)$ has a unique fixed point.

3.2 Uniqueness Result with Proper Priors

In this section, we provide a characterization of equilibria in the limiting perturbed game with general proper priors. In particular, we prove that in the perturbed game as $\sigma \to 0$, the set of period 1 equilibrium strategies converges to a unique strategy. We use the method of Laplacian beliefs introduced by Frankel et al. (2003) and reviewed by Morris and Shin (2003) to show our uniqueness result. In fact we show that the game described above is equivalent to a game discussed by Morris and Shin (2003). We then use their result to prove the following theorem:

Theorem 11 *Given the value function $V_2(\mu_2)$ given by (10), as $\sigma \to 0$ the set of first period equilibrium strategies in the game with proper priors converges to a unique strategy by the HH bank given by*

$$a_1(c_1) = \begin{cases} 1 & \text{if } c_1 \geq c_1^* \\ 0 & \text{if } c_1 < c_1^* \end{cases}$$

where c_1^* satisfies:

$$\hat{p}(\mu_1) - q + \beta \int_0^1 \left[\bar{\pi} V_2\left(\hat{\mu}_{s\bar{v}}\left(l\right)\right) + (1-\bar{\pi}) V_2\left(\hat{\mu}_{s0}\left(l\right)\right) - V_2\left(\hat{\mu}_h\left(l\right)\right)\right] dl = \bar{\pi}\bar{v} - q(1+r) - c_1^*$$

and

$$
\begin{aligned}
\hat{\mu}_{s\bar{v}}(l) &= \frac{\mu_1 \bar{\pi} l}{\mu_1 \bar{\pi} l + (1-\mu_1)\underline{\pi}} \\
\hat{\mu}_{s0}(l) &= \frac{\mu_1 (1-\bar{\pi}) l}{\mu_1 (1-\bar{\pi}) l + (1-\mu_1)(1-\underline{\pi})} \\
\hat{\mu}_{h}(l) &= \frac{\mu_1 \left[(1-\alpha)(1-l)+\alpha\right]}{\mu_1 \left[(1-\alpha)(1-l)+\alpha\right] + (1-\mu_1)\alpha}
\end{aligned}
$$

3.3 Fragility

We think of equilibrium outcomes as *fragile,* in three ways. One notion of fragility is simply that the economy has multiple equilibria so that sunspot-like fluctuations can induce changes in outcomes. A second notion of fragility is that small changes in fundamentals induce large changes in outcomes for a given decision maker. Third, small changes in fundamentals induce large changes in outcomes for a large number of decision makers.

Equilibrium outcomes in our unperturbed game are clearly fragile under the first notion because that game has multiple equilibria. They are also fragile under the second notion if agents in the model coordinate on different equilibria depending on the realization of the fundamental. Finally they are fragile under the third notion if all agents coordinate changes in the same way and do so discontinuously with the fundamentals.

We will argue that equilibrium outcomes in our perturbed game are fragile under the second notion. In the game with $\sigma > 0$, if we think of the fundamental as the signal, c_1, the equilibrium action of the bank changes discontinuously at $c_1^*(\sigma)$ so that the equilibrium is fragile under the second notion.

Consider the limiting case as $\sigma \to 0$. The limiting equilibrium can be thought of as a refinement concept for equilibria of the unperturbed game. The unique equilibrium which survives our refinement is sensitive to small changes in the costs of holding loans. Formally, as $\sigma \to 0$, the switching point of the equilibrium strategies in period 1, $c_1^*(\sigma)$, converges, say to c_1^*. In the limiting equilibrium, when $\bar{c}_1 < c_1^*$, the unique equilibrium strategy calls for the HH bank to hold its loan in period 1, for buyers to offer a price $p = \underline{\pi}\bar{v}$ in period 1, and for buyers in period 2 to decrease the reputation of any bank that sells in period 1 to 0 while increasing the reputation of any bank that sells in period 1. When $\bar{c}_1 \geq c_1^*$, the unique equilibrium strategy calls for the HH bank to sell its loan in period 1, for buyers to offer a price $\hat{p}(\mu_1)$in period 1, and for buyers in period 2 to reward high returns after a sale with a higher reputation, punish low returns after a sale with a lower reputation, and treat hold decisions as uninformative. As a comparative statics exercise, small changes in $\bar{c}_1$ away from the switching point, c_1^*, yield no change in period 1 equilibrium outcomes. Near c_1^*, however, small changes in $\bar{c}_1$ can potentially cause an abrupt change in equilibrium behavior. That is, a small change in $\bar{c}_1$ can induce the buyers to switch between offering high prices to low prices and cause the HH bank to switch from selling to holding its loan.

This sensitivity of the equilibrium outcome to small changes in bank costs of holding loans implies that reputation, as a mechanism to sustain non-opportunistic behavior is fragile under the second notion.

One concern with our argument that reputation induces fragility is that even in the static environment without any reputational concerns, equilibrium outcomes can change discontinuously in response to changes in holding costs. This feature of our static model arises because in our model, banks are restricted to a discrete choice set: sell or hold. We conjecture that in a version of our static model where banks can choose the proportion of their portfolio to sell, reputation will induce fragility. Notice in this version of our static model, banks choices will typically be continuous in their holding costs. We make this conjecture because in related work (Chari et al. (2009)) we have shown that reputation induces fragility under the second notion even when choices are continuous.

Finally, some numerical exercises we have conducted suggest that equilibrium outcomes in the perturbed game are also fragile under the third notion. To motivate these numerical exercises, consider the limit of our perturbed game as $\sigma \to 0$. In this limit, a bank with a given reputation level, μ_1, has a critical threshold c_1^* above which the bank sells and below which the bank holds. In Figure 3, we plot reputation levels against their associated critical threshold c_1^* for the limiting game as well as the analogous plot for the static model.

Consider a small change in the cost level from $\hat{c}$ to $\tilde{c}$. The figure shows that banks with reputation levels in a larger set changes their behavior in the limiting game than in the static game. In particular, the figure shows that in the limiting game, banks with reputation levels between $\tilde{\mu}$ and $\hat{\mu}$ change their decision while in the static game banks with reputation levels between $\tilde{\mu}_s$ and $\hat{\mu}_s$ change their decision. We have proved analytical results which suggests that the shape of the relationship between costs and reputation in our perturbed game is likely to be similar to that

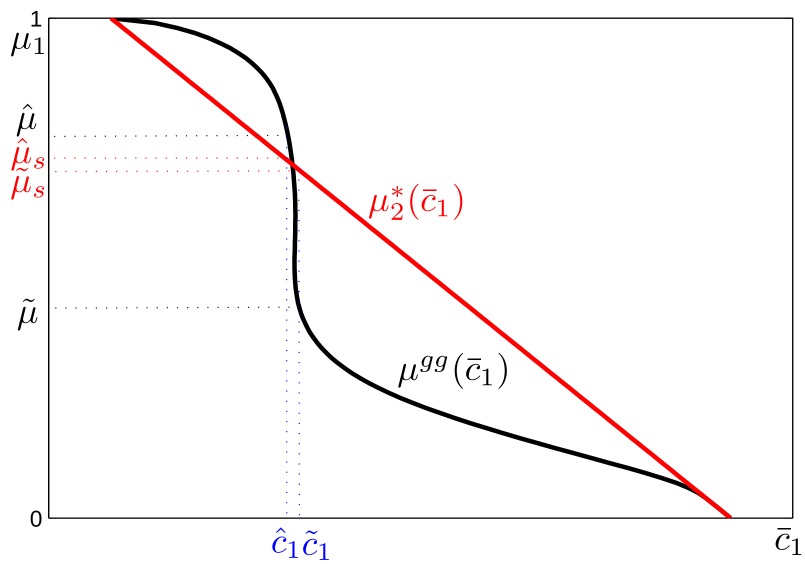

Figure 3: Static Cutoff vs. Dynamic Cutoff in the Perturbed Game

displayed in the figure. Specifically, we have proved that the slope of this graph at highest level of reputation, $\mu_1 = 1$, and the lowest level of costs is necessarily zero. Moreover, in the Appendix, we prove that when the probability that the bank draws a low value of cost, α, is not too low, the slope of this graph is less than the slope of the graph associated with the static game. That is, the graph of the limiting threshold lies above that of the static threshold for high values of reputation and lies below for low values of reputation. Thus, for an intermediate range of reputations, the limiting threshold must be steeper than the static threshold. In this sense, a given change in costs induces banks in a larger range of reputation levels to switch their behavior in the limiting game than in the static game, so that we have fragility under our third notion.

4 Policy Exercises

In this section, we use our model to evaluate the effects of various policies intended to remedy problems credit markets that have been proposed since the 2007 collapse of secondary loan markets in the U.S. We focus on the effects of policies in which the government would purchase asset backed securities at prices above existing market value, such as the Public-Private Partnership plan as well as on policies which decreased the costs of holding loans to maturity, including changes in the Fed

Funds target rate, the Term Asset-Backed Securities Loan Facility (TALF), and increased FDIC insurance.

We first consider policies in which the government attempts to purchase so-called toxic assets at above-market values. Consider the following government policy in the limiting version of the perturbed game as $\sigma \to 0$. The government offers to buy the asset at some price p in the first period.

Suppose first, that $p \leq \hat{p}(\mu_1)$. We claim that the unique equilibrium without government is also the unique equilibrium with this government policy. To see this claim, note that the equilibrium in the second period is the same with and without the government policy so that the reputational gains are the same with and without government policy. Consider the first period and a realization of first period costs $c_1 < c_1^*$. In the game without the government, the HH bank found it optimal not to sell at a price $\hat{p}(\mu_1)$. Since the reputational gains are the same with and without the government policy, in the game with the government, it is also optimal for the HH not to sell at this price. A similar argument implies that the equilibrium strategy of the HH bank is unchanged for $c_1 > c_1^*$. Thus, this government policy has no effect on the equilibrium strategy of the HH bank. Of course, under this policy, the government ends up buying the asset from low quality banks. The only effect of this policy is to make transfers to low quality banks.

Suppose next that the price set by the government, p, is sufficiently larger than $\hat{p}(\mu_1)$.Then, the HH bank will find it optimal to sell and will enjoy the reputational gain associated with a policy of selling. In this sense, if the government offers a sufficiently high price, it can ensure that reputational incentives work to overcome adverse selection problems. Note however that this policy necessarily implies that the government must earn negative profits.

Consider now a policy which reduces interest rates in period 1 and leaves period 2 interest rates unchanged. We begin the analysis with the unperturbed game. Such a policy increases the static payoff in period 1 from holding loans which worsens the static incentives for the HH bank to sell its loan. Specifically, this policy raises both the threshold $\underline{\mu}$ below which banks find it optimal to hold in the positive reputational equilibrium and the threshold $\bar{\mu}$ below which banks find it optimal to hold their loans in the negative reputational equilibrium. Thus, this policy serves only to aggravate the lemons problem in secondary loans markets.

Consider next a policy under which the government commits to reducing period 2 interest rates but leaves period 1 interest rates unchanged. Obviously, this policy increases incentives for banks to hold their loans in period 2 and thereby increases the threshold below which banks hold their loans, μ_2^*. In this sense, it makes period 2 allocations less efficient. We will show that this policy reduces the region of multiplicity in period 1 and in this sense can improve period 1 allocations. To show the reduction in the region of multiplicity, consider the reputational gain in the positive reputational equilibrium evaluated at $\underline{\mu}$:

$$\beta\left(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0}) - V_2(\mu_h)\right)$$

Using 4, it is straightforward to see that an arbitrarily small reduction in interest rates of dr in period 2 reduces $V_2(\mu_{s\bar{v}})$ by $\alpha q dr$ since $\mu_{s\bar{v}} > \mu_2^*$. Moreover, since μ_{s0} and μ_h are strictly less than μ_2^*, $V_2(\mu_{s0})$ and $V_2(\mu_h)$ fall by qdr. As a result, the reputational gain falls by $\beta\bar{\pi}(1-\alpha)qdr$. This decline in reputational gain induces an increase in the threshold $\underline{\mu}$. Similarly, we can show that the policy induces a fall in the threshold $\bar{\mu}$. Thus, the region of multiplicity shrinks and in this sense can improve period 1 allocations. Interestingly, such a policy is time inconsistent because the government has a strong incentive in period 2 not to make period 2 allocations less efficient.

An alternative policy which has not been proposed is to consider forced asset sales in which the government randomly forces banks to sell their loan. Such a policy in our model would mitigate the lemons problem in secondary loan markets by generating a pool of loans in secondary markets consistent with the ex-ante mix of loan types. While this is a standard intervention directed at increasing the price and volume of trade in markets that suffer from adverse selection, in our model such an intervention comes at the cost of misallocating loans to those without comparative advantage. Specifically, some banks with low costs of holding loans will be forced to sell to the marketplace.

As our discussion of the Dynamic Coordination Problem suggests, a policy under which the government commits to purchase assets in period 2 at prices which are contingent on the realization of the signals can eliminate the multiplicity of equilibria and support the positive reputational equilibrium. While such a policy would be desirable, the feasibility of such a policy can only be analyzed by developing a model in which private agents cannot commit but the government can.

5 Conclusion

We have argued that reputational incentives can be both a blessing and a curse. Elsewhere, Chari et al. (2009) we have constructed a model of defaults in which reputational incentives can lead to multiple equilibria. Our findings suggest that this observation may hold in a larger class of models. We have argued that the source of multiplicity in reputational models of the kind considered here lies in a Dynamic Coordination Problem and have shown how we can use techniques adapted from the literature on global games to select a unique, fragile equilibrium. In adapting these techniques, we have assumed that the source of randomness lies in costs. It is tedious, but relatively straightforward to adapt our technique to an environment where the source of randomness lies in default risk.

We have applied the observation that reputational incentives can operate in both beneficial and perverse ways to account for sudden large fluctuations in the volume of new issues in the secondary loan market. We have shown that a model that can account for these fluctuations can be used to analyze the effects of various policies. In particular, we have shown that asset purchase policies necessarily imply negative profits for the government and that it is possible to find a high enough price to eliminate the multiplicity of equilibria and to mitigate the adverse selection problem.

Appendix

A Proofs

A.1 Proof of Lemma 3

We first show that when $\mu_1 \leq \mu_g$, $\Delta^g(\mu_1) = 0$. Notice that in this case, $\mu_{s\bar{v}}, \mu_{s0}, \mu_h \leq \mu_2^*$. Since $V_2(\mu_2)$ is a flat function for $\mu_2 \leq \mu_2^*$, $\Delta^g(\mu_1)$ must be zero. Moreover, when $\mu_1 = 1$, we have $\mu_{s\bar{v}} = \mu_{s0} = \mu_h = 1$ and therefore, $\Delta^g(\mu_1) = 0$.

Now suppose $\mu_1 \in (\mu_g, 1)$. From Bayesian updating formulas we have

$$\frac{\bar{\pi}}{\mu_{s\bar{v}}} + \frac{1-\bar{\pi}}{\mu_{s0}} = \frac{1}{\mu_h} = \frac{1}{\mu_1}$$

Let $\hat{V}(x) = V_2(\frac{1}{x})$. Since $V_2(\cdot)$ and $\frac{1}{x}$ are convex functions, $\hat{V}(x)$ is a convex function. Moreover, when $x \in [1, \frac{1}{\mu_2^*}]$, it is strictly convex. Note that when $\mu_1 \in (\mu_g, 1)$, $\mu_{s\bar{v}} > \mu_2^*$ and $\mu_{s\bar{v}} > \mu_{s0}$. Therefore, we must have

$$\bar{\pi}\hat{V}\left(\frac{1}{\mu_{s\bar{v}}}\right) + (1-\bar{\pi})\hat{V}\left(\frac{1}{\mu_{s0}}\right) > \hat{V}(\frac{1}{\mu_h})$$

and hence,

$$\bar{\pi}V_2\left(\mu_{s\bar{v}}\right) + (1-\bar{\pi})V_2\left(\mu_{s0}\right) > \hat{V}(\mu_h)$$

This establishes the claim.

A.2 Proof of Lemma 4

Note that $V_2(\mu_2)$ is linear and strictly increasing when $\mu_2 \in [\mu_2^*, 1]$ and constant for $\mu_2 \in [0, \mu_2^*]$. Moreover, $\mu_1 \leq \mu_b$ if and only if, $\mu_h \leq \mu_2^*$. Therefore, $\Delta^b(\mu_1) = \beta(V_2(0) - V_2(\mu_h))$ is negative when $\mu_1 > \mu_b$ and zero when $\mu_1 \leq \mu_b$. In addition, since $\mu_h = \frac{\mu_1}{\mu_1 + (1-\mu_1)\alpha}$ is strictly concave in μ_1 and $V_2(\mu_2)$ is linear and strictly increase, $V_2(\mu_h)$ is a strictly concave function of μ_1 for $\mu_1 \in [\mu_b, 1]$ and therefore, $\Delta^b(\mu_1)$, is strictly convex for $\mu_1 \in [\mu_b, 1]$.

A.3 Proof of Theorem 11

We prove theorem 3 by mapping our environment into the environment described in Morris and Shin (2003) and show that their requirements for existence of a unique equilibrium in the limit are satisfied.

Given a value function $V_2(\mu_2)$, consider an equilibrium strategy profile in the first period $(a_1(\cdot), \hat{a}_1(\cdot), p_1)$. In a game with full information about liquidity costs, when agents in period 2 believe that HH bank sells with probability l in the first period 1, the HH bank's differential gain from selling is given by

$$\hat{\pi}(c_1, l) = \hat{p}(\mu_1) + qr + c_1 - \bar{\pi}\bar{v} + \beta\left[\bar{\pi}V_2(\hat{\mu}_{s\bar{v}}(l)) + (1-\bar{\pi})\, V_2(\hat{\mu}_{s\bar{v}}(l)) - V_2(\hat{\mu}_h(l))\right]$$

Then, in the game with private information, $l = \int \hat{a}_1(c_1)dH(c_1|\bar{c}_1)$ is a random variable. We, then, show that $\hat{\pi}$ satisfies the conditions A1-A3, A4*, A5, and A6. We then can apply Proposition 2.2 and that complete the proof of Theorem 11. It is easy to see that $\hat{\mu}_{s\bar{v}}(l)$ and $\hat{\mu}_{s0}(l)$ are increasing in l and $\hat{\mu}_h(l)$ is decreasing in l. Since, $V_2(\mu_2)$ is non-decreasing in μ_2, $\hat{\pi}(c_1, l)$ is non-decreasing in l - condition A1. Obviously $\hat{\pi}(c_1, l)$ is increasing in c_1- condition A2. Since $\hat{\pi}(c_1, l)$ is separable in c_1 and l, and $\hat{\pi}(c_1, l)$ is linearly increasing in c_1, there must exist a unique c_1^* such that $\int \hat{\pi}(c_1^*, l)dl = 0$ - condition A3. Since $V_2(\mu_2)$ is a continuous function over a compact set $[0, 1]$, $\beta\left[\bar{\pi}V_2(\hat{\mu}_{s\bar{v}}(l)) + (1-\bar{\pi})\, V_2(\hat{\mu}_{s\bar{v}}(l)) - V_2(\hat{\mu}_h(l))\right]$ is a bounded above and below by $\underline{\Delta}$ and $\bar{\Delta}$, respectively. Now let

$$\begin{aligned} \underline{c}_1 &= -\hat{p}(\mu_1) - qr + \bar{\pi}\bar{v} - \bar{\Delta} - \varepsilon \\ \hat{c}_1 &= -\hat{p}(\mu_1) - qr + \bar{\pi}\bar{v} - \underline{\Delta} + \varepsilon \end{aligned}$$

Then, if $c_1 \le \underline{c}_1$, $\hat{\pi}(c_1, l) \le -\varepsilon$ for all $l \in [0, 1]$. Moreover, if $c_1 \ge \hat{c}_1$, $\hat{\pi}(c_1, l) \ge -\varepsilon$ for all $l \in [0, 1]$ - condition A4*. Continuity of V_2 implies that $\hat{\pi}(c_1, l)$ is a continuous function of c_1 and l. Therefore, $\int_0^1 g(l)\hat{\pi}(c_1, l)dl$ is a continuous function of $g(\cdot)$ and c_1 - condition A5. Moreover, by definition of $F(\cdot)$ and $G(\cdot)$, noisy signal c_1 has a finite expectation, $E[c_1] \in \mathbb{R}$ - condition A6. Therefore, we can rewrite proposition 2.2 in Morris and Shin (2003) for our environment:

Proposition 12 *Let c_1^* satisfy $\int \hat{\pi}(c_1^*, l)dl = 0$. For any $\delta > 0$, there exists a $\bar{\sigma} > 0$ such that for* *all $\sigma \leq \bar{\sigma}$, if strategy a_1 survives iterated elimination of dominated strategies, then $a_1(c_1) = 1$ for all $c_1 \geq c_1^* + \delta$ and $a_1(c_1) = 0$ for all $c_1 \leq c_1^* - \delta$.*

This completes the proof of Theorem 11.

A.4 Equilibrium Strategies of the Perturbed Game

In this section we provide partial characterization for the equilibrium strategies of the perturbed game. We summarize the result in the following lemma:

Lemma 13 *Consider the cutoff $c_1^*(\mu_)$ in the unique equilibrium of the perturbed game and let $\mu^{gg}(\bar{c}_1) = (c_1^*)^(-1)(\bar{c}_1)$. Then*

1. $\frac{dc_1^*}{d\mu_1}|_{\mu_1=1} = \infty$. *Equivalently,* $\frac{d\mu^{gg}}{d\bar{c}_1}||_{\bar{c}_1=c_1^*(1)} = 0$
2. *If* $\frac{\bar{\pi}^2}{\underline{\pi}} + \frac{(1-\bar{\pi})^2}{(1-\underline{\pi})} > \frac{1+\alpha}{2\alpha}$, *then* $\frac{d\mu^{gg}}{d\bar{c}_1}|_{\mu_1=0} > \frac{d\mu_2^*}{d\bar{c}_1}|_{\mu_1=0}$.

Proof. To Be Completed. ■

A.5 Strategic Behavior by All Types

In this section, we study the game in which all types of banks are strategic. We provide conditions on fundamental parameters so that the constructed equilibrium strategies above for the HH bank and lenders together with imposed strategies on LH, HL, and LL banks, in fact, constitute an equilibrium in the new game. We show that we only need to assume that $\underline{c}$ is low enough. Notice first that the value function $V_2(\mu_2)$ is a linear function of $\underline{c}$ with slope $-\alpha$. This implies that for any μ, μ', $V_2(\mu) - V_2(\mu')$ is independent of $\underline{c}$. Therefore, the following expression is a well defined expression and independent of $\underline{c}$;

A.6 Other Proofs

Lemma 14 *If for any $c_1 > c_1'$, $\frac{g(c_1-\bar{c}_1)}{g(c_1'-\bar{c}_1)}$ is increasing in $\bar{c}_1$, then $H(\hat{c}_1|c_1)$ is a decreasing function of c_1 where $H(\hat{c}_1|c_1)$ is defined by*

$$H(\hat{c}_1|c_1) = \frac{\int_{-\infty}^{\hat{c}_1} f(\bar{c}_1) g\left(\frac{c_1-\bar{c}_1}{\sigma}\right) d\bar{c}_1}{\int_{-\infty}^{\infty} f(\bar{c}_1) g\left(\frac{c_1-\bar{c}_1}{\sigma}\right) d\bar{c}_1}.$$

Proof. First, note that if for any $c_1 > c_1'$, $\frac{g(c_1-\bar{c}_1)}{g(c_1'-\bar{c}_1)}$ is increasing in $\bar{c}_1$, then $\frac{g'(\cdot)}{g(\cdot)}$ is a decreasing function. Then,

$$\begin{aligned}
\frac{dH(\hat{c}_1|c_1)}{dc_1} &= \frac{1/\sigma}{\left(\int_{-\infty}^{\infty} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\right)^2}\left\{\int_{-\infty}^{\hat{c}_1} f(\bar{c}_1)g'\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\int_{-\infty}^{\infty} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\right. \\
&\quad \left. -\int_{-\infty}^{\hat{c}_1} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\int_{-\infty}^{\infty} f(\bar{c}_1)g'\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\right\} \\
&= \frac{1/\sigma}{\left(\int_{-\infty}^{\infty} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\right)^2}\left\{\int_{-\infty}^{\hat{c}_1} f(\bar{c}_1)g'\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\int_{\hat{c}_1}^{\infty} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\right. \\
&\quad \left. -\int_{-\infty}^{\hat{c}_1} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\int_{\hat{c}_1}^{\infty} f(\bar{c}_1)g'\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\right\}
\end{aligned}$$

Hence, to prove that $H(\hat{c}_1|c_1)$ is decreasing in c_1, is suffices to prove that

$$\int_{-\infty}^{\hat{c}_1} f(\bar{c}_1)g'\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\int_{\hat{c}_1}^{\infty} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1-\int_{-\infty}^{\hat{c}_1} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\int_{\hat{c}_1}^{\infty} f(\bar{c}_1)g'\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1 < 0$$

Note that

$$\begin{aligned}
&\int_{-\infty}^{\hat{c}_1} f(\bar{c}_1)g'\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\int_{\hat{c}_1}^{\infty} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1 \\
= &\int_{-\infty}^{\hat{c}_1} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)\frac{g'\left(\frac{c_1-\bar{c}_1}{\sigma}\right)}{g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)}d\bar{c}_1\int_{\hat{c}_1}^{\infty} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1 \\
< &\frac{g'\left(\frac{c_1-\hat{c}_1}{\sigma}\right)}{g\left(\frac{c_1-\hat{c}_1}{\sigma}\right)}\int_{-\infty}^{\hat{c}_1} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\int_{\hat{c}_1}^{\infty} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1
\end{aligned}$$

where the last inequality follows from the fact that $\frac{g'(\cdot)}{g(\cdot)}$ is a decreasing function. Similarly,

$$\begin{aligned}
&\int_{-\infty}^{\hat{c}_1} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\int_{\hat{c}_1}^{\infty} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1 \\
> &\frac{g'\left(\frac{c_1-\hat{c}_1}{\sigma}\right)}{g\left(\frac{c_1-\hat{c}_1}{\sigma}\right)}\int_{-\infty}^{\hat{c}_1} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1\int_{\hat{c}_1}^{\infty} f(\bar{c}_1)g\left(\frac{c_1-\bar{c}_1}{\sigma}\right)d\bar{c}_1
\end{aligned}$$

As a result,

$$\begin{aligned}
&\int_{-\infty}^{\hat{c}_1} f(\bar{c}_1) g'\left(\frac{c_1-\bar{c}_1}{\sigma}\right) d\bar{c}_1 \int_{\hat{c}_1}^{\infty} f(\bar{c}_1) g\left(\frac{c_1-\bar{c}_1}{\sigma}\right) d\bar{c}_1 - \int_{-\infty}^{\hat{c}_1} f(\bar{c}_1) g\left(\frac{c_1-\bar{c}_1}{\sigma}\right) d\bar{c}_1 \int_{\hat{c}_1}^{\infty} f(\bar{c}_1) g'\left(\frac{c_1-\bar{c}_1}{\sigma}\right) d\bar{c}_1 \\
< \; & \frac{g'\left(\frac{c_1-\hat{c}_1}{\sigma}\right)}{g\left(\frac{c_1-\hat{c}_1}{\sigma}\right)} \left\{ \int_{-\infty}^{\hat{c}_1} f(\bar{c}_1) g\left(\frac{c_1-\bar{c}_1}{\sigma}\right) d\bar{c}_1 \int_{\hat{c}_1}^{\infty} f(\bar{c}_1) g\left(\frac{c_1-\bar{c}_1}{\sigma}\right) d\bar{c}_1 \right. \\
& \left. - \int_{-\infty}^{\hat{c}_1} f(\bar{c}_1) g\left(\frac{c_1-\bar{c}_1}{\sigma}\right) d\bar{c}_1 \int_{\hat{c}_1}^{\infty} f(\bar{c}_1) g\left(\frac{c_1-\bar{c}_1}{\sigma}\right) d\bar{c}_1 \right\} \\
= \; & 0
\end{aligned}$$

■

Lemma 15 *There exists* $\underline{\Delta} < \bar{\Delta}$ *such that for all* $c_1 \in [\underline{c}, \infty)$ *and* $\hat{a}_1$*, we have* $\underline{\Delta} \leq \Delta(c_1; \hat{a}_1) \leq \bar{\Delta}$*.*

Proof. We first show that for all μ_2, $V_2(\mu_2)$ is well defined and continuous. Note that when $c_2 \geq \mu^{*-1}(\mu_2)$, $V_2(\mu_2, c_2) = [\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\bar{v} - q$ and if $c_2 < \mu^{*-1}(\mu_2)$, $V_2(\mu_2, c_2) = \bar{\pi}\bar{v} - q(1+r) - c_2$. Therefore,

$$\begin{aligned}
V_2(\mu_2) &= \int_{-\infty}^{\infty} \left[\int_{\mu^{*-1}(\mu_2)}^{\infty} \{[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\bar{v} - q\} \, dG\left(\frac{c_2-\bar{c}_2}{\sigma}\right) \right. \\
&\quad \left. + \int_{-\infty}^{\mu^{*-1}(\mu_2)} \{\bar{\pi}\bar{v} - q(1+r) - c_2\} \, dG\left(\frac{c_2-\bar{c}_2}{\sigma}\right) \right] dF(\bar{c}_2) \\
&= \{[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\bar{v} - q\} \\
&\quad + \int_{-\infty}^{\infty} \int_{-\infty}^{\mu^{*-1}(\mu_2)} \Big\{ (1-\mu_2)(\bar{\pi} - \underline{\pi})\bar{v} - qr - c_2 \Big\} \, dG\left(\frac{c_2-\bar{c}_2}{\sigma}\right) dF(\bar{c}_2)
\end{aligned}$$

The function inside the integral is $O(c_2)$ and therefore the inner integral is $O(\bar{c}_2 G(\bar{c}_2))$.Since $0 \leq G(\bar{c}_2) \leq 1$, the inner integrand is $O(\bar{c}_2)$ and therefore a finite number. Since all the operators above are continuous and G and F are continuous functions, $V_2(\mu_2)$ is a continuous function. Since V_2 is continuous function defined over a compact set $[0, 1]$, it should be bounded above and below. This implies that bounds $\underline{\Delta} \leq \bar{\Delta}$ such that for any $c_1, \hat{a}_1$

$$\underline{\Delta} \leq \bar{\pi} V_2(\mu_{s\bar{v}}(\bar{c}_1; \hat{a}_1)) + (1-\bar{\pi}) V_2(\mu_{s0}(\bar{c}_1; \hat{a}_1)) - V_2(\mu_h(\bar{c}_1; \hat{a}_1)) \leq \bar{\Delta}$$

Hence, $\underline{\Delta} \leq \Delta(c_1; \hat{a}_1) \leq \bar{\Delta}$. ■

Lemma 16 *$\Delta(c_1;\hat{a}_1)$ satisfies the following properties:*

1. *$\Delta(c_1;\hat{a}_1)$ is continuous in c_1 and $\hat{a}_1$. Furthermore, if $\hat{a}_1$ is point-wise higher than $\hat{a}_1'$, $\Delta(c_1;\hat{a}_1) \geq \Delta(c_1;\hat{a}_1')$. Moreover, if $\hat{a}_1(c_1) \neq \hat{a}_1'(c_1)$ for a positive measure subset of c_1's, $\Delta(c_1;\hat{a}_1) > \Delta(c_1;\hat{a}_1')$. In particular $\Delta(c_1;d_k)$ is decreasing in k.*

2. *If $\hat{a}_1$ is switching strategy, $\Delta(c_1;\hat{a}_1)$ is increasing in c_1.*

Proof.

1. Consider the set $A = \{c_1; \hat{a}_1(c_1) > \hat{a}_1'(c_1)\}$. Then

$$\int \hat{a}_1(c_1)dG\left(\frac{c_1-\bar{c}_1}{\sigma}\right) - \int \hat{a}_1'(c_1)dG\left(\frac{c_1-\bar{c}_1}{\sigma}\right) = \int_A dG\left(\frac{c_1-\bar{c}_1}{\sigma}\right) \geq 0$$

with equality only if A is measure zero. Given the Bayesian updating formulas, it implies that for any $c_1 \in \mathbb{R}$,

$$\mu_{s\bar{v}}(c_1;\hat{a}_1) \geq \mu_{s\bar{v}}(c_1;\hat{a}_1'), \mu_{s0}(c_1;\hat{a}_1) \geq \mu_{s0}(c_1;\hat{a}_1'), \mu_h(c_1;\hat{a}_1) \leq \mu_h(c_1;\hat{a}_1')$$

with inequalities only if A is zero measure. Therefore, for each c_1, the integrand in the definition of Δ, is higher for $\hat{a}_1$and therefore $\Delta(c_1;\hat{a}_1) \geq \Delta(c_1;\hat{a}_1')$ with equality only if A is measure zero.

2. If $\hat{a}_1$ is a switching strategy with switching point k,

$$\begin{aligned}
\mu_{s\bar{v}}(\bar{c}_1;\hat{a}_1) &= \frac{\mu_1\bar{\pi}\left[1-G\left(\frac{k-\bar{c}_1}{\sigma}\right)\right]}{\mu_1\bar{\pi}\left[1-G\left(\frac{k-\bar{c}_1}{\sigma}\right)\right]+(1-\mu_1)\underline{\pi}} \\
\mu_{s0}(\bar{c}_1;\hat{a}_1) &= \frac{\mu_1(1-\bar{\pi})\left[1-G\left(\frac{k-\bar{c}_1}{\sigma}\right)\right]}{\mu_1(1-\bar{\pi})\left[1-G\left(\frac{k-\bar{c}_1}{\sigma}\right)\right]+(1-\mu_1)(1-\underline{\pi})} \\
\mu_h(\bar{c}_1;\hat{a}_1) &= \frac{\mu_1\left[(1-\alpha)G\left(\frac{k-\bar{c}_1}{\sigma}\right)+\alpha\right]}{\mu_1\left[(1-\alpha)G\left(\frac{k-\bar{c}_1}{\sigma}\right)+\alpha\right]+(1-\mu_1)\alpha}
\end{aligned}$$

Hence, $\mu_{s\bar{v}}(c_1;\hat{a}_1), \mu_{s0}(c_1;\hat{a}_1)$ are strictly increasing and $\mu_h(c_1;\hat{a}_1)$ is strictly decreasing. This means that the integrand in the definition of $\Delta(c_1;\hat{a}_1)$ is increasing in c_1. Since $H(\hat{c}|c_1)$

is decreasing in c_1, by definition of first order stochastic dominance, $\Delta(c_1;\hat{a}_1)$ is strictly increasing.

■

Lemma 17 *$b(k)$ satisfies the following:*

1. *$b(k)$ is continuous and strictly increasing in k.*

2. *There exists a unique c_1^*, such that $b(c_1^*) = c_1^*$.*

3. *For all $k > c_1^*$, $b(k) < k$ and for all $k < c_1^*$, $b(k) > k$.*

Proof.

1. $b(k)$ satisfies the following

$$\hat{p}(\mu_1) - q + \Delta(b(k);d_k) = \bar{\pi}\bar{v} - q(1+r) - b(k)$$

Since $\Delta(b;d_k)$ is continuous in b and k, it is obvious that $b(k)$ is continuous. An increase in k, causes the function $\Delta(c_1;d_k)$ to decrease by previous lemma. Therefore, in order to sustain the above equation, $b(k)$ must increase.

2. Any fixed point of $b(k)$, c_1^* must satisfy

$$\hat{p}(\mu_1) - q + \Delta(c_1^*;d_{c_1^*}) = \bar{\pi}\bar{v} - q(1+r) - c_1^*$$

Now, notice that under $d_{c_1^*}$, from the Bayesian updating rules, the updating rules are functions of only $1 - G\left(\frac{c_1^*-\bar{c}_1}{\sigma}\right)$. Therefore, we can rewrite $\Delta(c_1^*;d_{c_1^*})$ as the following

$$\Delta(c_1^*;d_{c_1^*}) = \beta\int_{-\infty}^{\infty}\left\{\bar{\pi}V_2\left(\hat{\mu}_{s\bar{v}}\left(1-G\left(\frac{c_1^*-\bar{c}_1}{\sigma}\right)\right)\right) + (1-\bar{\pi})V_2\left(\hat{\mu}_{s0}\left(1-G\left(\frac{c_1^*-\bar{c}_1}{\sigma}\right)\right)\right)\right.$$
$$\left. -V_2\left(\hat{\mu}_h\left(1-G\left(\frac{c_1^*-\bar{c}_1}{\sigma}\right)\right)\right)\right\}dG\left(\frac{c_1^*-\bar{c}_1}{\sigma}\right)$$

Let $l = 1 - G\left(\frac{c_1^* - \bar{c}_1}{\sigma}\right)$. Then the above integral becomes

$$\Delta(c_1^*; d_{c_1^*}) = \beta \int_0^1 \left[\bar{\pi} V_2\left(\hat{\mu}_{s\bar{v}}\left(l\right)\right) + (1-\bar{\pi}) V_2\left(\hat{\mu}_{s0}\left(l\right)\right) - V_2\left(\hat{\mu}_h\left(l\right)\right)\right] dl$$

and c_1^*must satisfy

$$\hat{p}(\mu_1) - q + \beta \int_0^1 \left[\bar{\pi} V_2\left(\hat{\mu}_{s\bar{v}}\left(l\right)\right) + (1-\bar{\pi}) V_2\left(\hat{\mu}_{s0}\left(l\right)\right) - V_2\left(\hat{\mu}_h\left(l\right)\right)\right] dl = \bar{\pi}\bar{v} - q(1+r) - c_1^*$$

The LHS of the above equation is fixed, while the RHS is strictly decreasing. Since $c_1^* \in \mathbb{R}$, there must exist a unique c_1^* that satisfies the above equation.

3. Suppose $k < c_1^*$ and $b(k) \leq k$. Since, $\lim_{k \to -\infty} b(k) = \hat{c}^0 > -\infty$. Then by continuity of $b(\cdot)$, there must exists $\hat{k} \in (-\infty, k]$ such that $b(\hat{k}) = \hat{k}$. Contradicting part 2. Similarly, we can show that for all $k > c_1^*$, $b(k) < k$.

■

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