Foreclosures and House Prices*

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Abstract

The empirical evidence from the last decade suggest that sizable increases in housing defaults can be the result of either income shocks (recession 2001) or changes in the market value of the house (2005-2007 period). The objective of the paper is to understand the double feedback mechanism between foreclosures and house prices. To understand the importance of this channel we develop an equilibrium theory of housing default. Housing investment requires a downpayment and long-term mortgage financing. However, at any point in time homeowners can default in their obligations, but they lose the property. The stationary version of the model is capable of generating house price increases that are consistent with the average capital gains realized between 1990 and 2005. The model can also rationalize declines in house prices that are consistent with the observed counterpart. The baseline model also replicates the observed decline in the user cost of housing defined as the ratio between the price index for rental property and owner-occupied housing that models based on arbitrage conditions are incapable of replicate.

Keywords: Housing default, deficiency provisions

J.E.L.: E2, E6

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Introduction

The boom in ownership in the United States that was initiated in 1994 started to have significant impact in house prices around 2002. Figure 1 describes the real appreciation rate measured by different house price indices.

The figure combines a relatively steady appreciation of house prices between 1998 and 2002, a much rapid increase in house prices between 2003 and 2005, with a final decline after the summer 2005. Changes in house prices have a significant impact in the homeowners portfolios since they change the level of equity accrued in the dwelling. In periods with high appreciation homeowners can borrow against a collateral with an increased value to increase consumption or can opt to sell the property and purchase a bigger one, but in periods with falling prices the outstanding debt can be larger than the market value of the property making default a viable option for some homeowners. The evidence seems to suggest a connection between house prices and housing foreclosures. Next figure, summarizes the evolution of seriously delinquent mortgages between 1990 and 2007.

1 The concept of "seriously delinquent mortgages" is calculated by adding the percentages of mortgage payments 90 days past due and the percentages of inventory of mortgages in foreclosure. "Inventory of Mortgages in foreclosure" refers to the total number of loans in the legal process of foreclosure as a percentage of the total number of mortgages in the pool during a quarter. The number of loans in the process of foreclosure during a quarter means that some foreclosures may have started in other quarters but have yet to be resolved.
Figure 2 clearly shows that periods with relatively steady grow in house prices are consistent with low delinquency rates, whereas as periods with a decline in house prices have a large impact in delinquencies. The evolution between 2001 and 2002 is contaminated by the recession in 2001, that had a positive impact in delinquents rates. This episode suggests that house price declines can have a positive impact in house defaults but so other economic variables. That can occur even when house prices are growing.

The objective of the paper is to understand the double feedback mechanism between foreclosures and house prices. A change in house prices has an effect in the level of equity accrued in the property. A drop in the house price can trigger an increase in mortgage delinquencies that ultimately results in foreclosures. The delinquent properties and then sold in the market affecting the underlying transacted priced. The larger the number of foreclosed units transacted the more likely that the house price will fall. This channel does not necessarily have to be trigger by a change in the house price. There can be other sources (i.e. income shocks, mortgaged with teaser rates) than increase the foreclosure rate without decrease in the value of the collateralized asset. However, the sale of the foreclosed properties could increase the stock of housing units for sale, and end up impacting house prices.

The understand the importance of this channel we develop an equilibrium theory of housing default. Households face uninsurable labor income, life uncertainty, and borrowing constraints. They make decisions with respect to consumption of goods and housing services. Housing investment is part of the household’s portfolio decision and differs from capital investment along several dimension. Housing investment is lumpy and indivisible, is subject to idiosyncratic capital gains shocks, requires a downpayment and long-term mortgage financing. However, at any point in time homeowners can default in their obligations, but they loose the property. Households have the option to purchase housing services in the rental market. Mortgage loans are available from a financial sector that receives deposits from households and also loans capital to private firms. The production sector considers neoclassical firms that use capital and labor to produce a consumption/investment good and housing. We estimate the structural parameters of the model to match certain moments in the U.S. economy. We show that the model replicates the key factors and the distributional
patterns of housing ownership, housing consumption, and distribution of landlords.

The primary findings in this paper are:

- The stationary version of the model is capable of generating house price increases that are consistent with the average capital gains realized between 1990 and 2005. These capital gains can be rationalized with the decline in mortgage rates and the introduction of new mortgage products that either reduce the downpayment constraint, or the structure of the repayment profile of mortgage contracts. The model can also rationalize declines in house prices that are consistent with the observed counterpart.

- The baseline model also replicates the observed decline in the user cost of housing defined as the ration between the price index for rental property and owner-occupied housing. The evidence suggest that between 1994 and 2005 this ratio drop by 18.7 percent, and the model is capable of generating declines of a similar magnitude. Our formulation with an elastic supply of rental property seem to reconcile a feature that model based on arbitrage conditions are incapable of replicate.

- Findings of default rates (to be completed....)

**Empirical Evidence**

**0.1 Foreclosures**

The mortgage industry is heavily regulated, and the markets are relatively segmented by the type of borrowers and the risk associated in each market is priced with premiums over a baseline mortgage rate. In this section we try to identify the nature of deliquent rates by markets considering conventional prime, conventional subprime, FHA loans. The next figures illustrates the evolution of delinquent loans by lender.
We clearly observed that the default rates are substantially large in the subprime market, and in the government loans provided through the Federal Housing Administration (FHA). By contrary, loans funded in the conventional prime market have a really low default rate, even in periods with declining house prices. The aggregate default rates seem to be entirely driven by the conventional subprime market and the FHA loans. All these lenders provide offer different type of loans that differ in the downpayment requirement, repayment schedule, and interest payments among other things. However, these contracts are often categorized as fixed rate mortgage loan (FRM) and adjustable rate mortgage (ARM). If the condition delinquency rates by loan type we observe that most defaults are associated to adjustable rate mortgages.
This is true in the conventional subprime and FHA loans, but is also true in the prime market. Even though the overall default rate in the prime market is relatively low. Delinquency rates have more than double. It is interesting to note that the default rates in the subprime market appear to be very large independently of the loan type. However, the market share of the subprime market has increase dramatically since 1995, with the largest increase between 2003 and 2005. This market is specially important since is geared towards more riskier types, so an increase in the market share also implies an increase in aggregate risk.

Next table summarizes the evolution of the market share of these different mortgage lenders:

Table 1: Market Shares for Home Purchase Loans, 1996-2005

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<tbody>
<tr>
<td>CONVENTIONAL</td>
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<tr>
<td>Prime</td>
<td>73.2</td>
<td>71.8</td>
<td>72.8</td>
<td>71.5</td>
<td>712</td>
<td>71.1</td>
<td>73.0</td>
<td>74.0</td>
<td>75.7</td>
<td>76.4</td>
</tr>
<tr>
<td>Subprime</td>
<td>1.6</td>
<td>2.6</td>
<td>3.7</td>
<td>4.5</td>
<td>5.9</td>
<td>5.0</td>
<td>6.6</td>
<td>9.3</td>
<td>12.0</td>
<td>14.5</td>
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<tr>
<td>GOVERNMENT</td>
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<tr>
<td>FHA</td>
<td>18.8</td>
<td>19.6</td>
<td>17.5</td>
<td>19.0</td>
<td>18.6</td>
<td>19.3</td>
<td>16.5</td>
<td>13.1</td>
<td>9.2</td>
<td>6.3</td>
</tr>
<tr>
<td>VA</td>
<td>6.0</td>
<td>5.4</td>
<td>5.3</td>
<td>4.5</td>
<td>3.9</td>
<td>4.2</td>
<td>3.6</td>
<td>3.1</td>
<td>2.6</td>
<td>2.4</td>
</tr>
</tbody>
</table>

The table illustrates an important increase in the market share of the conventional subprime market, at the expense of the government loan programs funded by FHA and VA. However, the largest market share growth for the subprime market occur around 2002, coinciding with rapid increase in house price and a decline in the default rates in that market.

0.2 House Prices

The objective of this section is to present some further evidence of the evolution of house prices. We look at a regional decomposition across the 4 regions in the United States.
As we observe in Figure 5, the general pattern of house appreciation has been generalized in the nation. The largest increase are in the West Coast and North East, but all prices have been declining since 2005. That suggest no substantial difference with broadly defined regions. However, one can expect larger levels of heterogeneity at the MSA.

### Housing Model with Foreclosure

We modify the formulation used by Chambers, Garriga, and Schlagenhauf (2007) to include foreclosure, and a different timing of the housing idiosyncratic house shock. Households are indexed by their asset holding, $a$, investment position in housing, $h$, mortgage choice, $z$, remaining periods on the mortgage, $n$, the idiosyncratic income shock, $\epsilon$, and age, $j$. We will summarize the household state by $x = (a, h, n, z, \epsilon, j)$. The timing on information with respect to foreclosure works as follows.

**Idiosyncratic capital gain (late revelation of uncertainty):** Given the current information summarized by the individual state variable, $x$, the households decides to sell the house. At this moment, the revelation of the house price shock, $\xi$, takes place. Given the observed realization the households chooses to default, if the option value of defaulting is higher than the one associated to sell the house and clear any outstanding balance with the financial intermediary. The advantage of this approach is computational, since it does not require to introduce an additional state variable.\footnote{There are alternative timing conventions that could have been used. One could consider a one time capital gain shock. After purchasing the house, the individual observes a one time idiosyncratic shock, $\xi$. The cost of this approach is to include an additional state variable, $x = (a, h, n, z, \epsilon, j, \xi)$. An extension of this timing could allow for an idiosyncratic capital gain with early revelation of uncertainty. The approach is similar to the previous one, but we allow the shocks to change every period according to an $iid$ shock with a probability distribution, $\pi_s$. The individuals observe the house price shock, $\xi$, and then they decide to sell or not.}
0.3 Households

Preferences: Households live a maximum of $J$ periods, and survival each period is subject to mortality risk. The probability of surviving from age $j$ to age $j+1$ is denoted by $\psi_{j+1} \in (0,1)$, with $\psi_1 = 1$. Household’s preferences are given by the expected value of the discounted sum of momentary utility functions,

$$E\sum_{j=1}^{J} \beta^{j-1} \psi_j u(c_j, d_j),$$

where $\beta \in (0,1)$ is the discount factor, $c_j$, is the consumption of goods at age $j$, $d_j$ is the amount of housing service consumption. The utility function is neoclassical and satisfies the standard properties of continuity and differentiability. The main difference is that we require a minimum consumption level of goods and housing services. We assume that the minimum levels are the same across all households levels and do not vary with age, or income status. We index the discount rate by age to include the survival probability, $\beta_j = \beta \psi_j$.

Housing: Housing investment is lumpy and indivisible, and the price of a unit of housing is $p$. Since we focus on stationary equilibrium $p' = p$ for all periods (we hope to change it in the future to capture the path of prices). The size of housing investment is restricted by the set $\mathcal{H}$ where $\mathcal{H} \equiv \{0\} \cup \{h, \ldots, \bar{h}\}$, $h < \ldots < \bar{h}$, is the minimum housing investment, and $\bar{h}$ is the upper bound on housing investment. Housing investment, $h > 0$, generates a flow of housing services, $s$, that can be consumed. We assume a linear technology, $s = g(h') = h'$, that transforms the housing investment in the current period into housing services. A household can choose a dwelling size that is equal or less than the housing investment position. The separation between housing investment and housing consumption allows us to formalize rental markets. Those households that have a positive housing investment can choose to consume all housing services $s = h' = d$, or pay a fixed cost $\bar{c} > 0$ and sell (lease) some services in the market equal to $h' - d$ at the rental price $R$. Homeowners that consume housing services equal to their housing investment position forgo rental income which captures the opportunity cost of owner-occupied housing explicitly in the budget constraint.

Household’s Income: Each household receives a time endowment that is inelastically supplied to the labor market until retirement. Households differ in their productivity for two reasons - age and period specific productivity shocks. We define $v_j$ as the labor productivity of an age $j$ individual. The age profile of labor productivity is $\{v_j\}_{j=1}^{\infty}$. Households also draw a period specific earnings component, $\epsilon$, from a probability space, where $\epsilon \in \mathcal{E}$. The realization of the current period productivity component evolves according to the transition law $\Pi_{\epsilon,\epsilon'}$. Thus, a worker’s labor earnings in a given period is $w\epsilon v_j$ where $w$ is the market wage rate. In addition to labor earnings, the gross return from the asset market investment is another source of income, and $r$ is the net interest rate. We define the household’s (non rental) income as:

$$y = \begin{cases} 
  w\epsilon v_j + (1 + r)a + tr + y_r & \text{if } j < j^*, \\
  \theta + (1 + r)a + tr + y_r & \text{if } j \geq j^*. 
\end{cases} \tag{1}$$

where $\theta$ is retirement benefit, $tr$ represents a lump-sum transfer from accidental bequests, and $y_r$ represents net rental income. Net rental income earned from the housing investment $y_r$ is defined as

$$y_r = \begin{cases} 
  R(h' - d) - \bar{c} - x(h', d) & \text{if } d < h' \text{ and } h' > 0 \\
  -x(h', d) & \text{if } d = h' \text{ and } h' > 0 \\
  0 & \text{if } h' = 0 
\end{cases}$$
where the term \( x(h', d) \) represents the housing maintenance expense. The rate that housing depreciates depends on whether housing is owner-occupied or rental-occupied. A homeowner that chooses a dwelling that is used to their housing investment position incurs a maintenance expense equal to \( x(h', d) = \delta_p h' \) where \( \delta_p \) represents the depreciation rate of owner-occupied housing. If a household chooses to pay the fixed cost to become a landlord, the maintenance expense depends on the fraction of services the household consumes and the fraction other households consume. Rental-occupied housing depreciates at \( \delta_r > \delta_p \). The different depreciation rates are a result of a moral hazard problem that occurs in rental markets as renters decide how intensively to utilize the dwelling. That is, \( x(h', d) = \delta噪 pd + \delta噪 (h' - d) \).

For renters \( (h' = 0) \), the implied rental income is zero. Households earn income in the labor market if they are under the age \( j^* \), or from retirement benefits if they are of age \( j^* \) or older.

- **Renters**: The state variable of a renter is \( x = (a, h, z, n, \epsilon, j) = (a, 0, 0, 0, \epsilon, j) \). The optimization problem of a renter is to continue being a renter or purchase a house and be a homeowner:

\[
v(x) = \max \{ v^r, v^o \}.
\]

The value function associated to continue renting is given by

\[
v^r(x) = \max_{(d, a') \in \mathbb{R}_+} \left\{ u(y - a' - Rd, d) + \beta_{j+1} \sum_{\epsilon' \in \mathcal{E}} \pi(\epsilon, \epsilon') v(x') \right\}
\]

where \( x' = (a', 0, 0, 0, \epsilon', j + 1) \), and \( Rd \) denote the cost of housing services purchased in the rental market. Their is no restriction on the size of housing services rented\(^3\).

The restriction in the choice set indicates that asset markets are incomplete since short-selling is precluded and only an noncontingent claim on capital is traded.

The value function associated to buy a house solves

\[
v^o(x) = \max_{(c, d, a', h', \phi, \chi, z') \in \mathbb{R}_+} \left\{ u(c, d) + \beta_{j+1} \sum_{\epsilon' \in \mathcal{E}} \pi(\epsilon, \epsilon') v(x') \right\},
\]

s.t. \( c + a' + (\phi_b + \chi(z')) ph' + m(k, z') = y \).

The household chooses to purchase a house with a a cost \( ph' \) financed using a mortgage mortgage \( z' \in \mathcal{Z} \). The current expenditure is \( (\phi_b + \chi(z')) ph' \) where \( \phi_b \) represents a transaction cost parameter and \( \chi(z') \) denotes the downpayment fraction associated to mortgage \( z' \). The period mortgage payment is \( m(k, z') \) where \( k = (p, h', \chi(z'), N(z'), r_m(z')) \).

The state variables for tomorrow are \( x' = (a', h', z', N - 1, \epsilon', j + 1) \).

- **Owners**: The state variables for a homeowner is given by \( x = (a, h, z, n, \epsilon, j) \). At every period, homeowners observe the realization of the capital gain shock, \( \xi \in \Xi \). Then, they can decide to maintain the same dwelling, change size, exit the owner-occupied housing market. There are two distinct ways to leave the market. In the first one, the individual sells the property, whereas in the second one defaults on the loan obligations.

\[
v(x) = \max \{ v^o, v^e, v^f \}
\]

\(^3\)Other housing papers impose some limits in the size of rental-occupied housing. In this paper, renters can consume any size of housing services.
**Maintain house:** The value function associated to remain homeowner \((h' = h > 0)\) in the same dwelling is given by

\[
v^m(x) = \max_{(c,d,a') \in \mathbb{R}_+, I_r \in \{0,1\}} \left\{ u(c, d) + \beta_j \sum_{\epsilon' \in \mathcal{E}} \pi(\epsilon, \epsilon') v(x') \right\},
\]

\[
s.t. \quad c = y - (a' + m(k, z')).
\]

**Change house size:** The value function associated to change the house size is

\[
v^c(x) = \max_{(c,s,a',h') \in \mathbb{R}_+, z' \in \mathbb{Z}, L_r \in \{0,1\}} \left\{ \sum_{\xi} \pi_\xi [u(c, d) + \beta_j \sum_{\epsilon' \in \mathcal{E}} \pi(\epsilon, \epsilon') v(x')] \right\}
\]

\[
s.t. \quad c = y + \Pi_s - (a' + (\phi_h + \chi(z'))ph' + m(k, z')),\]

where \(\Pi_s = (1 - \phi_s)p\xi_h - D(k, z)\). We assume that individuals that change house size a prevented from default.

**Exit housing market (sell or foreclosure):** Homeowners can exit the market and rent by taking two different actions. They can sell the property and cancel any outstanding debt with the financial intermediary \((I_f = 0)\), or they can foreclose the property. These decisions only affect the budget constraint. The value function is given by:

\[
v^e(x) = \max_{(c,s,a',h') \in \mathbb{R}_+, I_f \in \{0,1\}} \left\{ \sum_{\xi} \pi_\xi [u(c, d, \varphi I_f) + \beta_j \sum_{\epsilon' \in \mathcal{E}} \pi(\epsilon, \epsilon') v(x')] \right\},
\]

\[
s.t. \quad c = y + \max(\Pi_s, 0) - (a' + Rd).
\]

The \(\max(\Pi_s, 0)\) operator gives homeowners the option of foreclose the property, \(I_f = 1\). That occurs when the current value of the property net of selling costs is lower than the outstanding level of debt, \(\Pi_s < 0\). The penalty associated to foreclosure is that the individual is forced to rent for one period, and the potential utility penalty, \(\varphi I_f\).
The Decision Making Tree

**Renter** \((h = 0)\):

\[
v(x) = \max\{v^r, v^o\}
\]

Rent: \(c + a' + Rs = y(x)\)

Own: \(c + a' + \chi(z')ph' + m(z') = y(x)\)

**Owner** \((h > 0)\):

\[
v(x) = \max\{v^m, v^c, v^r, v^f\}
\]

Maintain: \(c + a' + m(z) = y(x)\)

Change: \(c + a' + \chi(z')ph' + m(z') = y(x) + \Pi_s\)

Rent: \(c + a' + Rs = y(x) + \Pi_s\)

Sell: \(c + a' + Rs = y(x) + \max(\Pi_s, 0)\)

**0.4 Mortgage Brokers**

We assume a competitive lending sector that maximizes expected profits per mortgage contract. The base interest rate per mortgage contract is given by \(r^* + \varphi(z)\), where \(\varphi(z)\) is a mortgage premium that depends on the default rate observed in contract \(z\). The computation of the premium solves an implicit function

\[
M + D + S - L = 0, \quad \forall z
\]

where

\[
M = \int \sum_{\xi \in \Xi} \pi_\xi \mu_j m(k, z) \Phi(d\Lambda)
\]

Mortgage payments

\[
D = \int \mu_j D(\Lambda, z) \Phi(d\Lambda) + \int \mu_j (1 - \psi_j) D(\Lambda, z) \Phi(d\Lambda)
\]

Cancellation principal (alive) \quad Cancellation principal (death)

\[
S = \int_{I_f} \sum_{\xi \in \Xi} \pi_\xi \mu_j \gamma(1 - \phi_s) p \xi h^2 \Phi(d\Lambda)
\]

Proceeds selling foreclosed properties

\[
L = \int \mu_j (1 - \chi(z)) ph'(\Lambda) \Phi(d\Lambda)
\]

Loans
The mortgage broker borrows in the international capital markets and the premium is used to cover the default rate probability. With the law of large numbers the expected level of profits is zero. For every contract, we need to determine \( \varphi^*(z) \) such that the mortgage broker makes zero profits per contract. With the equilibrium conditions we need to compute \( \{ \varphi^*(z) \}_{z=1}^{Z} \) that guarantee zero profits.

### 0.5 Firms

In this economy, a representative firm produces a good in a competitive environment that can be used either for consumption, government, capital purposes, or housing purposes. The representative firm produces goods using a constant returns to scale technology \( F(K, L) = K^\alpha L^{1-\alpha} \), where \( K \) and \( L \) denote the amount of capital and labor utilized. In the economy with global capital markets the interest rate is fixed, \( r^* \). Given the competitive nature of financial and labor markets, the optimal firm chooses \( \{K^*, L^*\} \) such that:

\[
\begin{align*}
r^* &= F_1(K, L) - \delta, \\
w &= F_2(K, L),
\end{align*}
\]

the demand for capital is determined by solving

\[
\begin{align*}
r^* + \delta &= F_1(K, L) = \alpha K^{\alpha-1} L^{1-\alpha}, \\
K^* &= \left( \frac{\alpha}{r^* + \delta} \right)^{\frac{1}{1-\alpha}} L,
\end{align*}
\]

with the optimal demand for capital, we can easily determine the implied equilibrium wage rate:

\[
\begin{align*}
w &= F_2(K^*, L) = (1 - \alpha)(K^*)^\alpha L^{-\alpha}, \\
w &= (1 - \alpha) \left( \frac{\alpha}{r^* + \delta} \right)^{\frac{\alpha}{1-\alpha}} L^{1-\alpha}.
\end{align*}
\]

Aggregate output can be easily calculated using \( \{K^*, L^*\} \), that is:

\[
Y = F(K^*, L^*) = \left( \frac{\alpha}{r^* + \delta} \right)^{\frac{\alpha}{1-\alpha}} L^{1-\alpha}.
\]

Given the global interest rate \( r^* \), using the firms problem we can calculate the stock of capital \( (K^*) \) used by domestic firms and the wage rate \( (w) \).

### 0.6 Government

In this economy, the government engages in a number of activities: finances some exogenous government expenditure; provides retirement benefits through a social security program; and redistributes the wealth of those individuals who die unexpectedly. We assume that the financing of government expenditure and social security are run under different budgets.

The government provides social security benefits to retired households. The benefit, \( \theta \), is based on a fraction, \( \bar{I} \), of the average income of workers. These payments are financed by taxing the wage income if employed households at the tax rate \( \tau_p \). Since this policy is
self-financing, the tax rate depends on the replacement ratio \( \bar{\theta} \). The social security benefit can be defined as:

\[
\theta \equiv \bar{\theta} \sum_{j=1}^{j^*} \sum_{i} \mu_j w v_j e / \sum_{j=1}^{j^*} \mu_j
\]

where \( \mu_j \) is the size of the age \( j \) cohorts. The social security budget constraint is:

\[
\tau_p \sum_{j=1}^{j^*} \sum_{i} (\mu_j w v_j e) = \theta \sum_{j=j^*}^{j} \mu_j. \tag{2}
\]

The government also has the responsibility to collect the physical and housing assets of those individual who unexpectedly die. Both of these assets are sold and any outstanding debt on housing is paid off. The remaining value of these assets is distributed to the surviving households as a lump sum payment, \( tr \). This transfer can be defined as

\[
tr = \frac{Tr}{1 - \mu_1}
\]

where \( Tr \) is the aggregate (net) value of assets accumulated over the state space from unexpected death and is defined as

\[
Tr = \int \mu_j (1 - \psi_j) a(\Lambda) \Phi(d\Lambda) + \sum_{\xi \in \Xi} \pi_\xi \int \mu_j (1 - \psi_j) [(1 - \phi_\xi) p \xi h(\Lambda) - D(\Lambda) \Phi(d\Lambda)]. \tag{3}
\]

where \( \Phi(d\Lambda) \equiv \Phi(da \times dh \times dn \times de \times dj) \).

### 0.7 Market Equilibrium

This economy has four competitive markets: the goods market, labor market, the rental of housing services market, and the housing market.

- **Housing market:** We assume that the aggregate supply of housing is fixed \( \bar{H} \). The market clearing condition is then given by

\[
\int_{I_x(\Lambda)=0} \mu_j h'(\Lambda) \Phi(d\Lambda) + \int_{I_x(\Lambda)=1} \sum_{\xi \in \Xi} \pi_\xi \mu_j h'_\xi(\Lambda) \Phi(d\Lambda) = \bar{H},
\]

or in compact notation

\[
\int \mu_j h'(\Lambda) \Phi(d\Lambda) \Phi(d\Lambda) = \bar{H},
\]

- **Rental market:** The equilibrium in this market is determined by the aggregate amount of housings services made available by landlords and the total demand of rental housing services. That is

\[
\int_{I_x(\Lambda)=0} \mu_j [h'(\Lambda) - s(\Lambda)] \Phi(d\Lambda) + \int_{I_x(\Lambda)=1} \sum_{\xi \in \Xi} \pi_\xi \mu_j [h'_\xi(\Lambda) - s_\xi(\Lambda)] \Phi(d\Lambda) = \bar{H}, \tag{4}
\]

\[
\int_{I_x(\Lambda)=0} \mu_j s(\Lambda) \Phi(\Lambda) + \int_{I_x(\Lambda)=1} \sum_{\xi \in \Xi} \pi_\xi \mu_j s_\xi(\Lambda) \Phi(\Lambda)
\]

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4In the formulation, the new born generation does not receive a lump sum transfer as we endow these individuals with capital assets as observed in data. In this model the aggregate mass of households of age 1 is \( \mu_1 \) and total population is normalized to one.
This definition accounts for the effect of the idiosyncratic capital gains shock for both the landlord and the renter that just sold a property.

- **Goods market:** The aggregate resource constraint is given by

\[ C + (1 + \rho)K' - (1 - \delta_K)K + pI_H + \Upsilon = F(K, L), \tag{5} \]

where \( C, K, G, I_H \) and \( \Upsilon \) represent aggregate consumption, the aggregate capital stock at the beginning of the next period, aggregate government spending, aggregate housing investment and various transactions costs, respectively.\(^5\) The parameter \( \delta_K \) denotes the depreciation rate for physical capital. Since housing depreciates with utilization, homeowners and landlords need to maintain the stock of housing by investing resources. We assume that \( p \) units of consumption goods can be transformed into 1 unit of housing, but the amount transformed cannot change the total size of the housing stock. Housing investment depends on the fraction that is used for owner and rental occupied housing. Formally, \( I_H \) represents the investment housing goods,

\[
I_H = \int_{I_s(\Lambda)=0}^{\mu_j h'(\Lambda)\Phi(d\Lambda)} + \int_{I_s(\Lambda)=1}^{\mu_j h'(\Lambda)\Phi(d\Lambda)} \sum_{\xi \in \Xi} \pi_{ij} \mu_j h'_{\xi}(\Lambda)\Phi(d\Lambda) \\
- \left[ \int_{I_s(\Lambda)=0}^{\mu_j h'(\Lambda)\Phi(d\Lambda)} - \left[ \int_{I_s(\Lambda)=0}^{\mu_j h'(\Lambda)\Phi(d\Lambda)} + \int_{I_s(\Lambda)=1}^{\mu_j h'(\Lambda)\Phi(d\Lambda)} \sum_{\xi \in \Xi} \pi_{ij} \mu_j h'_{\xi}(\Lambda)\Phi(d\Lambda) \right] \\
- \delta_r \left[ \int_{I_s(\Lambda)=0}^{\mu_j h'(\Lambda)\Phi(d\Lambda)} + \int_{I_s(\Lambda)=1}^{\mu_j h'(\Lambda)\Phi(d\Lambda)} \sum_{\xi \in \Xi} \pi_{ij} \mu_j h'_{\xi}(\Lambda)\Phi(d\Lambda) \right] \right]
\]

- **Labor market:** In the labor market, labor demand is determined by the marginal product of labor, \( F_2(K, L) \). Labor is inelastically supplied and determined by \( L = \sum_{j=1}^{\pi_j - 1} \mu_j v_j \).

### 1 Mapping the Model and the Data

In order to evaluate the model, parameters must be specified. We choose to estimate most of the parameters using an exactly-identified Method of Moments approach. That is, we solve for the parameters that are consistent with some key properties of U. S. economy observed in 1990. This choice allows to start with a baseline with stable house prices and default rates.

#### 1.1 Functional Forms and Parameters

**Functional Forms:** Our choice of the utility function departs from the usual specification of a constant relative risk aversion utility function with a homothetic aggregator between consumption of goods and housing services. This preference structure is not consistent an increasing ratio of housing services/ consumption ratio by age which is observed in the data, [see Jeske (2005) for a detailed discussion].\(^6\) We assume that preferences over the

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\(^5\)The definition of aggregate housing investment and total transactions cost are define in the appendix.

\(^6\)We also find that such a momentary utility function generates insufficient movements in housing position as well as introducing some counterfactual implications for the rental market.
consumption of goods and housing services can be represented by a period utility function of the form:

$$U(c,d) = c^{\sigma_1} + (1 - \gamma) d^{\sigma_2}$$

where $\sigma_1$ and $\sigma_2$ determine the curvature of the utility function with respect to consumption and housing services, respectively. The relative ratio of $\sigma_1$ and $\sigma_2$ determines the growth rate of the housing to consumption. A larger curvature on consumption implies that the marginal utility of consumption declines faster than the marginal utility of housing services. Consequently, when household income increases over the life-cycle, households choose to allocate a larger fraction of resources to housing services. We choose to set $\sigma_1 = 3$ and $\sigma_2 = 1$ and estimate and the preference parameter $\gamma$.

The representative firm uses a Cobb-Douglas technology to produce a good that can be used either for consumption, housing investment, or capital good investment. We assume that the aggregate production function is of the Cobb-Douglas form, $F(K,L) = K^\alpha L^{1-\alpha}$. The capital share parameter is set to 0.29. This value is calculated by dividing private fixed assets plus the stock of consumer durables less the stock of residential structures by output plus the service flows from consumer durables less the service flow from housing.

**Population structure:** Each period in the model is taken to be three years. An individual enters the labor force at age 20 (model period 1) and lives till age 83 (model period 23). Retirement is assumed to be mandatory at age 65 (model period 16). Individuals survive to the next period with probability $\psi_{j+1}$. These probabilities are set at survival rates from the National Center for Health Statistics, *United States Life Tables* (1994). The size of the age specific cohorts, $\mu_j$, needs to be specified. Because of our focus on steady state equilibrium, these shares must be consistent with the stationary population distribution. As a result, these shares are determined from $\mu_j = \psi_j \mu_{j-1}/(1 + \rho)$ for $j = 2, 3, \ldots, J$ and $\sum_{j=1}^J \mu_j = 1$, where $\rho$ denotes the rate of growth of population. Using resident population as the measure of the population, we set this the three year growth rate to 3.643 percent.

**Endowments:** Workers are assumed to have an inelastic labor supply, but the effective quality of their supplied labor depends on two components. One component is an age-specific, $v_j$, an is designed to capture the "hump" in life cycle earnings. We use data from U.S. Bureau of the Census, "Money, Income of Households, Families, and Persons in the United States, 1994," *Current Population Reports*, Series P-60 to construct this variable. The other component captures the stochastic component of earnings and is based on Storesletten, Telmer and Yaron (2004). We discretize this income process into a five state Markov chain using the methodology presented in Tauchen (1986). The values we report reflect the three year horizon employed in the model. As a result, the efficiency values associated with each possible productivity value $\epsilon$ are

$$\epsilon \in \mathcal{E} = \{4.41, 3.51, 2.88, 2.37, 1.89\}$$

and the transition matrix is:

$$\pi = \begin{bmatrix}
0.47 & 0.33 & 0.14 & 0.05 & 0.01 \\
0.29 & 0.33 & 0.23 & 0.11 & 0.03 \\
0.12 & 0.23 & 0.29 & 0.24 & 0.12 \\
0.03 & 0.11 & 0.23 & 0.33 & 0.29 \\
0.01 & 0.05 & 0.14 & 0.33 & 0.47 \\
\end{bmatrix}.$$
Each household is born with an initial asset position. The purpose of this assumption is to account for the fact that some of the youngest households who purchase housing have some wealth. Failure to allow for this initial asset distribution creates a bias against the purchase of homes in the earliest age cohorts. As a result we use the asset distribution observed in Panel Study on Income Dynamics (PSID) to match the initial distribution of wealth for the cohort of age 20 to 23. Each income state has assigned the corresponding level of assets to match the nonhousing wealth to earnings ratio.

**Housing:** The housing market introduces a number of parameters. The purchase of a house requires a mortgage and downpayment. In this paper we focus on 30 year fixed rate mortgage. As a result of the assumption that a period is three years, we set the mortgage length, $N$, to ten periods. The downpayment requirement, $\chi$, is set to twenty percent.\(^8\) Buying and selling property is subject a transaction costs. We assume that all these costs are incurred at purchase and set $\phi_s = 0$ and $\phi_b = 0.06$. Because of the lumpy nature of housing, the specification of the second point in the housing grid determines the minimum house size, $h$. The specification of this grid point has implications for the timing of the homeownership decision and thus wealth portfolio decisions. To avoid having the choice of this variable having inadvertent implications for the results, we determine the size of this grid point as part of the estimation problem. As previously explained, housing depreciates at rates which depend on whether the property is owner occupied or rented. The values for $\delta_o$ and $\delta_r$ are estimated. The parameter $\varpi$ affects the number of households that choose to become landlords. Determination of the this parameters is difficult as we have little direct evidence on the number of households who own rental property. An indirect measure is to calculate the number of homeowners that report rental income. In the AHS in 1995, approximately ten percent of the sampled homeowners claim to receive rental income. As a result, we choose to set $\varpi$ to 0.10.

We used data from the 1995 *American Housing Survey* to quantify the i.i.d. capital gain shock. To calculate the probability distribution for this shock we measure capital gains based on the purchase price of the property and what the property owner believes to be the current market value. This ratio is adjusted for the holding length to express the appreciation in annualized terms. Then, we estimate a kernel density and then discretize the density in three even partitions. The average annualized prices changes ranging from lowest to highest are -6.6, -1.4, and 10.5 percent. These values are adjusted to be consistent with a period being defined as three years. In order to test the robustness of the data from the *American Housing Survey*, we employed a similar approach using 1995 Tax Roll Data for Duval County in Florida. Jacksonville is the major city in Duval County. This data follows real estate properties as opposed to individuals. As a result, we can calculate annualized capital gains based in actual sales. We find very similar estimates for the idiosyncratic capital gains shock using this data source.

**Government:** The government enters the model in a number of ways. Income is provided to retired individuals through a social security program. We assume the retirement program is self-financed through a payroll tax on the earnings of workers. After retirement, households receive a transfer based on some fraction of the average labor income. The replacement ratio is set at thirty percent which results in a payroll tax on the worker of 5.25 percent.

\(^8\)The American Housing Survey in 1993-95 presents data that shows that the average downpayment is approximately twenty percent.
1.2 Performance of the Baseline Model

We estimate six parameters using an exactly-identified Method of Moments approach. The estimation of the structural parameters in not separated from the computation of equilibrium. This means three additional nonlinear equations (asset market, government budget constraint, and accidental bequest) have to satisfied along with the moments observed in the data. The parameters that need to be estimated are the depreciation rate of the capital stock, $\delta$, the depreciation rate for rental units, $\delta_r$, the depreciation rate for ownership units, $\delta_o$, the relative importance of consumption goods to housing services, $\gamma$, and the individual discount rate, $\beta$, the minimum size of the smallest housing investment position, $h$. We identify these parameter values so that the resulting aggregate statistics in the model economy are equal to seven targets observed in the U.S. economy.

1. The ratio of capital to gross domestic product: 2.541. This is the average for the period 1958-2001 where we define the capital stock as private fixed assets plus the stock of consumer durables less the stock of residential structures so as to be consistent with capital in the model. Output is GDP plus service flows from consumer durables less the service flow from housing.

2. The ratio of the housing capital stock to the nonhousing capital stock: 0.43. The housing capital stock is defined as the value of fixed assets in owner and tenant residential property.

3. The ratio of investment in capital goods to output: 0.135.

4. The ratio of the investment in residential structures to housing capital stock: 0.121.

5. Housing consumption relative to nonhousing consumption: 0.23. This is the average between 1990 and 2000 but the number does not vary greatly over the period. Housing services are defined as personal consumption expenditure for housing and non housing consumption is defined as nondurable and services consumption expenditures net of housing expenditures.

6. The homeownership rate in the period 1990 is 0.635 percent.

The estimated parameters expressed in annual terms are presented in Table 6. The model performs quite well in matching the seven targeted moments. The implied targets generated by the model solution are within one percent error for all the observed targets.

\footnote{We estimated services flows using procedures outlines in Cooley and Prescott (1995).}
Table 2: Estimation of Model (Annualized Values)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Target</th>
<th>Model</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio of wealth to gross domestic product ($K/Y$)</td>
<td>2.541</td>
<td>2.544</td>
<td>0.143</td>
</tr>
<tr>
<td>Ratio of housing stock to fixed capital stock ($H/K$)</td>
<td>0.430</td>
<td>0.426</td>
<td>-0.792</td>
</tr>
<tr>
<td>Housing investment to housing stock ratio ($x_H/H$)</td>
<td>0.040</td>
<td>0.0403</td>
<td>-0.388</td>
</tr>
<tr>
<td>Ratio housing services to consumption of goods ($Rd/c$)</td>
<td>0.230</td>
<td>0.2291</td>
<td>-0.411</td>
</tr>
<tr>
<td>Ratio fixed capital investment to output ($\delta K/Y$)</td>
<td>0.135</td>
<td>0.1353</td>
<td>0.339</td>
</tr>
<tr>
<td>Homeownership rate</td>
<td>0.635</td>
<td>0.6370</td>
<td>-0.468</td>
</tr>
</tbody>
</table>

Variable | Parameter | Estimate |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual discount rate</td>
<td>$\beta$</td>
<td>0.9749</td>
</tr>
<tr>
<td>Share of consumption goods in the utility function</td>
<td>$\gamma$</td>
<td>0.9541</td>
</tr>
<tr>
<td>Tax function coefficient</td>
<td>$\eta_0$</td>
<td>0.1974</td>
</tr>
<tr>
<td>Depreciation rate of owner occupied housing</td>
<td>$\delta_o$</td>
<td>0.0340</td>
</tr>
<tr>
<td>Depreciation rate of rental housing</td>
<td>$\delta_r$</td>
<td>0.0749</td>
</tr>
<tr>
<td>Depreciation rate of capital stock</td>
<td>$\delta_k$</td>
<td>0.0428</td>
</tr>
<tr>
<td>Minimum house size</td>
<td>$h$</td>
<td>1.4726</td>
</tr>
</tbody>
</table>

Since the model has been estimated to replicate the aggregate moments we explore whether reasonable housing statistics are generated. The model could be evaluated along a number of dimensions. We focus on the distribution of ownership rates by age; the distribution of housing consumption measured in square feet by age and household income; and the implications for the rental market. In Table 7 we summarize how the homeownership rate, the distribution of landlords, and housing consumption vary by age. We also report how housing size varies by income.

Table 3: Summary of Aggregate Results

<table>
<thead>
<tr>
<th>Variable</th>
<th>Homeownership Rate</th>
<th>by Age Cohorts</th>
<th>20-34</th>
<th>35-49</th>
<th>50-64</th>
<th>65-74</th>
<th>75-89</th>
</tr>
</thead>
<tbody>
<tr>
<td>AHS Data</td>
<td></td>
<td></td>
<td>33.2</td>
<td>67.7</td>
<td>78.4</td>
<td>80.3</td>
<td>73.5</td>
</tr>
<tr>
<td>Benchmark</td>
<td></td>
<td></td>
<td>33.1</td>
<td>73.5</td>
<td>86.4</td>
<td>91.3</td>
<td>66.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Distribution of Landlords</th>
<th>by Age Cohorts</th>
<th>20-34</th>
<th>35-49</th>
<th>50-64</th>
<th>65-74</th>
<th>75-89</th>
</tr>
</thead>
<tbody>
<tr>
<td>POMS Data</td>
<td></td>
<td></td>
<td>4.2</td>
<td>27.4</td>
<td>42.8</td>
<td>19.6</td>
<td>6.0</td>
</tr>
<tr>
<td>Benchmark</td>
<td></td>
<td></td>
<td>21.7</td>
<td>33.7</td>
<td>24.1</td>
<td>13.8</td>
<td>6.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sqft. Owners</th>
<th>by Income Quintiles</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5</th>
</tr>
</thead>
<tbody>
<tr>
<td>AHS Data</td>
<td></td>
<td>Q1</td>
<td>1,867</td>
<td>2,211</td>
<td>2,699</td>
<td>3,037</td>
<td>3,091</td>
</tr>
<tr>
<td>Benchmark</td>
<td></td>
<td>Q2</td>
<td>2,254</td>
<td>2,192</td>
<td>2,773</td>
<td>3,887</td>
<td>4,242</td>
</tr>
</tbody>
</table>

1 Owner occupied house size is measured in terms of square feet.
Data is from the *American Housing Survey* and *Property Owners and Manager Survey*. The model captures the hump-shaped behavior of ownership as the fraction of homeowners increases by age until retirement. Downsizing in the older cohorts is observed, but participation are overstated for the middle age cohorts. This can be due to the fact that in a model with only idiosyncratic labor income shock, most households eventually end up becoming homeowners. Since a focal point of housing policy is the participation of the younger households, we report the homeownership rate for households age 35 and under. The data indicates an ownership rate of 33.2 percent for all households under 35 while the model generates a corresponding homeownership rate of 33.1 percent. When studying the impact of the current tax treatment of homeowners and landlords we pay particular attention to this age group.

We are also interested in determining whether the model generates reasonable participation behavior in the supply of rental housing services. In Table 7 we report how the participation of household who are also landlord varies by age. The data for this distribution is based on data from the 1995 *Property Owners and Managers Survey*. We observe that the distribution of landlords exhibits a hump which occurs between age 50 and 64. The model generates a humped shaped participation pattern. However, the peak in the hump occurs somewhat earlier than in the model.

A frequent question is why have house sizes increased in the United States. One response is the current tax code which provides incentives to consume large units of owner-occupied housing. If this issue is to be addressed, it is important to inquire whether the model generates distributions of the consumption of housing services similar to what is actually observed. Some papers measure housing consumption using expenditure to measure housing services. Others just report the ratio with respect to goods consumption (defined in a broad sense). We report housing consumption in terms of square feet - the measure most frequently used to measure house size. We find that the model generates two important features observed in the data. The house size implied in the model are consistent with house sizes observed from either an age or income distribution perspective. The largest average size house occurs in households between age 50 and age 64. The average house size for this cohort is 2,301 square feet. The model generates an average house size for this cohort of 2,429 square feet. For the youngest age cohort, the average size of a house in the data is 1,854 square feet, whereas our model finds that the average size house for this cohort is 2,147 square feet. Hence, we do find evidence that the model overpredicts housing size. If housing size is examined by an income perspective, data shows that house size increases with income. The average size house of a homeowner in the lowest income quintal is 1,867 square feet while the average size house at the highest income quintal is 3,091 square feet. The model finds that home size tends to increase with income. However, the model once again overpredicts house size. The relatively smaller house size observed in the data for the top income quintiles may partially explained by the top coding, as well as the under sampling, of high income households in the *American Housing Survey*.

### 2 House Prices

This section summarizes some preliminary findings suggesting that the housing model can replicate movements in house prices and user cost of housing consistent with the empirical evidence.
The stationary version of the model is capable of generating house price increases that are consistent with the average capital gains realized between 1990 and 2005. These capital gains can be rationalized with the decline in mortgage rates and the introduction of new mortgage products that either reduce the downpayment constraint, or the structure of the repayment profile of mortgage contracts. The model can also rationalize declines in house prices that are consistent with the observed counterpart. The baseline model also replicates the observed decline in the user cost of housing defined as the ration between the price index for rental property and owner-occupied housing. The evidence suggest that between 1994 and 2005 this ratio drop by 18.7 percent, and the model is capable of generating declines of a similar magnitude. Our formulation with an elastic supply of rental property seem to reconcile a feature that model based on arbitrage conditions are incapable of replicate.

### 3 Foreclosures and House Prices

To be completed...

### 4 Dynamics of Foreclosures and House Prices

To be completed...

### 5 Conclusions

To be completed...