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- We will answer this question using a very simple aggregate (or economywide) model of economic growth.
- The model we will study is called the Solow model (after the Nobel Prize-winning economist Robert Solow at M.I.T.).

The Aggregate Production Function Revisited

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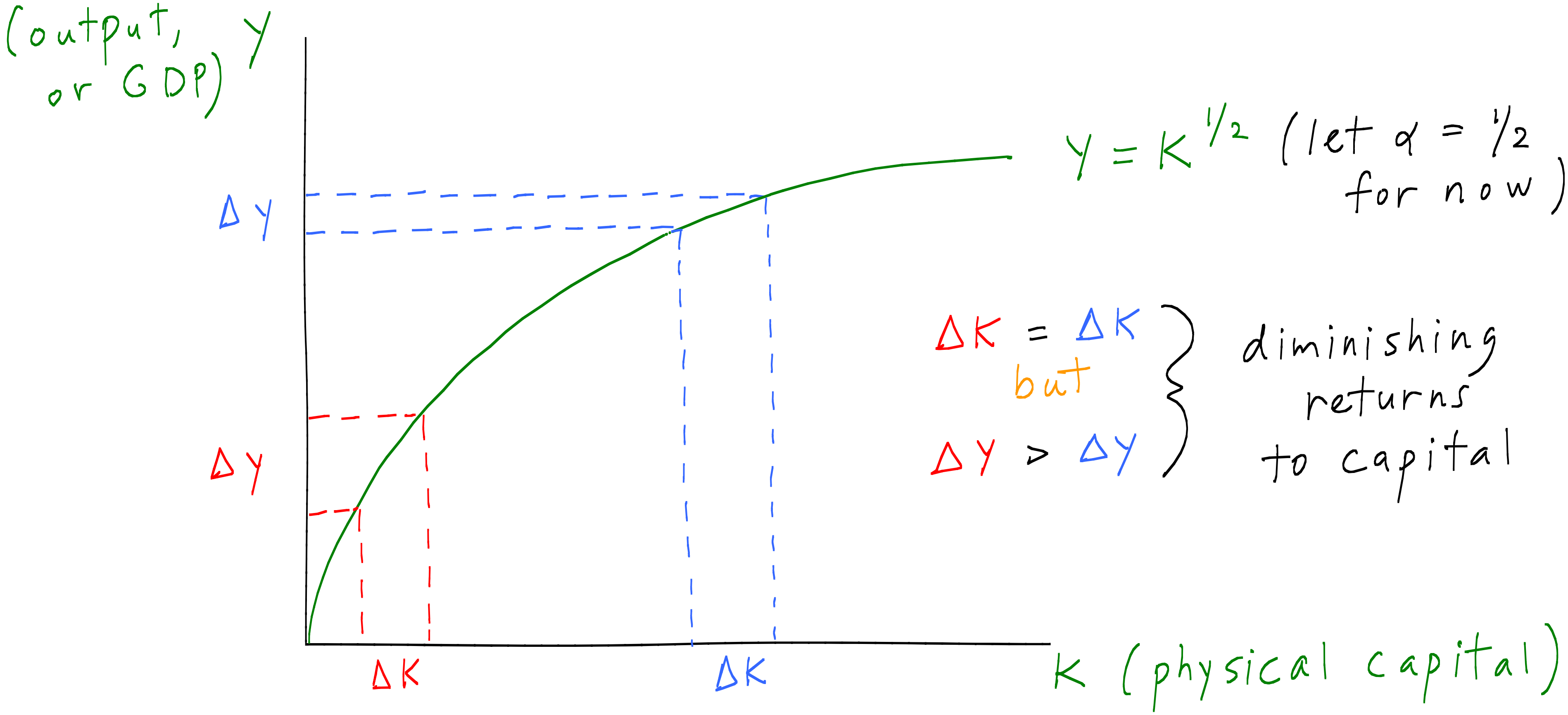
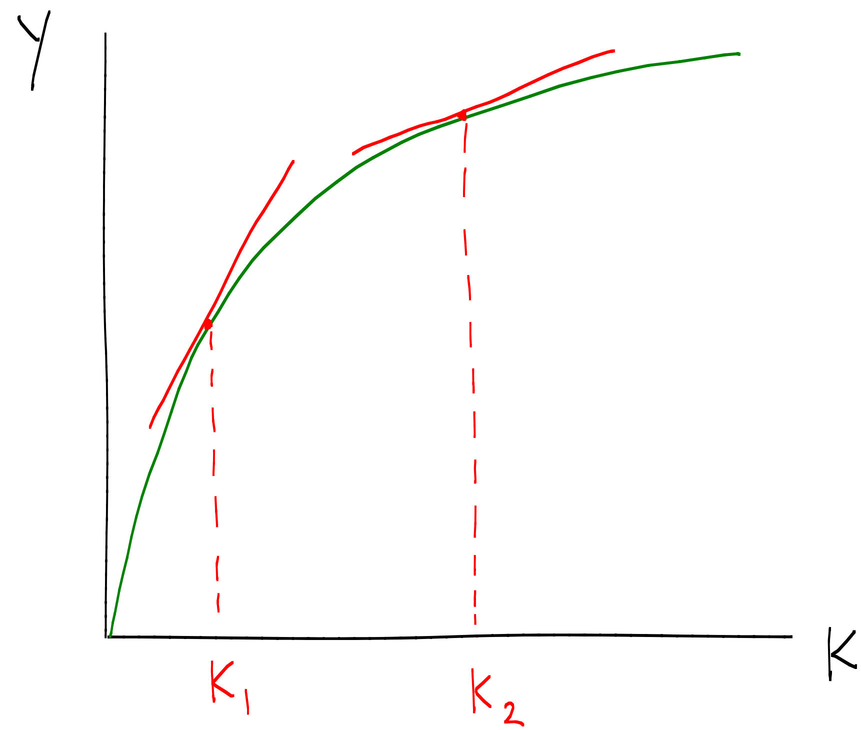
- Let's study the shape of the aggregate production function (again, holding technology and employment constant).



This production function exhibits diminishing returns to capital: the extra output from a little bit more capital decreases as K increases.

Put differently, diminishing returns to capital means that the slope of the production function decreases as capital increases:



The slope of the tangent line is higher at K_1 than at K_2.

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- The marginal product of capital is the amount by which output increases when capital increases by a (very) small amount.
- The declining marginal product of capital suggests that it will be difficult to generate sustained growth simply by increasing capital over time.

Sergey Brin, co-founder of Google,

on diminishing returns to capital

From:
New York Times
Business Section,
10/20/06

Mr. Brin said that he saw no end to other innovations. "You might imagine the lower-hanging fruit has been picked," he said, "but at the same time we have built ladders and are reaching for larger, higher-hanging fruit."

link to full article: http://www.nytimes.com/2006/10/20/technology/20google.html

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- In other words, if the economy does not invest today, there will be no capital with which to produce tomorrow.

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This is the law of motion for the economy's capital stock.

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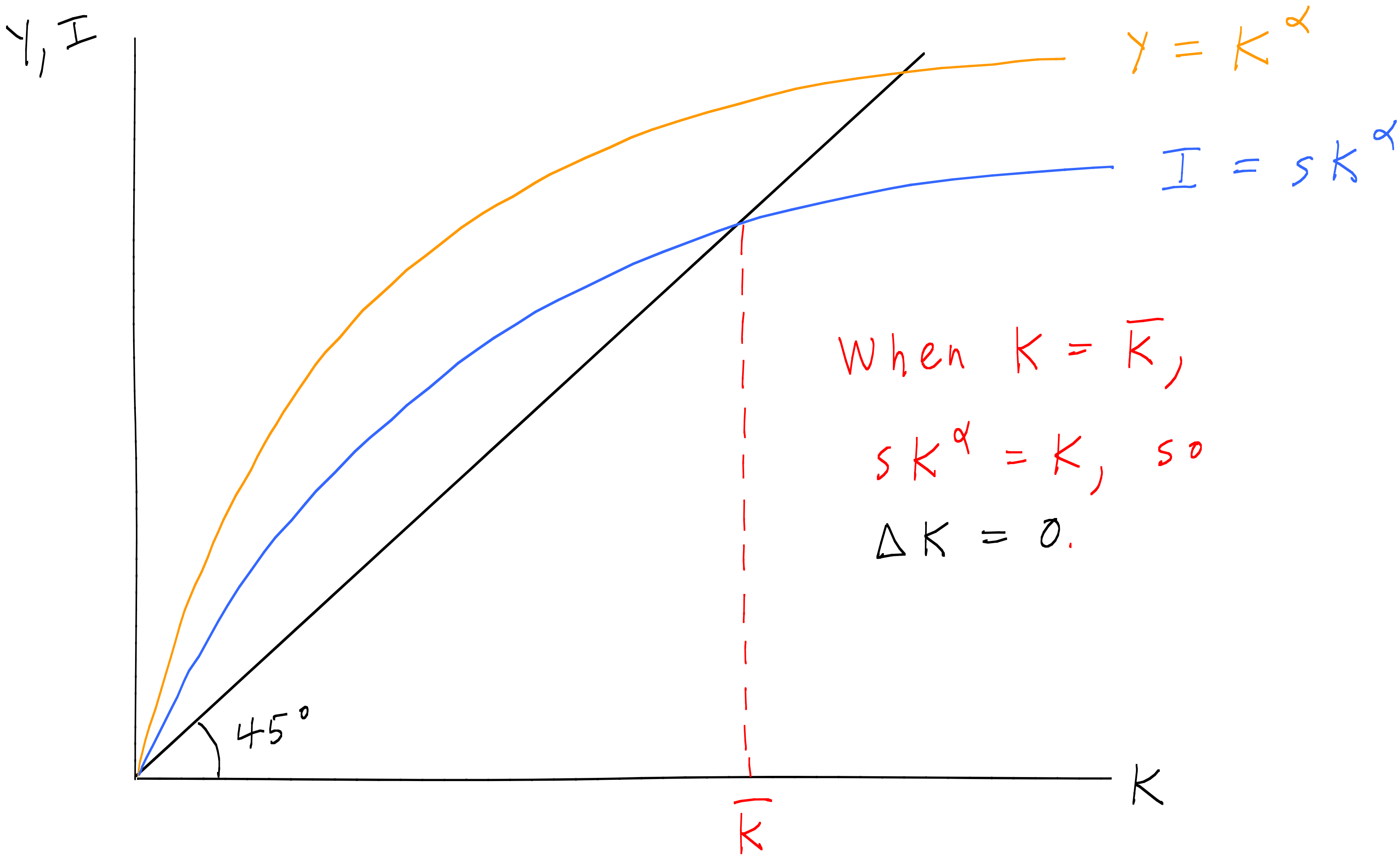
- ΔK_{t+1} is zero if $sK_t^{\alpha} = K_t$.

A Useful Graph



$\bar{K}$ is the steady state value of capital.

Let the initial period be $t = 0$.

IF $K_0 = \bar{K}$, THEN THE ECONOMY'S CAPITAL STOCK REMAINS AT $\bar{K}$.

period 0: $K_1 = sK_0^\alpha = \underbrace{s\bar{K}^\alpha = \bar{K}}$

this equation defines $\bar{K}$

period 1: $K_2 = sK_1^\alpha = s\bar{K}^\alpha = \bar{K}$

period 2: $K_3 = sK_2^\alpha = s\bar{K}^\alpha = \bar{K}$

periods 3, 4, 5, ... : more of the same

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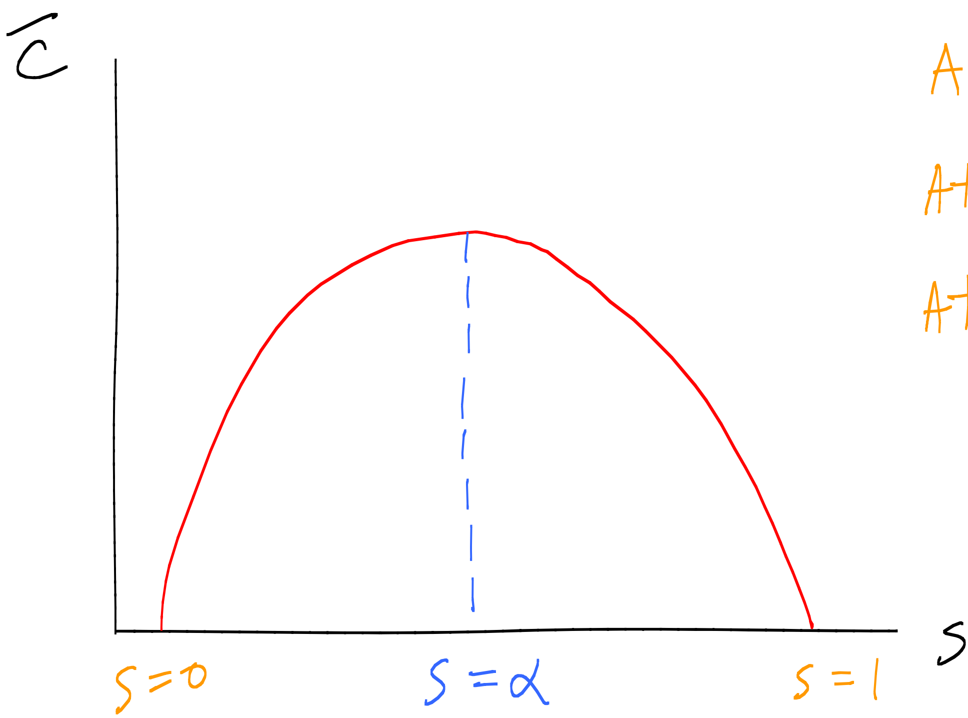
- Steady-state consumption is: $\overline{C} = (1 - s)\overline{Y}$.

THE GOLDEN RULE

(discovered by Edmund Phelps, last year's winner of the Nobel Prize in Economics)



At $s=0$, $\bar{K}=0$, so $\bar{c}=0$

At $s=1$, $\bar{c}=(1-s)\bar{y}=0$

At $s=\alpha$, $\bar{c}$ is maximized

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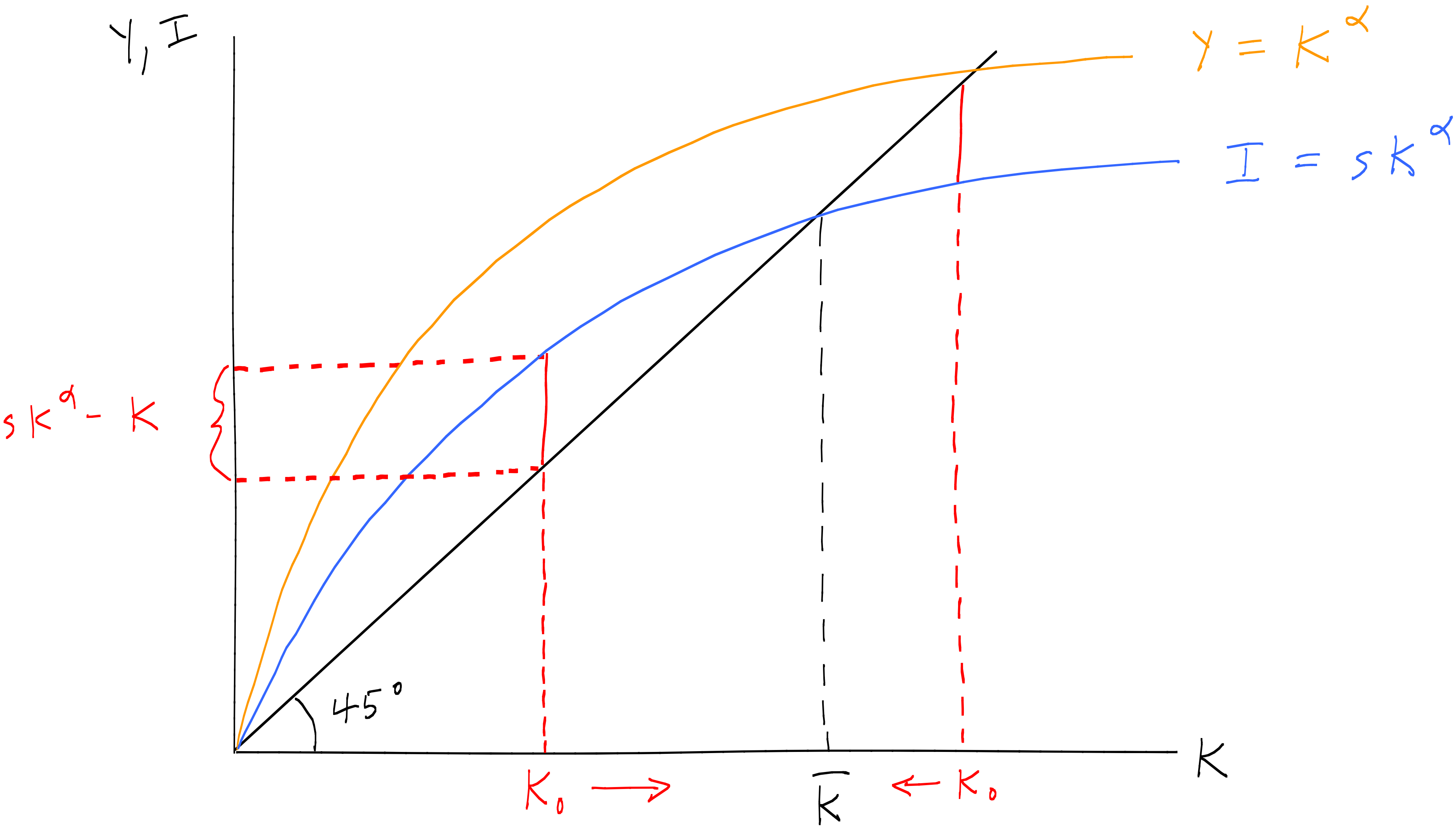
- Short answer: The economy always converges to $\overline{K}$ (as long as the initial capital stock is positive).
- However, it takes an infinite amount of time to get to the steady state.



When $K < \bar{K}$, $sK^\alpha - K > 0$, so K increases.

When $K > \bar{K}$, $sK^\alpha - K < 0$, so K decreases.

Dynamics using algebra

$$K_1 = sK_0^\alpha$$
$$K_2 = sK_1^\alpha$$
$$K_3 = sK_2^\alpha$$
etc.

$$C_0 = Y_0 - I_0 = K_0^\alpha - K_1$$
$$C_1 = Y_1 - I_1 = K_1^\alpha - K_2$$
$$C_2 = Y_2 - I_2 = K_2^\alpha - K_3$$
etc.

"Eventually", both K_t and C_t converge to their steady-state values $\bar{K}$ and $\bar{C}$.

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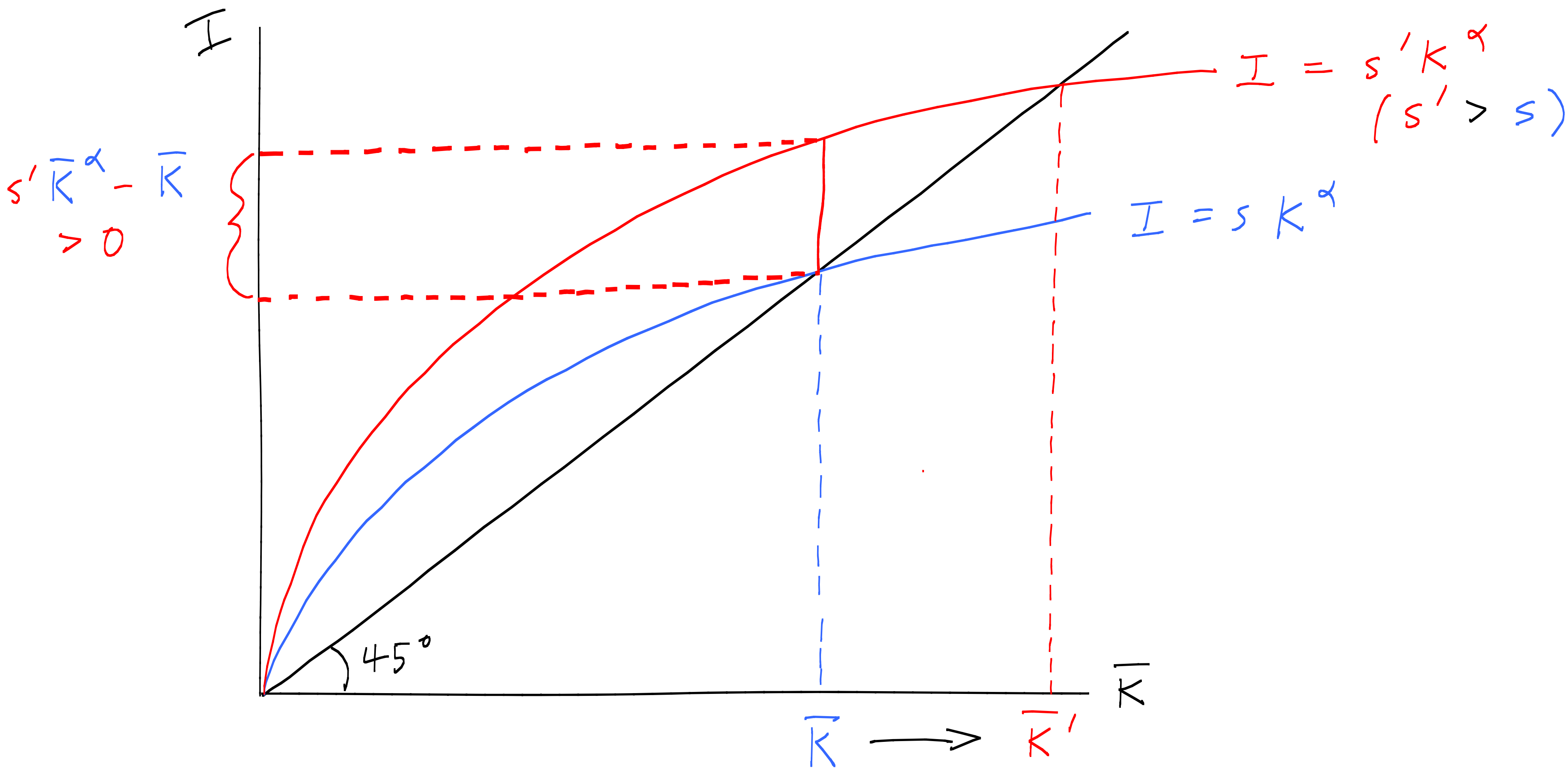
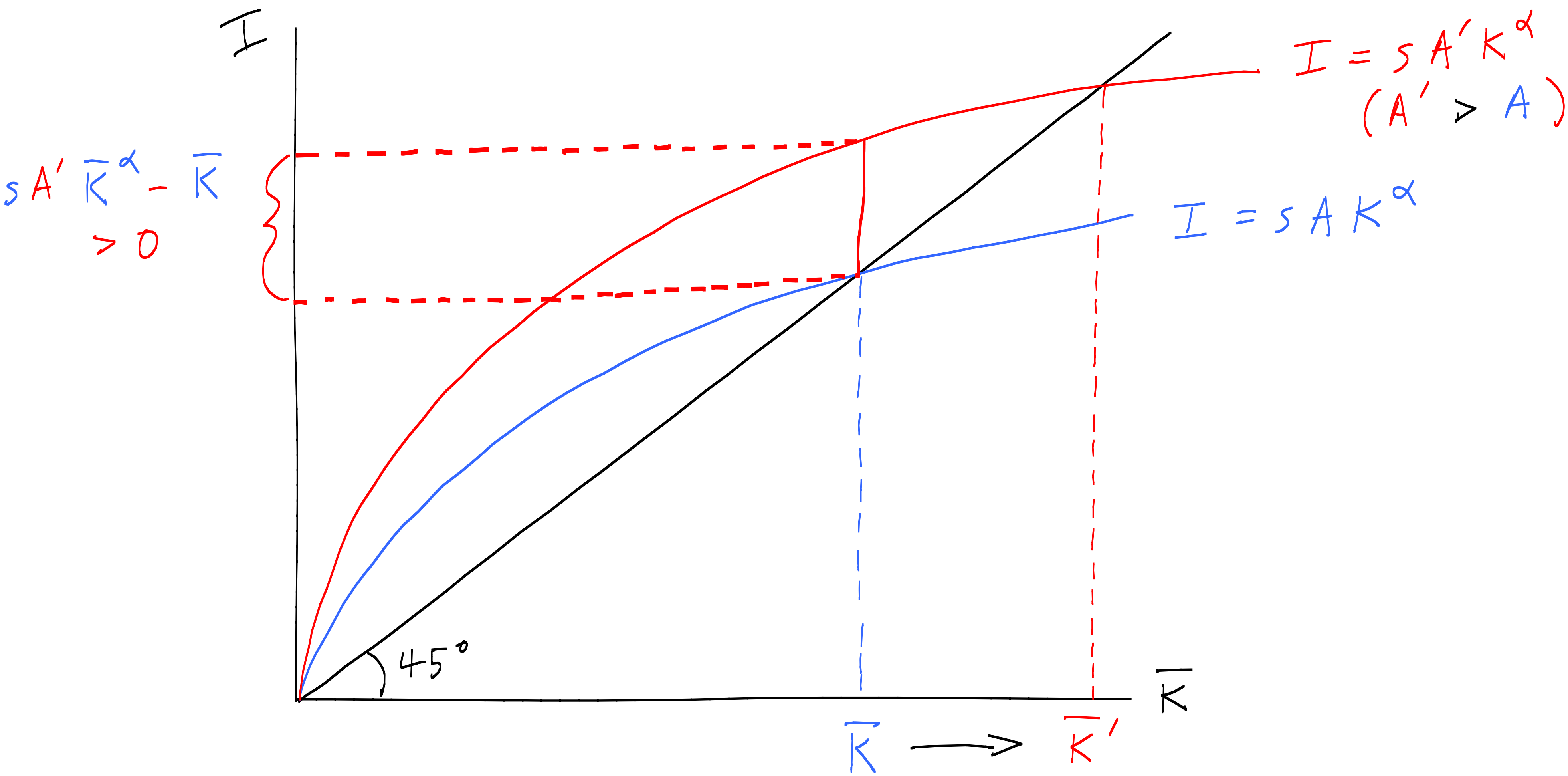
- The savings rate does NOT affect growth in the long run (that is, after the economy converges to its steady state).
- Increases in the savings rate DO affect growth in the short run but NOT in the long run.

An Increase in the Savings Rate



When s increases, the economy moves to a new higher steady state.

An Improvement in Technology



When technology improves (from A to A'), the economy moves to a new higher steady state.

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- This is what Sergey Brin means by "building ladders to larger, higher-hanging fruit."