1. This problem studies a version of the Solow growth model developed in Chapter 6 of the textbook. Suppose that aggregate output in period \( t \) is given by: \( Y_t = K_t^{0.3}N_t^{0.7} \).

(a) Calculate the steady-state capital stock implied by the law of motion for aggregate capital. Calculate the level of steady-state consumption implied by the steady-state capital stock...

**ANSWER:** The steady state capital stock is the value for capital \( K^* \) such that

\[
K^* = (1 - d)K^* + sY(K^*)
\]

Note that we have assumed \( N_t = N = 1 \), so we simply plug it in:

\[
K^* = (1 - 0.1)K^* + 0.2(K^*)^{0.3}(1)^{0.7}
\]

\[
K^*(0.1) = 0.2(K^*)^{0.3}
\]

\[
(K^*)^{0.7} = 0.2/0.1 = 2
\]

\[
K^* = (0.2/0.1)^{1/0.7} = 2.69
\]

To obtain consumption, simply find which part of output (the steady state output given by steady state capital \( K^* \)) is not saved but consumed:

\[
C^* = Y^* - I^* = (1 - s)Y^* = 0.8 \times 2.69^{0.3} = 0.8 \times 1.34 = 1.07
\]

(b) Suppose that \( s \) increases to 0.25. What is the new steady-state capital stock? What is the new level of steady-state consumption?

**ANSWER:** Clearly, substituting 0.25 instead of 0.2 in the equation above gives:

\[
K^* = 2.5^{1/0.7} = 3.70
\]

\[
C^* = 0.75 \times 3.70^{0.3} = 0.75 \times 1.48 = 1.11
\]
(c) Describe in qualitative terms the dynamic behavior of the capital stock after the increase in the savings rate (assume that the economy is in a steady state before the increase). Support your answer using an appropriate diagram.

**ANSWER:** Qualitatively, when the rate of saving increases, there is less output available for consumption and more for investment out of the original output, $Y(K^*)$, so that consumption falls first, while capital starts growing until it reaches its new steady state $\hat{K}^*$. Consumption in this new steady state might be above or below the original level of consumption. This is shown in the following diagrams:

![Figure 1: Qualitative effect of increase in savings](image1)

![Figure 2: Time path of Capital, Consumption, Investment](image2)

The first diagram shows output, saving, and consumption in each steady state. The second diagram shows the period-by-period path of capital from $K^*$ to $\hat{K}^*$. 

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The third diagram shows the paths of consumption, investment and capital in time. It also shows that depending on the original level of capital, consumption can end up being higher or lower than its original level, although in the numerical exercise above it ends up being higher.

(d) Show that the golden-rule savings rate (i.e., the value of the savings rate that maximizes steady-state consumption) is equal to the exponent on capital in the aggregate production function (i.e., 0.3).

**ANSWER:** To answer this question you first have to obtain consumption as a function of the savings rate $s$. Then you have to take the derivative of $C^*$ with respect to $s$ and find the point where it is 0:

$$K^*(s) = \left(\frac{s}{0.10}\right)^{1/0.7}$$
$$\Rightarrow C^*(s) = (1 - s)Y(K^*(s)) = (1 - s)\left(\frac{s}{0.10}\right)^{0.4}$$
$$\Rightarrow dC^*/ds = - \left(\frac{s}{0.10}\right)^{0.3} + (1 - s)\left(\frac{0.3}{0.7}\right)\left(\frac{s}{0.10}\right)^{0.3-1}\left(\frac{1}{0.10}\right)$$
$$= -1 + (1 - s)\left(\frac{0.3}{0.7}\right)\left(\frac{0.10}{s}\right)(1/0.1) = -1 + \left(\frac{0.3}{0.7}\right)\frac{1 - s}{s}$$
$$\Rightarrow \frac{s}{1 - s} = \frac{0.3}{0.7}$$
$$\Rightarrow s = 0.3$$

(e) Suppose again that $s = 0.2$ and that the economy is in a steady state. Explain what happens to the capital stock if the depreciation rate suddenly increases to 0.15.

**ANSWER:** As the depreciation rate increases, a given level of capital requires a higher investment to replace the capital lost to depreciation. Therefore, the steady state capital level for a given savings rate $s$ is lower. The actual capital level can be found substituting again in the equations above:

$$K^* = (0.20/0.15)^{1/0.7} = 1.50$$
This result shows that an increase in depreciation implies a large decrease in the steady state level of capital. Figure 2 presents this fact.

Figure 3: Increase in depreciation

2. This problem investigates the relationship between government budget deficits and trade deficits. (Although this problem is related to material in Chapter 5 of the textbook, Chapter 5 is not required reading and you should be able to work this problem without reading it.)...

(a) Find the equilibrium interest rate. Verify that net exports equal 0 in both countries...

**ANSWER:** To find the equilibrium interest rates, set total world investment equal to total world savings, and then solve for $r$, taking advantage of the fact that the two net export figures must add up to zero:

\[
S_1 + S_2 = Y_1 - C_1 - G_1 - NX_1 + Y_2 - C_2 - G_2 - NX_2 \\
= (1000 - 0.8(800) - 200) + (1000 - 0.8(800) - 200) = 320 \\
\]

\[
I_1 + I_2 = 210 - 1000r + 210 - 1000r = 420 - 2000r \\
\Rightarrow 320 = 420 - 2000r \\
\]
To verify that net exports are 0 for each country, we verify that their \( Y = C + I + G \) for each of them.

\[
1000 = 0.8(800) + 210 - 50 + 200 = 1000
\]

\[ \Rightarrow NX_i = 0, \quad i \in 1, 2 \]

(b) Suppose now that \( G_1 \) increases to 250 while \( T_1 \) remains fixed. (Assume that Ricardian equivalence does not hold.) Find the new equilibrium interest rate as well as net exports in each country...

**ANSWER:** We repeat the procedure above:

\[
S_1 + S_2 = Y_1 - C_1 - G_1 - NX_1 + Y_2 - C_2 - G_2 - NX_2
\]

\[ = (1000 - 0.8(800) - 250) + (1000 - 0.8(800) - 200) = 270 \]

\[
I_1 + I_2 = 210 - 1000r + 210 - 1000r = 420 - 2000r
\]

\[ \Rightarrow 270 = 420 - 2000r \]

\[ \Rightarrow r = 150/2000 = 0.075 \]

The increase in demand produced a higher equilibrium interest rate. We find \( NX \) for each country to show how this deficit is being financed.

\[
Y_1 = 0.8(800) + 210 - 75 + 250 + NX_1 = 1000
\]

\[ \Rightarrow NX_1 = -25 \]

\[
Y_2 = 0.8(800) + 210 - 75 + 200 + NX_2 = 1000
\]

\[ \Rightarrow NX_2 = +25 \]
Finally, we verify that $S_i = I_i + NX_i$ for each country

\[
S_1 = 1000 - 0.8(800) - 250 = 110 \\
I_1 + NX_1 = 210 - 75 - 25 = 110 \\
\Rightarrow S_1 = I_1 + NX_1 \\
S_2 = 1000 - 0.8(800) - 200 = 160 \\
I_2 + NX_2 = 210 - 75 + 25 = 160 \\
\Rightarrow S_2 = I_2 + NX_2
\]

The government deficit is financed by decreases in investment at home and abroad. The surplus savings of country 2 are spent by country 1 and appear as a current account deficit.

(c) Now suppose that government spending in both countries rises to 250 (again, taxes do not change and Ricardian equivalence does not hold in either country)...

**ANSWER:** We find equilibrium interest rates as before:

\[
S_1 + S_2 = Y_1 - C_1 - G_1 - NX_1 + Y_2 - C_2 - G_2 - NX_2 \\
= (1000 - 0.8(800) - 250) + (1000 - 0.8(800) - 250) = 220 \\
I_1 + I_2 = 210 - 1000r + 210 - 1000r = 420 - 2000r \\
\Rightarrow 220 = 420 - 2000r \\
\Rightarrow r = 200/2000 = 0.10
\]

We expect $NX=0$ if both countries behave alike:

\[
Y_1 = 0.8(800) + 210 - 100 + 250 + NX_1 = 1000 \\
\Rightarrow NX_1 = 0 \\
Y_2 = 0.8(800) + 210 - 100 + 250 + NX_2 = 1000 \\
\Rightarrow NX_2 = 0
\]

In this case investment in each country absorbs the full decrease in savings.
(d) Suppose now that the two countries are closed: they cannot trade with each other (net exports are forced to equal zero in both countries) and there is a separate goods market in each country, with possibly different equilibrium interest rates.
...

**ANSWER:** In this case only the closed economy equations matter, that is, \( NX = 0 \) and therefore \( S_i = I_i \).

\[
S_1 = Y_1 - C_1 - G_1 \\
I_1 = 210 - 1000r \\
\Rightarrow 110 = 210 - 1000r \\
\Rightarrow r_1 = \frac{100}{1000} = 0.10
\]

The interest rate is again 10%. In this case the government budget is again completely financed by a decrease in investment. The comparison to parts (b) and (c) suggests that an open economy can rely on foreign savings to increase its consumption, as opposed to a close one. However, part (c) shows that this does not imply that there are overall more savings in the world.