HOMEWORK #8

This homework assignment is due at the beginning of lecture on Monday, April 11.

1. Do numerical problem #4 on p. 425 in Chapter 11 of the textbook. In addition, do part (d) below:

(d) Suppose that the economy is the long-run equilibrium in part (c) and \( G \) increases from 100 to 110 (\( T \) remains fixed at 100). Determine the new short-run equilibrium values of \( Y \), \( r \), \( C \), and \( I \) (\( P \) remains fixed at its initial value). Calculate the multiplier on government spending (i.e., the change in \( Y \) divided by the change in \( G \)). In addition, graph the \( I^d \), \( S^d \), \( IS \), and \( AD \) curves and show how they shift when \( G \) increases from 100 to 110.

(e) Optional bonus question: How do your answers in part (d) change if \( T \) increases at the same time as \( G \) so as to keep the government’s budget balanced?

2. Do analytical problem #5 on p. 466 in Chapter 12 of the textbook.

3. This problem studies the dynamic behavior of a macroeconomic model that consists of three equations: Okun’s law, an aggregate demand curve, and a Phillips curve. Write Okun’s law as follows:

\[
u_t - u_{t-1} = -0.5(g_{yt} - 2), \tag{1}\]

where \( u_t \) is the unemployment rate in year \( t \), \( u_{t-1} \) is the unemployment rate in year \( t - 1 \), and \( g_{yt} \) is the growth rate of output in year \( t \) (the subscript ‘\( y \)’ denotes output). To keep things simple, suppose that the aggregate demand curve takes the form

\[
Y_t = \frac{\gamma M_t}{P_t},
\]

where \( Y_t \) is output in year \( t \), \( M_t \) is the money supply in year \( t \), \( P_t \) is the price level in year \( t \), and \( \gamma \) is a positive constant. Using the growth rate formulas in Appendix A.7 on p. 606 of the textbook, this aggregate demand curve can be written in terms of growth rates as follows:

\[
g_{yt} = g_{mt} - \pi_t, \tag{2}\]

where \( g_{mt} \) is the growth rate of the money supply in year \( t \) (the subscript ‘\( m \)’ denotes money) and \( \pi_t \) is the (actual) inflation rate in year \( t \). Finally, let the Phillips curve
take the form:

\[ \pi_t - \pi_t^e = -2(u_t - 5), \]  
(3)

where \( \pi_t^e \) is expected inflation in year \( t \). Throughout this problem, assume that \( \pi_t^e = \pi_{t-1} \), i.e., expected inflation in year \( t \) is equal to actual inflation in year \( t - 1 \). The unemployment rate, the inflation rate, and the growth rates of output and money are all expressed as percentages.

(a) Suppose that the growth rate of the money supply is 12% per year (i.e., \( g_{mt} = 12 \) for all \( t \)). In addition, suppose that \( u_t = u_{t-1} = 5, \ g_{yt} = 2, \) and \( \pi_t = \pi_{t-1} = 10 \). Show that this set of values constitutes a \textit{steady state} of the dynamic economy defined by equations (1), (2), and (3). In other words, show that if the economy begins in this position, then it will stay in this position forever.

(b) Suppose instead that \( g_{mt} = 6 \) for all \( t \). Show that the steady-state values of \( u_t \), \( g_{yt} \), and \( \pi_t \) are 5, 2, and 4, respectively.

(c) Now suppose that the economy begins (in year 0) in the steady state discussed in part (a) and that the monetary authority (the Fed) wants to move the economy from this steady state to the steady state in part (b). In particular, suppose that the Fed wants to reduce the inflation rate from 10% to 4% in increments of 2% per year; that is, the Fed wants the inflation rate, which is 10% in year 0, to be 8% in year 1, 6% in year 2, and 4% in year 3 and in all subsequent years. The Fed can achieve this goal by varying the growth rate of money appropriately. How should the Fed choose the growth rate of money in years 1, 2, 3, 4, 5, and 6? (Recall that \( g_{m0} = 12 \).)

(d) Calculate the sacrifice ratio associated with the transition path that you calculated in part (d). (Be sure to read Box 12.4 on p. 461 in Chapter 12 of the textbook.) In this problem, define the sacrifice ratio to be total point-years of excess unemployment divided by the total decrease in inflation. In a given year, excess unemployment is defined to be the difference between the actual unemployment rate and the natural rate of unemployment (i.e., the rate of unemployment that obtains when inflation is in a steady state). Total point-years of excess unemployment is defined to be the sum of the values for excess unemployment during the time it takes for the economy to transit from one steady state to another. How does the sacrifice ratio compare to the values reported in Box 12.4?

(e) Suppose that inflation expectations are “rational” rather than “adaptive.” That is, rather than set \( \pi_t^e = \pi_{t-1} \), assume instead that \( \pi_t^e \) jumps immediately to its value in the new steady state (after the Fed announces the new steady-state growth rate of the money supply). What is the sacrifice ratio in this case? (Hint: How long does the transition to the new steady state take in this case?)