Econ 510a (second half)
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Solutions for Homework #2

Question 1

(a) The state variable for the planner's problem will be each period's capital stock K. The decision variables will be $K_i, L_i,$ and K_c, L_c. Given however the total labor supply L and the periods capital stock K, the decision variables reduce to one of the two pairs, here we will assume w.l.o.g. that they are K_i and L_i. Therefore the planners problem can be written as:

$$V(K) = \max_{K_i, L_i} \{u(F(K - K_i, L - L_i)) + \beta V(K')\}$$

where:

$$K' = (1 - \delta)K + G(K_i, L_i)$$

(b) The first-order conditions for this problem are given by:

$$u'(F(K_c, L_c))F_K(K_c, L_c) = \beta V_K(K')G_K(K_i, L_i)$$
$$u'(F(K_c, L_c))F_L(K_c, L_c) = \beta V_K(K')G_L(K_i, L_i)$$

Using the 2 equations above we easily get that:

$$\frac{F_K(K_c, L_c)}{G_K(K_i, L_i)} = \frac{F_L(K_c, L_c)}{G_L(K_i, L_i)}$$

and the envelope condition is:

$$V_K(K) = u'(F(K_c, L_c))F_K(K_c, L_c) + (1 - \delta)\beta V_K(K')$$

(c) In the steady state we have that $K = K' = \overline{K}$. Substituting this into the envelope condition we get that:

$$V_K(\overline{K}) = u'(F(K_c, L_c))F_K(K_c, L_c) + (1 - \delta)\beta V_K(\overline{K}) \Leftrightarrow$$
$$V_K(\overline{K}) = \frac{u'(F(K_c, L_c))F_K(K_c, L_c)}{1 - (1 - \delta)\beta}$$

Substituting the above into the first FOC we get:

$$u'(F(K_c,L_c))F_K(K_c,L_c) = \beta\frac{u'(F(K_c,L_c))F_K(K_c,L_c)}{1-(1-\delta)\beta}G_K(K_i,L_i) \Leftrightarrow$$

$$1 = \frac{\beta G_K(K_i,L_i)}{1-(1-\delta)\beta}$$

and from the second FOC we get:

$$u'(F(K_c,L_c))F_L(K_c,L_c) = \beta\frac{u'(F(K_c,L_c))F_K(K_c,L_c)}{1-(1-\delta)\beta}G_L(K_i,L_i) \Leftrightarrow$$

$$F_L(K_c,L_c) = \frac{\beta F_K(K_c,L_c)G_L(K_i,L_i)}{1-(1-\delta)\beta}$$

For the case of $F(K_{c,}L_c) = K_c^a L_c^{1-a}$ and $F = G,$ we have that:

$$\frac{F_K(K_c,L_c)}{G_K(K_i,L_i)} = \frac{F_L(K_c,L_c)}{G_L(K_i,L_i)} \Rightarrow$$

$$\frac{aK_c^{a-1}L_c^{1-a}}{aK_i^{a-1}L_i^{1-a}} = \frac{(1-a)K_c^a L_c^{-a}}{(1-a)K_i^a L_i^{-a}} \Leftrightarrow$$

$$\frac{L_c}{L_i} = \frac{K_c}{K_i} \Leftrightarrow \frac{L-L_i}{L_i} = \frac{\overline{K}-K_i}{K_i} \Leftrightarrow$$

$$L_i(K-K_i) = L_i\overline{K} - L_iK_i \Leftrightarrow$$

$$L_i = \frac{LK_i}{\overline{K}}$$

The expression we got for the first FOC above, becomes:

$$1 = \frac{\beta a}{1-(1-\delta)\beta}\left(\frac{K_i}{L_i}\right)^{a-1}$$

and substituting in for L_i this becomes:

$$1 = \frac{\beta a}{1-(1-\delta)\beta}\left(\frac{\overline{K}}{L}\right)^{a-1} \Leftrightarrow$$

$$\frac{\overline{K}}{L} = \left[\frac{\beta a}{1-(1-\delta)\beta}\right]^{\frac{1}{1-a}}$$

and we if we normalize $L = 1,$ we have:

$$\overline{K} = \left[\frac{\beta a}{1-(1-\delta)\beta}\right]^{\frac{1}{1-a}}$$

(d) We have already derived one of the equations needed:

$$\frac{F_K(K_c,L_c)}{G_K(K_i,L_i)} = \frac{F_L(K_c,L_c)}{G_L(K_i,L_i)}$$

Now we're going to derive the other one. Using the first-order condition we can solve for $V_K(K')$:

$$V_K(K') = \frac{u'(c)F_K}{\beta G_K(K_i,L_i)}$$

Plugging this into the envelope condition gives us:

$$\begin{aligned}V_K(K) &= u'(F(K_c,L_c))F_K(K_c,L_c) + (1-\delta)\beta\frac{u'(c)F_K}{\beta G_K(K_i,L_i)} = \\ &= u'(F(K_c,L_c))F_K(K_c,L_c)\left(1 + (1-\delta)\frac{1}{G_K(K_i,L_i)}\right)\end{aligned}$$

We update what we found by one period:

$$V_K(K') = u'(F(K_c',L_c'))F_K(K_c',L_c')\left(1 + (1-\delta)\frac{1}{G_K(K_i',L_i')}\right)$$

Now we can replace $V_K(K')$ in the first-order condition which now becomes:

$$\begin{aligned}u'(F(K_c,L_c))F_K(K_c,L_c) &= \beta G_K(K_i,L_i)u'(F(K_c',L_c'))F_K(K_c',L_c')\left(1 + (1-\delta)\frac{1}{G_K(K_i',L_i')}\right) \Leftrightarrow \\ u'(F(K_c,L_c))F_K(K_c,L_c) &= \beta u'(F(K_c',L_c'))F_K(K_c',L_c')\frac{G_K(K_i,L_i)}{G_K(K_i',L_i')}(G_K(K_i',L_i') + 1 - \delta)\end{aligned}$$

(e) Now we are going to write the two equations in terms of the state variable, namely K. If the decision rules are given by:

$$\begin{aligned}K_i &\equiv h(K) \\ Li &\equiv l(K)\end{aligned}$$

we have:

$$\frac{F_K(K-h(K),L-l(K))}{G_K(h(K),l(K))} = \frac{F_L(K-h(K),L-l(K))}{G_L(h(K),l(K))}$$

and:

$$u'(F(K-h(K),L-l(K)))F_K(K-h(K),L-l(K)) =$$
$$\beta u'(F(K'-h(K'),L-l(K')))F_K(K'-h(K'),L-l(K'))\frac{G_K(h(K),l(K))}{G_K(h(K'),l(K'))}(G_K(h(K'),l(K'))+1-\delta) \Leftrightarrow$$
$$u'(F(K-h(K),L-l(K)))F_K(K-h(K),L-l(K)) =$$
$$\beta u'(F((1-\delta)K+G(h(K),l(K))-h((1-\delta)K+G(h(K),l(K))),L-l((1-\delta)K+G(h(K),l(K))))\cdot$$
$$F_K((1-\delta)K+G(h(K))-h((1-\delta)K+G(h(K)))),L-l((1-\delta)K+G(h(K))))\cdot$$
$$\frac{G_K(h(K),l(K))}{G_K(h((1-\delta)K+G(h(K))),l((1-\delta)K+G(h(K))))}\cdot$$
$$(G_K(h((1-\delta)K+G(h(K))),l((1-\delta)K+G(h(K))))+1-\delta)$$

Differentiating these equations with respect to K and evaluating the derivatives at the steady state, yields 2 equations with 2 unknowns, $h'(\overline{K})$ and $l'(\overline{K})$.

Question 2

(a) When $A = A_H$, the planner's problem is given by the following value function:

$$V_H(K) = \max_{K'} \{u(A_H F(K) + (1-\delta)K - K') + \beta V_L(K')\}$$

when $A = A_L$ the planner's problem is going to be given by:

$$V_L(K) = \max_{K'} \{u(A_L F(K) + (1-\delta)K - K') + \beta V_H(K')\}$$

(b) If $u(c) = \log c$, $\delta = 1$ and $F(K_t) = K_t^a$, then the above value functions become:

$$V_H(K) = \max_{K'} \{\log(A_H K^a - K') + \beta V_L(K')\}$$
$$V_L(K) = \max_{K'} \{\log(A_L K^a - K') + \beta V_H(K')\}$$

We know guess that the value functions are of the form:

$$V_H = E + F\log K$$
$$V_L = G + J\log K$$

In that case, the equation above for V_H becomes:

$$E + F\log K = \max_{K'} \{\log(A_H K^a - K') + \beta(G + J\log K')\}$$

The first-order conditions are:

$$-\frac{1}{A_H K^a - K'} + \frac{\beta J}{K'} = 0 \Leftrightarrow$$

$$\frac{\beta J}{K'} = \frac{1}{A_H K^a - K'} \Leftrightarrow$$

$$K' = A_H \beta J K^a - \beta J K' \Leftrightarrow$$

$$g_H(K) = K' = \frac{\beta J A_H K^a}{1+\beta J}$$

Plugging the decision rule derived above back in the original equation:

$$E + F\log K = \log\left(A_H K^a - \frac{J A_H \beta K^a}{1+\beta J}\right) + \beta\left(G + J\log\frac{J A_H \beta K^a}{1+\beta J}\right) =$$

$$= \log A_H + a\log K - \log(1+\beta J) + \beta G + \beta J\log J A_H \beta - \beta J\log(1+\beta J) + \beta J a\log K$$

Therefore:

$$E = \log A_H - (1+\beta J)\log(1+\beta J) + \beta G + \beta J\log J A_H \beta$$

and:

$$F = a + \beta a J$$

Similarly the value function when $A = A_L$, is given by:

$$G + J\log K = \max_{K'^{\prime}} \{\log(A_L K^a - K') + \beta(E + F\log K')\}$$

The first-order condition gives the following decision rule:

$$g_L(K) = K' = \frac{\beta F A_L K^a}{1+\beta F}$$

Plugging the decision rule back into the equation above, we get:

$$G + J\log K = \log(A_L K^a - \frac{\beta F A_L K^a}{1+\beta F}) + \beta\left(E + F\log\frac{\beta F A_L K^a}{1+\beta F}\right) =$$

$$= \log A_L + a\log K - \log(1+\beta F) + \beta E + \beta F\log\beta F A_L + \beta F a\log K - \beta F\log(1+\beta F)$$

Therefore:

$$G = \log A_L - (1+\beta F)\log(1+\beta F) + \beta E + \beta F\log\beta F A_L$$

and

$$J = a + \beta a F$$

combining the expression above for J with the one we found for F, gives us:

$$J = a + \beta a(a + \beta aF) \Leftrightarrow$$
$$J = \frac{a + \beta a^2}{1 - \beta^2 a^2} = \frac{a(1 + \beta a)}{(1 + \beta a)(1 - \beta a)} = \frac{a}{1 - \beta a}$$

and thus:

$$F = a + \beta a \frac{a}{1 - \beta a} = \frac{a}{1 - \beta a} = J$$

Therefore the equation for E now becomes:

$$E = \log A_H - \left(1 + \frac{\beta a}{1 - \beta a}\right)\log\left(1 + \frac{\beta a}{1 - \beta a}\right) + \beta G + \frac{\beta a}{1 - \beta a}\log A_H \frac{\beta a}{1 - \beta a} =$$
$$= \frac{1}{1 - \beta a}\log A_H + \beta G + \frac{\beta a}{1 - \beta a}\log\frac{\beta a}{1 - \beta a} - \frac{1}{1 - \beta a}\log\frac{1}{1 - \beta a}$$

Let $M = \frac{\beta a}{1-\beta a}\log\frac{\beta a}{1-\beta a} - \frac{1}{1-\beta a}\log\frac{1}{1-\beta a}$. Then E becomes:

$$E = \frac{1}{1 - \beta a}\log A_H + \beta G + M$$

And similarly plugging in for E, in the equation above we had for G we have:

$$G = \frac{1}{1 - \beta a}\log A_L + \beta E + M$$

Thus solving the above system of 2 equations and 2 unknowns, we get:

$$E = (1 - \beta^2)^{-1}\left((1 - \beta a)^{-1}(\log A_H + \beta \log A_L) + M + \beta M\right)$$
$$G = (1 - \beta^2)^{-1}\left((1 - \beta a)^{-1}(\log A_L + \beta \log A_H) + M + \beta M\right)$$

Therefore the decision rule when $A = A_H$ is given by:

$$g_H(K) = \beta a A_H K^a$$

and when $A = A_{L,}$ is given by:

$$g_L(K) = \beta a A_L K^a$$

We will now show that there is a ”global convergence” to a ”two-cycle” in which K_t oscillates between two values. These 2 values are:

$$K_H = \beta a A_H K_H^a \Leftrightarrow$$
$$K_H = (\beta a A_H)^{\frac{1}{1-a}}$$

when $A = A_H$ and similarly:

$$K_L = (\beta a A_L)^{\frac{1}{1-a}}$$

when $A = A_{L.}$ We will show that the assumptions of Question 2 of Homework #1 hold indeed in this case and thus there is a global convergence to a "two-cycle". We have:

•$g_H(0) = g_L(0) = 0$

• Moreover we have that: $g'_H(K) = \beta a^2 A_H K^{a-1}$ and $g'_L(K) = \beta a^2 A_L K^{a-1}$ and thus $g'_H(0) = \infty$ and $g'_L(0) = \infty$ since $a - 1 < 0.$ Therefore:

$$g'_H(0)g'_L(0) = \infty > 1$$

• Furthermore:

$$g_H(g_L(K)) = \beta^{a+1} a^{a+1} A_H A_L^a K^{a^2}$$
$$g'_H(g_L(K)) = \beta^{a+1} a^{a+3} A_H A_L^a K^{a^2-1}$$

and:

$$g'_H(g_L(K))g'_L(K) = \beta^{a+1} a^{a+3} A_H A_L^a K^{a^2-1} \beta a^2 A_L K^{a-1} =$$
$$= \beta^{a+2} a^{a+5} A_H A_L^{a+1} K^{a^2+a-2}$$

and since $a^2 + a - 2 < 0$ (remember $0 < a < 1$) we have that:

$$\lim_{K\to\infty} g'_H(g_L(K))g'_L(K) = 0$$

Similarly:

$$g'_L(g_H(K))g'_H(K) = \beta^{a+2} a^{a+5} A_L A_H^{a+1} K^{a^2+a-2}$$

and:

$$\lim_{K\to\infty}= g'_L(g_H(K))g'_H(K) = 0$$

• Finally g_H and g_L are strictly increasing and strictly concave

Therefore as we saw in Question 2 of Homework #1, there is a global convergence to a "two-cycle" in which K oscillates between the 2 values we derived above.

Question 3

(a) A Competitive Equilibrium with date-0 trading for the economy $\{u_A, u_B\}$, $\{\omega_{it}\}_{t=0}^{\infty}$ is a vector of prices $\{p_t\}_{t=0}^{\infty}$ and a vector of quantities $\{c_{it}^*\}_{t=0}^{\infty}$ for $i = A, B$ such that

(1) For $i = A, B$,

$$\{c_{it}^*\}_{t=0}^{\infty} = \arg\max \sum_{t=0}^{\infty} \beta^t u(c_{it})$$

$$s.t.$$

$$\sum_{t=0}^{\infty} p_t c_{it} = \sum_{t=0}^{\infty} p_t \omega_{it}$$

(2) $c_{At} + c_{Bt} = \omega_{At} + \omega_{Bt}\ for\ t = 0, 1, 2\ldots$

(b) The F.O.C. for consumer i is

$$\frac{\beta^j u'(c_{i,t+j})}{u'(c_{i,t})} = \frac{p_{t+j}}{p_t} \text{ for } \forall t, j$$

This together with budget constraint and market clearing condition determines the competitive equilibrium. Here there are two ways to solve for the equilibrium. One way is to solve for the system of simultaneous equations; another way is to make a guess of solution and check the feasibility for each equations. Due to the special structure of the model, here it is easier to proceed with the second way. Now guess that $c_{it} = c_i$ for $\forall t$. Plug into the F.O.C. and normalize $p_0 = 1$ we have

$$p_t = \beta^t$$

Plug into the budget constraint for each individual, we have

$$\sum_{t=0}^{\infty} p_t c_{it} = \sum_{t=0}^{\infty} p_t \omega_{it}$$

$$\Rightarrow \sum_{t=0}^{\infty} \beta^t c_i = \sum_{t=0}^{\infty} \beta^t \omega_i$$

$$\Rightarrow c_i = \omega_i$$

t is easy to check that it satisfies the market clearing condition. Therefore, the competitive equilibrium for this economy is

$$c_{At} = \omega_A = 2$$

$$c_{Bt} = \omega_B = 1$$

$$p_t = \beta^t$$

(c) Again guess that $c_{it} = c_i$ for $\forall t$. Plug into the F.O.C. and normalize $p_0 = 1$ we have

$$p_t = \beta^t$$

Plug into the budget constraint for each individual, we have

$$
\begin{aligned}
& \sum_{t=0}^{\infty} p_t c_{it} = \sum_{t=0}^{\infty} p_t \omega_{it} \\
\Rightarrow\ & \sum_{t=0}^{\infty} \beta^t c_i = \sum_{t=0}^{\infty} \beta^t \omega_i \\
\Rightarrow\ & \sum_{t=0}^{\infty} \beta^t c_i = \sum_{t=0}^{\infty} \beta^{2t} \omega_i^e + \beta \sum_{t=0}^{\infty} \beta^{2t} \omega_i^o \\
\Rightarrow\ & c_i = \frac{1}{1+\beta} \omega_i^e + \frac{\beta}{1+\beta} \omega_i^o
\end{aligned}
$$

where ω_i^e and ω_i^o means w_i in even and odd period, respectively.

It is easy to check that it satisfies the market clearing condition. Therefore, the competitive equilibrium for this economy is

$$
\begin{aligned}
c_{At} &= \frac{2+\beta}{1+\beta} \\
c_{Bt} &= \frac{1+2\beta}{1+\beta} \\
p_t &= \beta^t
\end{aligned}
$$

(d) Now guess the equilibrium as

$$
\begin{aligned}
c_{At} &= c_A^o \text{ for } t \text{ odd} \\
c_{At} &= c_A^e \text{ for } t \text{ even} \\
c_{Bt} &= c_A^o \text{ for } t \text{ odd} \\
c_{Bt} &= c_B^e \text{ for } t \text{ even} \\
p_t &= \beta^{t-1} p^o \text{ for } t \text{ odd} \\
p_t &= \beta^t \text{ for } t \text{ even}
\end{aligned}
$$

Plug into the F.O.C. we get

$$
\begin{aligned}
& \frac{\beta^j u'(c_{i,t+j})}{u'(c_{i,t})} = \frac{p_{t+j}}{p_t} \text{ for } \forall t, j \\
\Rightarrow\ & \frac{\beta u'(c_i^o)}{u'(c_i^e)} = p^o \\
\Rightarrow\ & \frac{\beta c_i^e}{c_i^o} = p^o
\end{aligned}
$$

Therefore, we have

$$
p^o = \beta \frac{c_A^e}{c_A^o} = \beta \frac{c_B^e}{c_B^o} = \beta \frac{c_A^e + c_B^e}{c_A^o + c_B^o} = \beta \frac{\omega_A^e + \omega_B^e}{\omega_A^o + \omega_B^o} = \frac{3\beta}{5}
$$

where the third equality comes from the market clearing condition. Moreover:

$$\frac{c_i^e}{c_i^o} = \frac{3}{5}$$

So the equilibrium price is

$$p_t = \frac{3}{5}\beta^t \text{ for } t \text{ odd}$$
$$p_t = \beta^t \text{ for } t \text{ even}$$

Plug p_t into budget constraint, we have

$$\begin{aligned}
&\sum_{t=0}^{\infty} p_t c_{it} = \sum_{t=0}^{\infty} p_t \omega_{it} \\
\Rightarrow\ &\sum_{t=0}^{\infty} \beta^{2t} c_i^e + \frac{3}{5}\sum_{t=0}^{\infty} \beta^{2t+1} c_i^o = \sum_{t=0}^{\infty} \beta^{2t}\omega_i^e + \frac{3}{5}\sum_{t=0}^{\infty} \beta^{2t+1}\omega_i^o \\
\Rightarrow\ &c_i^e + \frac{3\beta}{5} c_i^o = \omega_i^e + \frac{3\beta}{5}\omega_i^o \\
\Rightarrow\ &\frac{3}{5} c_i^o + \frac{3\beta}{5} c_i^o = \omega_i^e + \frac{3\beta}{5}\omega_i^o \quad \left(\textit{plug into } \frac{c_i^e}{c_i^o} = \frac{3}{5}\right) \\
\Rightarrow\ &c_i^o = \frac{5\omega_i^e + 3\beta\omega_i^o}{3(1+\beta)}
\end{aligned}$$

i.e.

$$c_A^o = \frac{10 + 6\beta}{3(1+\beta)}$$
$$c_B^o = \frac{5 + 9\beta}{3(1+\beta)}$$

and so

$$c_A^e = \frac{3}{5} c_A^o = \frac{10 + 6\beta}{5(1+\beta)}$$
$$c_B^e = \frac{3}{5} c_B^o = \frac{5 + 9\beta}{5(1+\beta)}$$

(e) A Competitive Equilibrium with sequential trading for the economy $\{u_A, u_B\}$, $\{\omega_{it}\}_{t=0}^{\infty}$ is a sequence $\{c_{it}^*\}_{t=0}^{\infty}, \{a_{i,t+1}^*\}_{t=0}^{\infty}, \{R_t^*\}_{t=0}^{\infty}$ (where R_t^* means interest rate from t to $t+1$) for $i = A, B$ such that

(1) For $i = A, B$,

$$\{c_{it}^*, a_{i,t+1}^*\}_{t=0}^{\infty} = \arg\max \sum_{t=0}^{\infty} \beta^t u(c_{it})$$

$$s.t.$$

$$c_{it} + a_{i,t+1} = R_t^* a_{i,t} + \omega_{it}$$

$$\lim_{t\to\infty} a_{i,t+1}\left(\prod_{j=0}^{t+1} R_j^{-1}\right) \geq 0$$

$$a_{i,0} = 0, c_{it} \geq 0$$

(2) $c_{At}^* + c_{Bt}^* = \omega_{At} + \omega_{Bt}$ *for* $t = 0,1,2\ldots$

(3) $a_{A,t}^* + a_{B,t}^* = 0$ *for* $t = 0,1,2\ldots$

Now we start to solve for equilibrium interest rate and asset holdings for different examples. In each example, it is easy to see that

$$R_t = \frac{p_{t-1}}{p_t}$$

$$a_{i,t+1} = R_t^* a_{i,t} + \omega_{it} - c_{it}$$

We start with part (b). Plug in the solution, we have

$$R_t = \frac{1}{\beta}$$

$$a_{i,t} = 0$$

In part (c), we have

$$R_t = \frac{1}{\beta}$$

$$a_{i,0} = 0$$

$$a_{i,1} = \omega_{i0} - c_{i0}$$

$$a_{i,2} = R_2^* a_{i,1} + \omega_{i1} - c_{i1} = \frac{1}{\beta}(\omega_{i0} - c_{i0}) + \omega_{i1} - c_{i1}$$

$$\ldots$$

Plug in the equilibrium solution, we have

$$R_t = \frac{1}{\beta}$$

$$a_A^o = -a_B^o = \frac{\beta}{1+\beta} \text{ for } t = 1,3,5,\ldots$$

$$a_A^e = -a_B^e = 0 \text{ for } t = 0,2,4,\ldots$$

In part (d), we have

$$
\begin{aligned}
R^o &= \frac{1}{p^o} = \frac{5}{3}\frac{1}{\beta} \ \text{ for } t = 1, 3, 5, \ldots \\
R^e &= \frac{p^o}{\beta^2} = \frac{3}{5}\frac{1}{\beta} \ \text{ for } t = 2, 4, 6, \ldots \\
a_{i,0} &= 0 \\
a_{A,1} &= \omega_{A0} - c_{A0} = 2 - c_A^e = \frac{4\beta}{5(1+\beta)} \\
a_{B,1} &= -a_{A,1} = -\frac{4\beta}{5(1+\beta)} \\
a_{A,2} &= R_1 a_{A,1} + \omega_{A1} - c_{A1} = 0 \\
a_{B,2} &= -a_{A,2} = 0 \\
a_{A,3} &= R_2 a_{A,2} + \omega_{A2} - c_{A2} = \frac{4\beta}{5(1+\beta)} \\
a_{B,3} &= -a_{A,3} = -\frac{4\beta}{5(1+\beta)} \\
&\ldots
\end{aligned}
$$

Therefore, the equilibrium interest rate and asset holdings are

$$
\begin{aligned}
R^o &= \frac{5}{3}\frac{1}{\beta} && \text{for } t = 1, 3, 5, \ldots \\
R^e &= \frac{3}{5}\frac{1}{\beta} && \text{for } t = 2, 4, 6, \ldots \\
a_A^o &= \frac{4\beta}{5(1+\beta)} && \text{for } t = 1, 3, 5, \ldots \\
a_A^e &= 0 && \text{for } t = 0, 2, 4, \ldots \\
a_B^o &= -\frac{4\beta}{5(1+\beta)} && \text{for } t = 1, 3, 5, \ldots \\
a_B^e &= 0 && \text{for } t = 0, 2, 4, \ldots
\end{aligned}
$$