Suggested Solutions to Homework #2
Econ 511b (Part I), Spring 2004

1. **Consider the planning problem for a simple finite-horizon neoclassical growth model:**

$$\max_{\{c_t,k_{t+1}\}_{t=0}^{T}} \sum_{t=0}^{T} \boldsymbol{\beta}^t \ln(c_t)$$

given $k_0 = 10$ and subject to the constraint that $\mathbf{c}_t + k_{t+1} = Ak_t^{\alpha} + (1-\delta)k_t$. Set $\beta = 0.95$, δ= 0.1, and α= 0.4. Choose A so that the steady-state value of capital in the corresponding infinite-horizon model is 100.

Solve the model numerically (say, in Matlab) using the "shooting" method described in lecture on January 14: start by guessing a value for k_1, solve for k_2 from the Euler equation at time 0, then solve for k_3 from the Euler equation at time 1, and so on, until k_{T+1} is found. Then vary k_1 and repeat until the appropriate value of k_{T+1} (i.e., 0) is found. Find the lowest value for T such that the highest value of capital between periods 0 and T exceeds 90.

We first derive the Euler equation for this problem, then we solve the question numerically. We can solve for Euler equation either from nonlinear programming or dynamic programming method. Here we use dynamic programming. Define $f(k_t) = Ak_t^{\alpha} + (1-\delta)k_t$. The recursive formulation for this problem is

$$v_t(k_t) = \max_{k_{t+1}} \ln(f(k_t) - k_{t+1}) + \beta v_{t+1}(k_{t+1})$$

Note that here value function depends on the time subscript t. F.O.C. for $t \le T-1$ is

$$-\frac{1}{c_t} + \beta v_{t+1}'(k_{t+1}) = 0$$

From Envelope theorem, we have

$$v_t'(k_t) = \frac{1}{c_t} f'(k_t)$$

Iterate forward and plug back time subscripts, we get the Euler equation

$$\beta \frac{1}{c_{t+1}} f'(k_{t+1}) = \frac{1}{c_t}$$

Plug in $c_t = f(k_t) - k_{t+1}$ and $f(k_t) = Ak_t^{\alpha} + (1-\delta)k_t$, we have

$$\begin{aligned}
& k_{t+2} = f(k_{t+1}) - \beta(f(k_t) - k_{t+1}) f'(k_{t+1}) \\
\Rightarrow \quad & k_{t+2} = \left(Ak_{t+1}^{\alpha} + (1-\delta)k_{t+1}\right) - \beta\left(Ak_t^{\alpha} + (1-\delta)k_t - k_{t+1}\right)\left(A\alpha k_{t+1}^{\alpha-1} + (1-\delta)\right) \\
\Rightarrow \quad & k_{t+2} = A(1+\alpha\beta)k_{t+1}^{\alpha} + (1-\delta)(1+\beta)k_{t+1} - \beta\left(Ak_t^{\alpha} + (1-\delta)k_t\right)\left(A\alpha k_{t+1}^{\alpha-1} + (1-\delta)\right)
\end{aligned}$$

We can see that it is a second-order nonlinear difference equation, which has boundary condition $k_0 = 10, k_{T+1} = 0$.

Before we go on to the numerical step, we solve for A. The infinite horizon steady state k^* solves

$$f'(k^*) = \beta^{-1} \Rightarrow A = \frac{\beta^{-1} - (1-\delta)}{\alpha (k^*)^{\alpha-1}} = \frac{29}{76} 10^{\frac{6}{5}} \approx 6.0476$$

Now we start to solve for it numerically by using "shooting" method. For an error bound $|k_{T+1}| < 0.01$, when time horizon T exceeds 37, the highest value of capital between periods 0 and T exceeds 90. The approximate period-1 capital stock for $T = 37$ is $k_1 = 16.3833123$. If you use grid point search, to get such a precision you have to define the step size of grid as fine as 10^{-7}.

2. **Consider a neoclassical growth model with two sectors, one producing consumption goods and one producing investment goods. Consumption is given by $C_t = F(K_{Ct}, L_{Ct})$ and investment is given by $I_t = G(K_{It}, L_{It})$, where K_{jt} is the amount of capital in sector j at the beginning of period t and L_{jt} is the amount of labor used in sector j in period t. The total amount of labor in each period is equal to L (leisure is not valued). Labor can be freely allocated in each period between the two sectors: $L = L_{Ct} + L_{It}$. Capital, on the other hand, is sector-specific: once it is installed in a given sector, it cannot be moved to the other sector. Investment goods, however, can be used to augment the capital stock in either sector. In particular, the capital stocks in the two sectors evolve according to:**

$$K_{j,t+1} = (1-\delta) K_{jt} + I_{jt}, \ j = C, I,$$

where $I_t = I_{Ct} + I_{It}$.

The social planner seeks to maximize $\sum_{t=0}^{\infty} \beta^t u(C_t)$, given K_{C0} and K_{I0}, subject to the constraints on technology. Note that although leisure is not valued (i.e., the total amount of labor supply L does not appear in the planner's objective), the planner must nonetheless decide in each period how to allocate L across the two sectors.

(a) **Formulate the planner's optimization problem as a dynamic programming problem. Be sure to distinguish clearly between state variables and control (or choice) variables.**

Given the technology of the two sector growth model, the capital accumulation equation becomes

$$K_{C,t+1} + K_{I,t+1} = (1-\delta) K_{Ct} + (1-\delta) K_{It} + G(K_{It}, L - L_{Ct})$$

Therefore, one recursive formulation of the problem is

$$\begin{aligned} v(K_{Ct}, K_{It}) \quad = \quad & \max_{\{C_t, L_t, K_{C,t+1}, K_{I,t+1}\}_{t=0}^{\infty}} u(C_t) + \beta v(K_{C,t+1}, K_{I,t+1}) \\ & s.t. \\ & C_t = F(K_{Ct}, L_{Ct}) \\ & K_{C,t+1} + K_{I,t+1} = (1-\delta) K_{Ct} + (1-\delta) K_{It} + G(K_{It}, L - L_{Ct}) \end{aligned}$$

or equivalently,

$$v(K_{Ct},K_{It}) = \max_{\{L_t,K_{C,t+1}\}_{t=0}^{\infty}} u(F(K_{Ct},L_{Ct}))+\beta v\begin{pmatrix} K_{C,t+1},(1-\delta)K_{Ct}+(1-\delta)K_{It} \\ +G(K_{It},L-L_{Ct})-K_{C,t+1}\end{pmatrix}$$

(b) Find a set of Euler equations and first-order conditions that an optimal solution to the planning problem must satisfy.

We have the F.O.C. as

$$\begin{aligned} \{L_{Ct}\} &: u'(C_t)\frac{F_2(K_{Ct},L_{Ct})}{G_2(K_{It},L-L_{Ct})} = \beta v_2(K_{C,t+1},K_{I,t+1}) \\ \{K_{C,t+1}\} &: v_1(K_{C,t+1},K_{I,t+1}) = v_2(K_{C,t+1},K_{I,t+1}) \end{aligned}$$

i.e.

$$v_1(K_{C,t+1},K_{I,t+1}) = v_2(K_{C,t+1},K_{I,t+1}) = \frac{u'(C_t)}{\beta}\frac{F_2(K_{Ct},L_{Ct})}{G_2(K_{It},L-L_{Ct})}$$

From Envelope Theorem, we have

$$\begin{aligned} v_1(K_{C,t},K_{I,t}) &= u'(C_t)F_1(K_{Ct},L_{Ct})+\beta v_2(K_{C,t+1},K_{I,t+1})(1-\delta) \\ v_2(K_{C,t},K_{I,t}) &= \beta v_2(K_{C,t+1},K_{I,t+1})(1-\delta+G_1(K_{It},L-L_{Ct})) \end{aligned}$$

Iterate forward for one period, it becomes

$$\begin{aligned} 1 &= u'(C_{t+1})\frac{F_1(K_{C,t+1},L_{C,t+1})}{v_1(K_{C,t+1},K_{I,t+1})}+\beta(1-\delta)\frac{v_2(K_{C,t+2},K_{I,t+2})}{v_1(K_{C,t+1},K_{I,t+1})} \\ 1 &= \beta\frac{v_2(K_{C,t+2},K_{I,t+2})}{v_2(K_{C,t+1},K_{I,t+1})}(1-\delta+G_1(K_{I,t+1},L-L_{C,t+1})) \end{aligned}$$

Plug F.O.C. into it, we get two Euler Equations:

$$\begin{aligned} EE1 &: 1=\frac{\beta u'(C_{t+1})}{u'(C_t)}\left[\begin{array}{c} F_1(K_{C,t+1},L_{C,t+1})\frac{G_2(K_{It},L-L_{Ct})}{F_2(K_{Ct},L_{Ct})}+ \\ (1-\delta)\frac{F_2(K_{C,t+1},L_{C,t+1})/G_2(K_{I,t+1},L-L_{C,t+1})}{F_2(K_{C,t},L_{C,t})/G_2(K_{I,t},L-L_{C,t})}\end{array}\right] \\ EE2 &: 1=\frac{\beta u'(C_{t+1})}{u'(C_t)}\frac{F_2(K_{C,t+1},L_{C,t+1})/G_2(K_{I,t+1},L-L_{C,t+1})}{F_2(K_{C,t},L_{C,t})/G_2(K_{I,t},L-L_{C,t})}\cdot \\ &\quad [(1-\delta)+G_1(K_{I,t+1},L-L_{C,t+1})] \end{aligned}$$

where $C_t = F(K_{Ct},L_{Ct})$. These two Euler equations together with budget constraint (state-variable evolution equations) and transversality condition determine the solution path.

(c) Suppose that $F(K_{Ct},L_{Ct}) = K_{Ct}^{\alpha}L_{Ct}^{1-\alpha}$ and $G(K_{It},L_{It}) = K_{It}^{\gamma}L_{It}^{1-\gamma}$. Use your answer from part (b) to find the steady state for this economy as a function of the structural parameters.

In the steady state, the Euler equations become

$$\begin{aligned}
\{K_{C,t+1}\} &: \beta\left[F_1\left(K_C^*, L_C^*\right)\frac{G_2\left(K_I^*, L-L_C^*\right)}{F_2\left(K_C^*, L_C^*\right)}+(1-\delta)\right]=1 \\
\{K_{I,t+1}\} &: \beta\left[(1-\delta)+G_1\left(K_I^*, L-L_C^*\right)\right]=1
\end{aligned}$$

With $F\left(K_{Ct}, L_{Ct}\right)=K_{Ct}^{\alpha}L_{Ct}^{1-\alpha}$ **and** $G\left(K_{It}, L_{It}\right)=K_{It}^{\gamma}L_{It}^{1-\gamma}$, we have

$$\begin{aligned}
F_1\left(K_C^*, L_C^*\right) &= \alpha\left(\frac{K_C^*}{L_C^*}\right)^{\alpha-1} \\
F_2\left(K_C^*, L_C^*\right) &= (1-\alpha)\left(\frac{K_C^*}{L_C^*}\right)^{\alpha} \\
G_1\left(K_I^*, L-L_C^*\right) &= \gamma\left(\frac{K_I^*}{L-L_C^*}\right)^{\gamma-1} \\
G_2\left(K_I^*, L-L_C^*\right) &= (1-\gamma)\left(\frac{K_I^*}{L-L_C^*}\right)^{\gamma}
\end{aligned}$$

Plug into the steady-state Euler equations, we get

$$\begin{aligned}
\{K_{C,t+1}\} &: \beta\left[\frac{\alpha(1-\gamma)}{1-\alpha}\left(\frac{K_C^*}{L_C^*}\right)^{-1}\left(\frac{K_I^*}{L-L_C^*}\right)^{\gamma}+(1-\delta)\right]=1 \\
\{K_{I,t+1}\} &: \beta\left[(1-\delta)+\gamma\left(\frac{K_I^*}{L-L_C^*}\right)^{\gamma-1}\right]=1
\end{aligned}$$

After some calculation, it simplifies to

$$\begin{aligned}
\frac{K_C^*}{L_C^*} &= \frac{\alpha}{1-\alpha}\frac{1-\gamma}{\gamma}\frac{K_I^*}{L-L_C^*}=\frac{\alpha}{1-\alpha}\frac{1-\gamma}{\gamma}\left(\frac{\frac{1}{\beta}-(1-\delta)}{\gamma}\right)^{\frac{1}{\gamma-1}} \\
\frac{K_I^*}{L-L_C^*} &= \left[\frac{\frac{1}{\beta}-(1-\delta)}{\gamma}\right]^{\frac{1}{\gamma-1}}
\end{aligned}$$

Plug these back into steady-state budget constraint

$$\begin{aligned}
& K_C^*+K_I^*=(1-\delta)K_C^*+(1-\delta)K_I^*+G\left(K_I^*, L-L_C^*\right) \\
\Rightarrow\ & \delta\left(K_C^*+K_I^*\right)=\left(K_I^*\right)^{\gamma}\left(L-L_C^*\right)^{1-\gamma}
\end{aligned}$$

we have

$$\begin{aligned}
& \delta\left(K_C^*+K_I^*\right)=\left(K_I^*\right)^{\gamma}\left(L-L_C^*\right)^{1-\gamma} \\
\Rightarrow\ & \delta\left(\frac{K_C^*}{L-L_C^*}+\frac{K_I^*}{L-L_C^*}\right)=\left(\frac{K_I^*}{L-L_C^*}\right)^{\gamma} \\
\Rightarrow\ & \frac{K_C^*}{L_C^*}\frac{L_C^*}{L-L_C^*}=\frac{1}{\delta}\left(\frac{K_I^*}{L-L_C^*}\right)^{\gamma}-\frac{K_I^*}{L-L_C^*} \\
\Rightarrow\ & L_C^*=\frac{A}{1+A}L \\
\textit{where } A &= \frac{1-\alpha}{\alpha}\frac{\gamma}{1-\gamma}\left[\frac{1}{\gamma\delta}\left(\frac{1}{\beta}-(1-\delta)\right)-1\right]
\end{aligned}$$

Finally we get the solution, i.e.

$$
\begin{aligned}
L_C^* &= \frac{A}{1+A}L \\
L_I^* &= \frac{1}{1+A}L \\
K_C^* &= \frac{\alpha}{1-\alpha}\frac{1-\gamma}{\gamma}\left(\frac{\frac{1}{\beta}-(1-\delta)}{\gamma}\right)^{\frac{1}{\gamma-1}} L_C^* \\
K_I^* &= \left[\frac{\frac{1}{\beta}-(1-\delta)}{\gamma}\right]^{\frac{1}{\gamma-1}} L_I^*
\end{aligned}
$$

3. **Consider an exchange economy with two consumers named A and B. The two consumers have identical preferences: they each value consumption streams according to $\sum_{t=0}^{\infty}\beta^t u(c_t)$, where u has a constant elasticity of intertemporal substitution σ^{-1}. Consumer i's endowment of consumption goods is $\{\omega_{it}\}_{t=0}^{\infty}, i=A,B$.Consumption goods are perishable (i.e., they cannot be stored and used for consumption in future periods).**

   (a) **Carefully define a competitive equilibrium with date-0 trading for this economy.**

   A Competitive Equilibrium with date-0 trading for the economy $\{u_A,u_B\},\{\omega_{it}\}_{t=0}^{\infty}$ is a vector of prices $\{p_t\}_{t=0}^{\infty}$ and a vector of quantities $\{c_{it}^*\}_{t=0}^{\infty}$ for $i=A,B$ such that

   (1) For $i=A,B$,

$$
\begin{aligned}
\{c_{it}^*\}_{t=0}^{\infty} &= \arg\max\sum_{t=0}^{\infty}\beta^t u(c_{it}) \\
& s.t. \\
& \sum_{t=0}^{\infty}p_t c_{it} = \sum_{t=0}^{\infty}p_t\omega_{it}
\end{aligned}
$$

   (2) $c_{At}+c_{Bt}=\omega_{At}+\omega_{Bt}\ for\ t=0,1,2...$

   (b) **Suppose that $\omega_{At}=3$ for all t and $\omega_{Bt}=1$ for all t. Find the competitive equilibrium allocations and prices.**

   The F.O.C. for consumer i is

$$
\frac{\beta^j u'(c_{i,t+j})}{u'(c_{i,t})} = \frac{p_{t+j}}{p_t} \text{ for } \forall t,j
$$

   This together with budget constraint and market clearing condition determines the competitive equilibrium. Here there are two ways to solve for the equilibrium. One way is to solve for the system of simultaneous equations; another way is to make a guess of solution and check the feasibility for each equations. Due to the special structure of the model, here it is easier to proceed with the second way.

Now guess that $c_{it} = c_i$ for $\forall t$. Plug into the F.O.C. and normalize $p_0 = 1$ we have

$$p_t = \beta^t$$

Plug into the budget constraint for each individual, we have

$$\begin{aligned} & \sum_{t=0}^{\infty} p_t c_{it} = \sum_{t=0}^{\infty} p_t \omega_{it} \\ \Rightarrow \quad & \sum_{t=0}^{\infty} \beta^t c_i = \sum_{t=0}^{\infty} \beta^t \omega_i \\ \Rightarrow \quad & c_i = \omega_i \end{aligned}$$

It is easy to check that it satisfies the market clearing condition. Therefore, the competitive equilibrium for this economy is

$$\begin{aligned} c_{At} &= \omega_A = 3 \\ c_{Bt} &= \omega_B = 1 \\ p_t &= \beta^t \end{aligned}$$

(c) **Suppose now that the endowments fluctuate deterministically: consumer A's endowment stream is $\{3, 1, 3, 1, 3, 1, . . .\}$ and consumer B's endowment stream is $\{1, 3, 1, 3, 1, 3, . . .\}$. Find the competitive equilibrium allocations and prices. (Hint: Guess that each consumer's consumption is constant across time and verify that this guess is correct.)**

Again guess that $c_{it} = c_i$ for $\forall t$. Plug into the F.O.C. and normalize $p_0 = 1$ we have

$$p_t = \beta^t$$

Plug into the budget constraint for each individual, we have

$$\begin{aligned} & \sum_{t=0}^{\infty} p_t c_{it} = \sum_{t=0}^{\infty} p_t \omega_{it} \\ \Rightarrow \quad & \sum_{t=0}^{\infty} \beta^t c_i = \sum_{t=0}^{\infty} \beta^t \omega_i \\ \Rightarrow \quad & \sum_{t=0}^{\infty} \beta^t c_i = \sum_{t=0}^{\infty} \beta^{2t} \omega_i^e + \beta \sum_{t=0}^{\infty} \beta^{2t} \omega_i^o \\ \Rightarrow \quad & c_i = \frac{1}{1+\beta}\omega_i^e + \frac{\beta}{1+\beta}\omega_i^o \end{aligned}$$

where ω_i^e and ω_i^o means w_i in even and odd period, respectively.

It is easy to check that it satisfies the market clearing condition. Therefore, the competitive equilibrium for this economy is

$$\begin{aligned} c_{At} &= \frac{3+\beta}{1+\beta} \\ c_{Bt} &= \frac{1+3\beta}{1+\beta} \\ p_t &= \beta^t \end{aligned}$$

(d) In parts (b) and (c) there is no variation in the aggregate endowment across time. Suppose that, as in part (b), consumer A's endowment is 3 in every period but that consumer B's endowment fluctuates: his endowment stream is {1/2, 3/2, 1/2, 3/2, 1/2, 3/2, . . .}. Find the competitive equilibrium allocations and prices. To simplify the algebra, set $\sigma = 1$ (i.e., let the felicity function u be logarithmic).

Now guess the equilibrium as

$$\begin{aligned} c_{At} &= c_A^o \text{ for } t \text{ odd} \\ c_{At} &= c_A^e \text{ for } t \text{ even} \\ c_{Bt} &= c_A^o \text{ for } t \text{ odd} \\ c_{Bt} &= c_B^e \text{ for } t \text{ even} \\ p_t &= \beta^{t-1} p^o \text{ for } t \text{ odd} \\ p_t &= \beta^t \text{ for } t \text{ even} \end{aligned}$$

Plug into the F.O.C. we get

$$\begin{aligned} & \frac{\beta^j u'\left(c_{i,t+j}\right)}{u'\left(c_{i,t}\right)} = \frac{p_{t+j}}{p_t} \text{ for } \forall t, j \\ \Rightarrow \quad & \frac{\beta u'\left(c_i^o\right)}{u'\left(c_i^e\right)} = p^o \\ \Rightarrow \quad & \frac{\beta c_i^e}{c_i^o} = p^o \end{aligned}$$

Therefore, we have

$$p^o = \beta \frac{c_A^e}{c_A^o} = \beta \frac{c_B^e}{c_B^o} = \beta \frac{c_A^e + c_B^e}{c_A^o + c_B^o} = \beta \frac{\omega_A^e + \omega_B^e}{\omega_A^o + \omega_B^o} = \frac{7\beta}{9}$$

and

$$\frac{c_i^e}{c_i^o} = \frac{7}{9}$$

So the equilibrium price is

$$\begin{aligned} p_t &= \frac{7}{9}\beta^t \text{ for } t \text{ odd} \\ p_t &= \beta^t \text{ for } t \text{ even} \end{aligned}$$

Plug p_t into budget constraint, we have

$$\begin{aligned} & \sum_{t=0}^{\infty} p_t c_{it} = \sum_{t=0}^{\infty} p_t \omega_{it} \\ \Rightarrow \quad & \sum_{t=0}^{\infty} \beta^{2t} c_i^e + \frac{7}{9} \sum_{t=0}^{\infty} \beta^{2t+1} c_i^o = \sum_{t=0}^{\infty} \beta^{2t} \omega_i^e + \frac{7}{9} \sum_{t=0}^{\infty} \beta^{2t+1} \omega_i^o \\ \Rightarrow \quad & c_i^e + \frac{7\beta}{9} c_i^o = \omega_i^e + \frac{7\beta}{9} \omega_i^o \\ \Rightarrow \quad & \frac{7}{9} c_i^o + \frac{7\beta}{9} c_i^o = \omega_i^e + \frac{7\beta}{9} \omega_i^o \quad \left(\textit{plug into } \frac{c_i^e}{c_i^o} = \frac{7}{9} \right) \\ \Rightarrow \quad & c_i^o = \frac{9\omega_i^e + 7\beta\omega_i^o}{7(1+\beta)} \end{aligned}$$

i.e.

$$\begin{aligned} c_A^o &= \frac{27+21\beta}{7(1+\beta)} \\ c_B^o &= \frac{9+21\beta}{14(1+\beta)} \end{aligned}$$

and so

$$\begin{aligned} c_A^e &= \frac{7}{9}c_A^o = \frac{9+7\beta}{3(1+\beta)} \\ c_B^e &= \frac{7}{9}c_B^o = \frac{3+7\beta}{6(1+\beta)} \end{aligned}$$

(e) **Carefully define a competitive equilibrium with sequential trading for this economy. Use your results from parts (b), (c), and (d) to determine the equilibrium interest rates for each pair of endowment streams. In addition, for each case determine how each consumer's asset holdings vary over time (assume that each consumer starts with zero assets in period 0).**

A Competitive Equilibrium with sequential trading for the economy $\{u_A, u_B\}$, $\{\omega_{it}\}_{t=0}^{\infty}$ is a sequence $\{c_{it}^*\}_{t=0}^{\infty}, \{a_{i,t+1}^*\}_{t=0}^{\infty}, \{R_t^*\}_{t=0}^{\infty}$ (where R_t^* means interest rate from t to $t+1$) for $i = A, B$ such that

(1) For $i = A, B$,

$$\begin{aligned} \{c_{it}^*, a_{i,t+1}^*\}_{t=0}^{\infty} &= \arg\max \sum_{t=0}^{\infty} \beta^t u(c_{it}) \\ & s.t. \\ & c_{it} + a_{i,t+1} = R_t^* a_{i,t} + \omega_{it} \\ & \lim_{t\to\infty} a_{i,t+1}\left(\prod_{t=0}^{\infty} R_{t+1}\right) \geq 0 \\ & a_{i,0} = 0, c_{it} \geq 0 \end{aligned}$$

(2) $c_{At}^* + c_{Bt}^* = \omega_{At} + \omega_{Bt}$ for $t = 0, 1, 2...$

(3) $a_{i,t}^* = 0$ for $t = 0, 1, 2...$

Now we start to solve for equilibrium interest rate and asset holdings for different examples. In each example, it is easy to see that

$$\begin{aligned} R_t &= \frac{p_{t-1}}{p_t} \\ a_{i,t+1} &= R_t^* a_{i,t} + \omega_{it} - c_{it} \end{aligned}$$

We start with part (b). Plug in the solution, we have

$$\begin{aligned} R_t &= \frac{1}{\beta} \\ a_{i,t} &= 0 \end{aligned}$$

In part (c), we have

$$\begin{aligned}
R_t &= \frac{1}{\beta} \\
a_{i,0} &= 0 \\
a_{i,1} &= \omega_{i0} - c_{i0} \\
a_{i,2} &= R_2^* a_{i,1} + \omega_{i1} - c_{i1} = \frac{1}{\beta}\left(\omega_{i0} - c_{i0}\right) + \omega_{i1} - c_{i1} \\
&...
\end{aligned}$$

Plug in the equilibrium solution, we have

$$\begin{aligned}
R_t &= \frac{1}{\beta} \\
a_A^o &= -a_B^o = \frac{2\beta}{1+\beta} \quad \text{for } t = 1, 3, 5, ... \\
a_A^e &= -a_B^e = 0 \quad \text{for } t = 0, 2, 4, ...
\end{aligned}$$

In part (d), we have

$$\begin{aligned}
R^o &= \frac{1}{p^o} = \frac{9}{7}\frac{1}{\beta} \quad \text{for } t = 1, 3, 5, ... \\
R^e &= \frac{p^o}{\beta^2} = \frac{7}{9}\frac{1}{\beta} \quad \text{for } t = 2, 4, 6, ... \\
a_{i,0} &= 0 \\
a_{A,1} &= \omega_{A0} - c_{A0} = 3 - c_A^e = \frac{2\beta}{3\left(1+\beta\right)} \\
a_{B,1} &= -a_{A,1} = -\frac{2\beta}{3\left(1+\beta\right)} \\
a_{A,2} &= R_1 a_{A,1} + \omega_{A1} - c_{A1} = 0 \\
a_{B,2} &= -a_{A,2} = 0 \\
a_{A,3} &= R_2 a_{A,2} + \omega_{A2} - c_{A2} = \frac{2\beta}{3\left(1+\beta\right)} \\
a_{B,3} &= -a_{A,3} = -\frac{2\beta}{3\left(1+\beta\right)} \\
&...
\end{aligned}$$

Therefore, the equilibrium interest rate and asset holdings are

$$\begin{aligned}
R^o &= \frac{9}{7}\frac{1}{\beta} \qquad \text{for } t = 1, 3, 5, ... \\
R^e &= \frac{7}{9}\frac{1}{\beta} \qquad \text{for } t = 2, 4, 6, ... \\
a_A^o &= \frac{2\beta}{3\left(1+\beta\right)} \quad \text{for } t = 1, 3, 5, ... \\
a_A^e &= 0 \qquad \text{for } t = 0, 2, 4, ... \\
a_B^o &= -\frac{2\beta}{3\left(1+\beta\right)} \quad \text{for } t = 1, 3, 5, ... \\
a_B^e &= 0 \qquad \text{for } t = 0, 2, 4, ...
\end{aligned}$$