Suggested Solutions to Homework #4
Econ 511b (Part I), Spring 2004

1. **Consider a neoclassical growth model with valued leisure. The (representative) consumer values streams of consumption and leisure according to $\sum_{t=0}^{\infty}\beta^t U(c_t, l_t)$, where l_t is hours of leisure in period t. The felicity function U is strictly concave and strictly increasing in both of its arguments. Output is produced according to $y_t = F(k_t, n_t)$, where $n_t = L - l_t$ is hours of labor supply in period t (F is the total number of hours available for either leisure or work in period t) and F exhibits constant returns to scale. Output in period t can be used for either consumption c_t or investment i_t and capital accumulates according to $k_{t+1} = (1-\delta)k_t + i_t$.**

   (a) **Display Bellman's equation for the problem faced by the social planner in this economy. Identify clearly the state and control (or choice) variable(s).**

   The recursive formulation for the planning problem is

$$
\begin{aligned}
v\left(\overline{k}\right) &= \max_{\left\{\overline{c},\overline{l},\overline{k}'\right\}} U\left(\overline{c},\overline{l}\right) + \beta v\left(\overline{k}'\right) \\
& \quad s.t. \\
& \quad \overline{c} + \overline{k}' = F(\overline{k}, L-\overline{l}) + (1-\delta)\overline{k}
\end{aligned}
$$

   or

$$
v\left(\overline{k}\right) = \max_{\left\{l,k'\right\}} U\left(F(\overline{k}, L-\overline{l}) + (1-\delta)\overline{k} - \overline{k}', \overline{l}\right) + \beta v\left(\overline{k}'\right)
$$

   From the way we write it, we can see that the state variable is $\overline{k}$, and control variables are l, k'.

   (b) **Derive the first-order and envelope conditions for the planning problem.**

   The F.O.C. is

$$
\begin{aligned}
\left\{\overline{l}\right\} &: U_1\left(\overline{c},\overline{l}\right) F_2(\overline{k}, L-\overline{l}) = U_2\left(\overline{c},\overline{l}\right) \\
\left\{\overline{k}'\right\} &: U_1\left(\overline{c},\overline{l}\right) = \beta v'\left(\overline{k}'\right)
\end{aligned}
$$

   From Envelope Theorem, we have

$$
v'\left(\overline{k}\right) = U_1\left(\overline{c},\overline{l}\right)\left(F_1(\overline{k}, L-\overline{l}) + (1-\delta)\right)
$$

   Iterate forward for one period, it becomes

$$
v'\left(\overline{k}'\right) = U_1\left(\overline{c}',\overline{l}'\right)\left(F_1(\overline{k}', L-\overline{l}') + (1-\delta)\right)
$$

   Plug it into F.O.C., we get the final optimality conditions:

$$
\begin{aligned}
\left\{\overline{l}_t\right\} &: U_1\left(\overline{c}_t,\overline{l}_t\right) F_2(\overline{k}_t, L-\overline{l}_t) = U_2\left(\overline{c}_t,\overline{l}_t\right) \\
\left\{\overline{k}_{t+1}\right\} &: U_1\left(\overline{c}_t,\overline{l}_t\right) = \beta U_1\left(\overline{c}_{t+1},\overline{l}_{t+1}\right)\left(F_1(\overline{k}_{t+1}, L-\overline{l}_{t+1}) + (1-\delta)\right)
\end{aligned}
$$

(c) Use the conditions from part (b) to determine the economy's steady state. Show how the steady state depends on primitives and compare your results to those for a growth model without valued leisure.

In steady state, the optimality condition becomes

$$\begin{aligned} \left\{\overline{l}^*\right\} &: U_1\left(\overline{c}^*, \overline{l}^*\right) F_2(\overline{k}^*, L - \overline{l}^*) = U_2\left(\overline{c}^*, \overline{l}^*\right) \\ \left\{\overline{k}^*\right\} &: F_1(\overline{k}^*, L - \overline{l}^*) = \frac{1}{\beta} - (1-\delta) \end{aligned}$$

where $\overline{c}^* = F(\overline{k}^*, L - \overline{l}^*) - \delta \overline{k}^*$. We can see that $\overline{k}^*$ and $\overline{l}^*$ depends on both β, δ, production techonology $F(k_t, n_t)$, and utility function $U(c_t, l_t)$.

In a growth model without valued leisure, the steady state is determined by the equation

$$F_1(\overline{k}^*, L) = \frac{1}{\beta} - (1-\delta)$$

which does not depend on the utility function $U(c_t, l_t)$.

Now let's compare two models. First, in the model with leisure choice we add an additional equation which states that the marginal rate of substitution between consumption and leisure must equal to the marginal rate of transformation. Second, the equation about $\overline{k}^*$ is the same, except that the level of steady-state leisure is different. Third, for the equation $F_1(\overline{k}^*, L - \overline{l}^*) = \frac{1}{\beta} - (1-\delta)$, due to the difference in the steady-state leisure level, the steady-state capital stock is also different. For example, if $F_{12} > 0$ as in the case of Cobb-Douglas production function, the capital stock in the model with leisure will be lower than that without leisure choice (since $L - \overline{l}^* < L$).

(d) Let $F(k,n) = k^\alpha n^{(1-\alpha)}$ and $U(c,l) = \lambda \log(c) + (1-\lambda)\log(l)$, where $0 < \alpha < 1$ and $0 < \lambda < 1$. Solve explicitly for the steady state in terms of parameters.

With $F(k,n) = k^\alpha n^{(1-\alpha)}$ and $U(c,l) = \lambda \log(c) + (1-\lambda)\log(l)$, the steady-state condition becomes

$$\begin{aligned} \left\{\overline{l}^*\right\} &: \left(\frac{\lambda}{\overline{k}^{*\alpha}\left(L - \overline{l}^*\right)^{(1-\alpha)} - \delta \overline{k}^*}\right)(1-\alpha)\left(\frac{\overline{k}^*}{L - \overline{l}^*}\right)^\alpha = \frac{1-\lambda}{\overline{l}^*} \\ \left\{\overline{k}^*\right\} &: \alpha\left(\frac{\overline{k}^*}{L - \overline{l}^*}\right)^{\alpha-1} = \frac{1}{\beta} - (1-\delta) \end{aligned}$$

which leads to

$$\begin{cases} \overline{n}^* = \dfrac{1}{\frac{1-\lambda}{\lambda}\frac{\alpha}{1-\alpha}\left(\frac{1}{\alpha} - \frac{\delta}{\frac{1}{\beta}-(1-\delta)}\right)+1} L \\ \overline{l}^* = L - \overline{n}^* \\ \overline{k}^* = \left(\dfrac{\frac{1}{\beta}-(1-\delta)}{\alpha}\right)^{\frac{1}{\alpha-1}} \overline{n}^* \\ \overline{c}^* = \overline{k}^{*\alpha}\left(\overline{n}^*\right)^{(1-\alpha)} - \delta \overline{k}^* \end{cases}$$

2. **(a) Carefully define a recursive competitive equilibrium for the neoclassical growth model with valued leisure. (Hint: You need two functions to describe the behavior of the aggregate economy.)**

A recursive competitive equilibrium for the neoclassical growth model with valued leisure is a set of functions:

$$\begin{aligned} \textit{price function} &: r\left(\overline{k}\right), w\left(\overline{k}\right) \\ \textit{policy function} &: k^{'} = g_k\left(k, \overline{k}\right), l = g_l\left(k, \overline{k}\right) \\ \textit{value function} &: v\left(k, \overline{k}\right) \\ \textit{aggregate state} &: \overline{k}^{'} = G\left(\overline{k}\right), \overline{l} = \overline{l}(\overline{k}) \end{aligned}$$

such that:

(1) Given $\overline{k}^{'} = G\left(\overline{k}\right)$, $k^{'} = g_k\left(k, \overline{k}\right), l = g_l\left(k, \overline{k}\right)$ and $v\left(k, \overline{k}\right)$ solves consumer's problem:

$$\begin{aligned} v\left(k, \overline{k}\right) &= \max_{\{c,l,k'\}} U(c,l) + \beta v\left(k^{'}, \overline{k}^{'}\right) \\ & s.t. \\ & c + k^{'} = r\left(\overline{k}\right) k + (1-\delta) k + w\left(\overline{k}\right)(L - l) \\ & \overline{k}^{'} = G\left(\overline{k}\right) \end{aligned}$$

(2) Price is competitively determined:

$$\begin{aligned} r\left(\overline{k}\right) &= F_1\left(\overline{k}, L - \overline{l}(\overline{k})\right) \\ w\left(\overline{k}\right) &= F_2\left(\overline{k}, L - \overline{l}(\overline{k})\right) \end{aligned}$$

(3) Consistency:

$$\begin{aligned} G\left(\overline{k}\right) &= g_k\left(\overline{k}, \overline{k}\right) \\ \overline{l}(\overline{k}) &= g_l\left(\overline{k}, \overline{k}\right) \end{aligned}$$

(b) Find the (functional) first-order conditions of a typical consumer who takes as given the economy's aggregate laws of motion.

Solve a typical consumer's problem

$$\begin{aligned} v\left(k, \overline{k}\right) &= \max_{\{l,k'\}} U\left(r\left(\overline{k}\right) k + (1-\delta) k + w\left(\overline{k}\right)(L-l) - k^{'}, l\right) + \beta v\left(k^{'}, \overline{k}^{'}\right) \\ & s.t. \\ & \overline{k}^{'} = G\left(\overline{k}\right) \end{aligned}$$

We get the F.O.C. as

$$\begin{aligned} \{l\} &: U_1(c,l)\, w\left(\overline{k}\right) = U_2(c,l) \\ \left\{k^{'}\right\} &: U_1(c,l) = \beta v_1\left(k^{'}, \overline{k}^{'}\right) \end{aligned}$$

Use the envelope condition

$$v_1\left(k, \overline{k}\right) = U_1(c, l)\left(r\left(\overline{k}\right) + (1-\delta)\right)$$

We get the optimality condition:

$$\begin{aligned}
\{l_t\} &: U_1(c_t, l_t)\, w\left(\overline{k}_t\right) = U_2(c_t, l_t) \\
\{k_{t+1}\} &: U_1(c_t, l_t) = \beta U_1(c_{t+1}, l_{t+1})\left(r\left(\overline{k}_{t+1}\right) + (1-\delta)\right)
\end{aligned}$$

Correspondingly, the functional F.O.C. is

$$\begin{aligned}
\{l_t\} &: U_1\left(c_t, g_l\left(k_t, \overline{k}_t\right)\right) w\left(\overline{k}_t\right) = U_2\left(c_t, g_l\left(k_t, \overline{k}_t\right)\right) \\
\{k_{t+1}\} &: U_1\left(c_t, g_l\left(k_t, \overline{k}_t\right)\right) = \beta U_1\left(c_{t+1}, g_l\left(k_{t+1}, \overline{k}_{t+1}\right)\right)\left(r\left(G\left(\overline{k}_t\right)\right) + (1-\delta)\right)
\end{aligned}$$

where $c_t = r\left(\overline{k}_t\right) k_t + (1-\delta) k_t + w\left(\overline{k}_t\right)\left(L - g_l\left(k_t, \overline{k}_t\right)\right) - g_k\left(k_t, \overline{k}_t\right)$, $r\left(\overline{k}\right) = F_1\left(\overline{k}, L - \overline{l}\right) = F_1\left(\overline{k}, L - g_l\left(\overline{k}, \overline{k}\right)\right)$, and $w\left(\overline{k}\right) = F_2\left(\overline{k}, L - \overline{l}\right) = F_2\left(\overline{k}, L - g_l\left(\overline{k}, \overline{k}\right)\right)$.

(c) **Impose equilibrium conditions on the first-order conditions from part (b) and verify that the resulting equations are identical to the first-order conditions associated with the planning problem for this economy.**

Impose the equilibrium conditions $k_t = \overline{k}_t, l_t = \overline{l}_t$, we have

$$\begin{aligned}
\{l_t\} &: U_1\left(\overline{c}_t, \overline{l}_t\right) F_2\left(\overline{k}_t, L - \overline{l}_t\right) = U_2\left(\overline{c}_t, \overline{l}_t\right) \\
\{k_{t+1}\} &: U_1\left(\overline{c}_t, \overline{l}_t\right) = \beta U_1\left(\overline{c}_{t+1}, \overline{l}_{t+1}\right)\left(F_1\left(\overline{k}_t, L - \overline{l}_t\right) + (1-\delta)\right)
\end{aligned}$$

where

$$\begin{aligned}
\overline{c}_t &= F_1\left(\overline{k}_t, L - \overline{l}_t\right)\overline{k}_t + (1-\delta)\overline{k}_t + F_2\left(\overline{k}_t, L - \overline{l}_t\right)\left(L - \overline{l}_t\right) - \overline{k}_{t+1} \\
&= F\left(\overline{k}_t, L - \overline{l}_t\right) + (1-\delta)\overline{k}_t - \overline{k}_{t+1}
\end{aligned}$$

which are identical to the first-order conditions associated with the planning problem for this economy.

3. **Consider a competitive equilibrium one-sector growth model without valued leisure in which consumers own capital and labor and rent their services to firms. There is no uncertainty and the felicity function of a typical consumer has constant elasticity of intertemporal substitution σ^{-1}.**

   (a) **Show that the decision rule of a typical consumer takes the form:**

$$k' = \mu\left(\overline{k}\right) + \lambda\left(\overline{k}\right) k$$

where the functions μ and λ satisfy the following pair of functional equations (i.e., these two equations must hold for all values of $\overline{k}$):

$$\mu\left(\overline{k}\right) + \frac{w\left(\overline{k}'\right) - \mu\left(\overline{k}'\right)}{r\left(\overline{k}'\right) - \lambda\left(\overline{k}'\right)} = \frac{w\left(\overline{k}\right) - \mu\left(\overline{k}\right)}{r\left(\overline{k}\right) - \lambda\left(\overline{k}\right)}\lambda\left(\overline{k}\right)$$

and

$$\frac{1}{\beta r\left(\overline{k}'\right)} = \left(\frac{\left(r\left(\overline{k}'\right)-\lambda\left(\overline{k}'\right)\right)\lambda\left(\overline{k}\right)}{r\left(\overline{k}\right)-\lambda\left(\overline{k}\right)}\right)^{-\sigma},$$

where $k' = \mu\left(\overline{k}\right)+\lambda\left(\overline{k}\right)k$**. In these equations,** k **is the individual's holdings of capital,** $\overline{k}$ **is aggregate capital,** $r\left(\overline{k}\right)$ **is the rental rate of capital plus one minus the depreciation rate, and** $w\left(\overline{k}\right)$ **is the wage rate. (Hint: Obtain the Euler equation for a typical consumer, guess that the consumer's decision rule takes the conjectured form, and then find restrictions that the "coefficients"** $\mu\left(\overline{k}\right)$ **and** $\lambda\left(\overline{k}\right)$ **must satisfy in order for the Euler equation to hold for all values of** k **and** $\overline{k}$**.)**

A recursive competitive equilibrium for this economy is a set of functions:

$$\begin{aligned} price\ function &: r\left(\overline{k}\right), w\left(\overline{k}\right) \\ policy\ function &: k' = g\left(k,\overline{k}\right) \\ value\ function &: v\left(k,\overline{k}\right) \\ transition\ function &: \overline{k}' = G\left(\overline{k}\right) \end{aligned}$$

such that:

(1) $k' = g\left(k,\overline{k}\right)$ and $v\left(k,\overline{k}\right)$ solves consumer's problem:

$$\begin{aligned} v\left(k,\overline{k}\right) &= \max_{\{c,k'\}} \frac{c^{1-\sigma}}{1-\sigma}+\beta v\left(k',\overline{k}'\right) \\ & s.t. \\ & c+k' = r\left(\overline{k}\right)k+w\left(\overline{k}\right) \\ & \overline{k}' = G\left(\overline{k}\right) \end{aligned}$$

(2) Price is competitively determined:

$$\begin{aligned} r\left(\overline{k}\right) &= F_1\left(\overline{k},1\right)+1-\delta \\ w\left(\overline{k}\right) &= F_2\left(\overline{k},1\right) \end{aligned}$$

(3) Consistency:

$$G\left(\overline{k}\right) = g(k,\overline{k})$$

By taking F.O.C. and using envelope condition, we can get the Euler equation as

$$\begin{aligned} & \frac{\beta u'\left(c_{t+1}\right)}{u'\left(c_t\right)} r\left(\overline{k}_{t+1}\right) = 1 \\ \Rightarrow\ & \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} r\left(\overline{k}_{t+1}\right) = 1 \\ \Rightarrow\ & c_{t+1} = \left[\beta r\left(\overline{k}_{t+1}\right)\right]^{\frac{1}{\sigma}} c_t \\ \Rightarrow\ & r\left(\overline{k}_{t+1}\right)k_{t+1}+w\left(\overline{k}_{t+1}\right)-k_{t+2} \\ =\ & \left[\beta r\left(\overline{k}_{t+1}\right)\right]^{\frac{1}{\sigma}}\cdot\left[r\left(\overline{k}_t\right)k_t+w\left(\overline{k}_t\right)-k_{t+1}\right] \end{aligned}$$

Since this is a second-order linear difference equation in k_t, we conjecture the solution form as

$$k^{'} = \mu\left(\overline{k}\right) + \lambda\left(\overline{k}\right)k$$

Plug back into Euler equation, we get

$$\begin{aligned} LHS &= r\left(\overline{k}_{t+1}\right)k_{t+1} + w\left(\overline{k}_{t+1}\right) - k_{t+2} \\ &= r\left(\overline{k}_{t+1}\right)\left(\mu\left(\overline{k}_t\right) + \lambda\left(\overline{k}_t\right)k_t\right) + w\left(\overline{k}_{t+1}\right) \\ &\quad -\left(\mu\left(\overline{k}_{t+1}\right) + \lambda\left(\overline{k}_{t+1}\right)\left(\mu\left(\overline{k}_t\right) + \lambda\left(\overline{k}_t\right)k_t\right)\right) \\ &= \left[\left(r\left(\overline{k}_{t+1}\right) - \lambda\left(\overline{k}_{t+1}\right)\right)\lambda\left(\overline{k}_t\right)\right]k_t + \\ &\quad \left[\mu\left(\overline{k}_t\right)\left(r\left(\overline{k}_{t+1}\right) - \lambda\left(\overline{k}_{t+1}\right)\right) + w\left(\overline{k}_{t+1}\right) - \mu\left(\overline{k}_{t+1}\right)\right] \end{aligned}$$

and

$$\begin{aligned} RHS &= \left[\beta r\left(\overline{k}_{t+1}\right)\right]^{\frac{1}{\sigma}}\left[r\left(\overline{k}_t\right)k_t + w\left(\overline{k}_t\right) - \left(\mu\left(\overline{k}_t\right) + \lambda\left(\overline{k}_t\right)k_t\right)\right] \\ &= \left[\beta r\left(\overline{k}_{t+1}\right)\right]^{\frac{1}{\sigma}}\left[\left(r\left(\overline{k}_t\right) - \lambda\left(\overline{k}_t\right)\right)k_t + \left(w\left(\overline{k}_t\right) - \mu\left(\overline{k}_t\right)\right)\right] \end{aligned}$$

To make LHS and RHS equate for every k_t and k_{t+1}, we must have

$$\begin{aligned} (1) &: \left(r\left(\overline{k}_{t+1}\right) - \lambda\left(\overline{k}_{t+1}\right)\right)\lambda\left(\overline{k}_t\right) = \left[\beta r\left(\overline{k}_{t+1}\right)\right]^{\frac{1}{\sigma}}\left(r\left(\overline{k}_t\right) - \lambda\left(\overline{k}_t\right)\right) \\ (2) &: \mu\left(\overline{k}_t\right)\left(r\left(\overline{k}_{t+1}\right) - \lambda\left(\overline{k}_{t+1}\right)\right) + w\left(\overline{k}_{t+1}\right) - \mu\left(\overline{k}_{t+1}\right) \\ &= \left[\beta r\left(\overline{k}_{t+1}\right)\right]^{\frac{1}{\sigma}}\left(w\left(\overline{k}_t\right) - \mu\left(\overline{k}_t\right)\right) \end{aligned}$$

i.e.

$$(1) \Rightarrow \frac{1}{\beta r\left(\overline{k}^{'}\right)} = \left(\frac{\left(r\left(\overline{k}^{'}\right) - \lambda\left(\overline{k}^{'}\right)\right)\lambda\left(\overline{k}\right)}{r\left(\overline{k}\right) - \lambda\left(\overline{k}\right)}\right)^{-\sigma}$$

$$\begin{aligned} (2) \Rightarrow \mu\left(\overline{k}\right) + \frac{w\left(\overline{k}^{'}\right) - \mu\left(\overline{k}^{'}\right)}{r\left(\overline{k}^{'}\right) - \lambda\left(\overline{k}^{'}\right)} &= \left[\beta r\left(\overline{k}^{'}\right)\right]^{\frac{1}{\sigma}}\frac{w\left(\overline{k}\right) - \mu\left(\overline{k}\right)}{r\left(\overline{k}^{'}\right) - \lambda\left(\overline{k}^{'}\right)} \\ \Rightarrow \mu\left(\overline{k}\right) + \frac{w\left(\overline{k}^{'}\right) - \mu\left(\overline{k}^{'}\right)}{r\left(\overline{k}^{'}\right) - \lambda\left(\overline{k}^{'}\right)} &= \left[\frac{\left(r\left(\overline{k}^{'}\right) - \lambda\left(\overline{k}^{'}\right)\right)\lambda\left(\overline{k}\right)}{r\left(\overline{k}\right) - \lambda\left(\overline{k}\right)}\right]\frac{w\left(\overline{k}\right) - \mu\left(\overline{k}\right)}{r\left(\overline{k}^{'}\right) - \lambda\left(\overline{k}^{'}\right)} \\ \Rightarrow \mu\left(\overline{k}\right) + \frac{w\left(\overline{k}^{'}\right) - \mu\left(\overline{k}^{'}\right)}{r\left(\overline{k}^{'}\right) - \lambda\left(\overline{k}^{'}\right)} &= \frac{w\left(\overline{k}\right) - \mu\left(\overline{k}\right)}{r\left(\overline{k}\right) - \lambda\left(\overline{k}\right)}\lambda\left(\overline{k}\right) \end{aligned}$$

This finishes the proof.

(b) Suppose now that there are two (types of) consumers in the economy who differ only in their initial capital holdings. Each consumer represents half of the economy's population. Use the result from part (a) to argue that a redistribution of capital (holding aggregate capital constant) across the two consumers at time 0 has no effect on equilibrium interest rates and wages.

To prove the result, we need to see what is the difference between a representative agent economy and heterogeneous agent economy. So we first clearly define a recursive competitive equilibrium for heterogeneous agent economy.

A recursive competitive equilibrium for the economy with two types of agents is a set of functions:

$$\begin{aligned} price\ function &: r\left(\overline{k}\right), w\left(\overline{k}\right) \\ policy\ function &: k_i^{'} = g_i\left(k_i, \overline{k}_1, \overline{k}_2\right) \\ value\ function &: v_i\left(k_i, \overline{k}_1, \overline{k}_2\right) \\ transition\ function &: \overline{k}_i^{'} = G_i\left(\overline{k}_1, \overline{k}_2\right) \end{aligned}$$

where $\overline{k} = \frac{1}{2}(k_1 + k_2)$, such that:

(1) Given $G_1\left(\overline{k}_1, \overline{k}_2\right)$ and $G_2\left(\overline{k}_1, \overline{k}_2\right)$, $k_i^{'} = g_i\left(k_i, \overline{k}_1, \overline{k}_2\right)$ and $v_i\left(k_i, \overline{k}_1, \overline{k}_2\right)$ solve consumer's problem:

$$\begin{aligned} v_i\left(k_i, \overline{k}_1, \overline{k}_2\right) &= \max_{\{c_i, k_i^{'}\}} u_i(c_i) + \beta v_i\left(k_i^{'}, \overline{k}_1^{'}, \overline{k}_2^{'}\right) \\ & \quad s.t. \\ & \quad c_i + k_i^{'} = r\left(\overline{k}\right) k_i + w\left(\overline{k}\right) \\ & \quad \overline{k}_1^{'} = G_1\left(\overline{k}_1, \overline{k}_2\right) \\ & \quad \overline{k}_2^{'} = G_2\left(\overline{k}_1, \overline{k}_2\right) \end{aligned}$$

(2) Price is competitively determined:

$$\begin{aligned} r\left(\overline{k}\right) &= F_1\left(\overline{k}, 1\right) + 1 - \delta \\ w\left(\overline{k}\right) &= F_2\left(\overline{k}, 1\right) \end{aligned}$$

(3) Consistency:

$$G_i\left(\overline{k}_1, \overline{k}_2\right) = g_i(\overline{k}_i, \overline{k}_1, \overline{k}_2)$$

Notice the difference between the definition of recursive competitive equilibrium we met before: in heterogeneous agent economy, normally the whole distribution of state variable counts.

Now we start to prove the result. Due to the special preference relation here, first we assume that only average capital level matters for the evolution of the economy, i.e. $\overline{k}^{'} = G\left(\overline{k}\right)$. Then the result from part (a) tells us that in recursive competitive equilibrium we will have

$$k_i^{'} = \mu\left(\overline{k}\right) + \lambda\left(\overline{k}\right) k_i$$

Therefore, we have

$$\overline{k}^{'} = \frac{1}{2}\left(\overline{k}_1^{'} + \overline{k}_2^{'}\right) = \mu\left(\overline{k}\right) + \lambda\left(\overline{k}\right)\overline{k}$$

Therefore, indeed the evolution of average capital stock does not depend on the distribution of capital. In other words, this verifies our guess that $\overline{k}' = G\left(\overline{k}\right)$. Since equilibrium interest rates and wages depend only on the aggregate average capital stock (remember that $r\left(\overline{k}\right) = F_1\left(\overline{k}, 1\right) + 1 - \delta$ and $w\left(\overline{k}\right) = F_2\left(\overline{k}, 1\right)$), a redistribution of capital (holding aggregate capital constant) across the two consumers at time 0 has no effect on equilibrium interest rates and wages.