Suggested Solutions to Homework #6
Econ 511b (Part I), Spring 2004

1.

(a) **Find the planner's optimal decision rule in the stochastic one-sector growth model without valued leisure by linearizing the Euler equation. Let the production function take the form $f(k_t, n_t, z_t) = e^{z_t} k_t^{\alpha} n_t^{1-\alpha}$, let the consumer's felicity function have a constant elasticity of intertemporal substitution γ^{-1}, and let z_t follow an AR(1) process: $z_{t+1} = \rho z_t + \epsilon_{t+1}$, $\epsilon_{t+1}\tilde{}\, iidN(0, \sigma^2)$. Set $\alpha = 0.36$, $\rho = 0.95$, $\sigma = 0.007$, the discount factor $\beta = 0.99$, and the depreciation rate $\delta = 0.025$. Solve the model for two different values of γ: 1 (the log case) and 2.**

The recursive formulation of central planning problem is

$$\begin{aligned} v(k, z) &= \max_{k'} \frac{c^{1-\gamma} - 1}{1-\gamma} + \beta E_z v(k', z') \\ & s.t. \\ & c + k' = e^z k^{\alpha} + (1-\delta) k \\ & z' = \rho z + \epsilon,\ \epsilon \sim N\left(0, \sigma^2\right) \end{aligned}$$

or

$$\begin{aligned} v(k, z) &= \max_{k'} \frac{\left(e^z k^{\alpha} + (1-\delta) k - k'\right)^{1-\gamma} - 1}{1-\gamma} + \beta E_z v(k', z') \\ & s.t. \\ & z' = \rho z + \epsilon,\ \epsilon \sim N\left(0, \sigma^2\right) \end{aligned}$$

We can easily get the Stochastic Euler equation as

$$\begin{aligned} & u'(c_t) = \beta E_z \left[u'(c_{t+1}) \left(e^{z_{t+1}} f'(k_{t+1}) + 1 - \delta\right)\right] \\ \Rightarrow\ & c_t^{-\gamma} = \beta E_z \left[c_{t+1}^{-\gamma} \left(\alpha e^{z_{t+1}} k_{t+1}^{\alpha-1} + 1 - \delta\right)\right] \\ \Rightarrow\ & \left(e^{z_t} k_t^{\alpha} + (1-\delta) k_t - k_{t+1}\right)^{-\gamma} \\ =\ & \beta E_z \left[\left(e^{z_{t+1}} k_{t+1}^{\alpha} + (1-\delta) k_{t+1} - k_{t+2}\right)^{-\gamma} \left(\alpha e^{z_{t+1}} k_{t+1}^{\alpha-1} + 1 - \delta\right)\right] \end{aligned}$$

The steady state for the deterministic model is

$$\begin{aligned} & u'(c^*) = \beta \left[u'(c^*) \left(f'(k^*) + 1 - \delta\right)\right] \\ \Rightarrow\ & \beta \left(\alpha (k^*)^{\alpha-1} + 1 - \delta\right) = 1 \\ \Rightarrow\ & k^* = \left[\frac{1}{\alpha}\left(\frac{1}{\beta} - (1-\delta)\right)\right]^{\frac{1}{\alpha-1}} \\ \Rightarrow\ & k^* = \left(\frac{\alpha\beta}{1 - \beta(1-\delta)}\right)^{\frac{1}{1-\alpha}} \end{aligned}$$

Now we start to linearize the system. We deal with LHS and RHS separately. First, expanding LHS around steady-state leads to

$$\begin{aligned}
\widehat{LHS} &= \widehat{\left(e^{z_t}k_t^{\alpha} + (1-\delta)\,k_t - k_{t+1}\right)^{-\gamma}} \\
&= -\gamma\left(k^{*\alpha} - \delta k^*\right)^{-\gamma-1} k^{*\alpha}\widehat{z}_t - \gamma\left(k^{*\alpha} - \delta k^*\right)^{-\gamma-1}\left(\alpha k^{*\alpha-1} + 1 - \delta\right)\widehat{k}_t \\
&\quad +\gamma\left(k^{*\alpha} - \delta k^*\right)^{-\gamma-1}\widehat{k}_{t+1}
\end{aligned}$$

where we use the fact $\overline{z} = 0$ and define $\widehat{z}_t = z_t - \overline{z}$, $\widehat{k}_t = k_t - k^*$, and $\widehat{k}_{t+1} = k_{t+1} - k^*$. Second, RHS is equal to

$$\begin{aligned}
\widehat{RHS} &= \beta E_t\left[\widehat{\left(e^{z_{t+1}}k_{t+1}^{\alpha} + (1-\delta)\,k_{t+1} - k_{t+2}\right)^{-\gamma}\left(\alpha z_{t+1}k_{t+1}^{\alpha-1} + 1 - \delta\right)}\right] \\
&= \beta\left(k^{*\alpha} - \delta k^*\right)^{-\gamma-1} k^{*\alpha}\left[\alpha(1-\gamma)\,k^{*\alpha-1} - \alpha\delta - \gamma(1-\delta)\right]E_t\left(\widehat{z}_{t+1}\right) + \\
&\quad \beta\left(k^{*\alpha} - \delta k^*\right)^{-\gamma-1}\left[-\gamma\left(\alpha k^{*\alpha-1} + 1 - \delta\right)^2 + \alpha(\alpha-1)\,k^{*\alpha-1}\left(k^{*\alpha-1} - \delta\right)\right]\widehat{k}_{t+1} \\
&\quad +\beta\gamma\left(k^{*\alpha} - \delta k^*\right)^{-\gamma-1}\left(\alpha k^{*\alpha-1} + 1 - \delta\right)E_t\left(\widehat{k}_{t+2}\right)
\end{aligned}$$

Let $\widehat{LHS} = \widehat{RHS}$, we have

$$\begin{aligned}
& \widehat{LHS} = \widehat{RHS} \\
\Rightarrow\; & -\gamma k^{*\alpha}\widehat{z}_t - \gamma\left(\alpha k^{*\alpha-1} + 1 - \delta\right)\widehat{k}_t + \gamma\widehat{k}_{t+1} \\
=\; & \beta k^{*\alpha}\left[\alpha(1-\gamma)\,k^{*\alpha-1} - \alpha\delta - \gamma(1-\delta)\right]E_t\left(\widehat{z}_{t+1}\right) \\
& +\beta\left[-\gamma\left(\alpha k^{*\alpha-1} + 1 - \delta\right)^2 + \alpha(\alpha-1)\,k^{*\alpha-1}\left(k^{*\alpha-1} - \delta\right)\right]\widehat{k}_{t+1} \\
& +\beta\gamma\left(\alpha k^{*\alpha-1} + 1 - \delta\right)E_t\left(\widehat{k}_{t+2}\right)
\end{aligned}$$

which is a second-order linear difference equation with expectations. We can simplify it a little bit by using the steady-state equation, which leads to

$$\begin{aligned}
& \widehat{LHS} = \widehat{RHS} \\
\Rightarrow\; & -\gamma k^{*\alpha}\widehat{z}_t - \gamma\beta^{-1}\widehat{k}_t + \gamma\widehat{k}_{t+1} \\
=\; & k^{*\alpha}\left[(1-\gamma) - \beta + \beta(1-\alpha)\,\delta\right]E_t\left(\widehat{z}_{t+1}\right) \\
& +\left[-\gamma\beta^{-1} + \left(1-\alpha^{-1}\right)\left(\beta^{-1} - (1-\delta)\right)(1-\beta)\right]\widehat{k}_{t+1} + \gamma E_t\left(\widehat{k}_{t+2}\right) \qquad (1)
\end{aligned}$$

To save notation, now we express the equation as

$$a_1\widehat{z}_t + a_2\widehat{k}_t + a_3\widehat{k}_{t+1} = b_1E_t\left(\widehat{z}_{t+1}\right) + b_2\widehat{k}_{t+1} + b_3E_t\left(\widehat{k}_{t+2}\right) \qquad (2)$$

where

$$\begin{aligned}
a_1 &= -\gamma k^{*\alpha} \\
a_2 &= -\gamma\beta^{-1} \\
a_3 &= \gamma \\
b_1 &= k^{*\alpha}\left[(1-\gamma) - \beta + \beta(1-\alpha)\,\delta\right] \\
b_2 &= -\gamma\beta^{-1} + \left(1-\alpha^{-1}\right)\left(\beta^{-1} - (1-\delta)\right)(1-\beta) \\
b_3 &= \gamma
\end{aligned}$$

We solve this equation using Method of Undetermined Coefficients as taught in the lectures. Conjecture the form of solution as

$$\begin{cases} \widehat{k}_{t+1} = a\widehat{k}_t + b\widehat{z}_t \\ \widehat{z}_{t+1} = \rho\widehat{z}_t + \epsilon_{t+1} \end{cases}$$

Plug this into the equation (1), we have

$$\begin{aligned} & a_1\widehat{z}_t + a_2\widehat{k}_t + a_3\left(a\widehat{k}_t + b\widehat{z}_t\right) \\ = \quad & b_1 E_t\left(\rho\widehat{z}_t + \epsilon_{t+1}\right) + b_2\left(a\widehat{k}_t + b\widehat{z}_t\right) + b_3 E_t\left(a\left(a\widehat{k}_t + b\widehat{z}_t\right) + b\left(\rho\widehat{z}_t + \epsilon_{t+1}\right)\right) \\ \Rightarrow \quad & \left[b_1\rho + b_2 b + b_3 b\left(a+\rho\right) - a_1 - a_3 b\right]\widehat{z}_t + \left[ab_2 + b_3 a^2 - a_2 - a_3 a\right]\widehat{k}_t = 0 \\ \Rightarrow \quad & \left[b_1\rho - a_1 + \left(b_2 + b_3\left(a+\rho\right) - a_3\right)b\right]\widehat{z}_t + \left[b_3 a^2 + \left(b_2 - a_3\right)a - a_2\right]\widehat{k}_t = 0 \end{aligned}$$

To make this an identity, we must have

$$\begin{cases} b_1\rho - a_1 + \left(b_2 + b_3\left(a+\rho\right) - a_3\right)b = 0 \\ b_3 a^2 + \left(b_2 - a_3\right)a - a_2 = 0 \end{cases}$$

Therefore, the final solution is

$$\begin{cases} a = \frac{(a_3-b_2)\pm\sqrt{(b_2-a_3)^2+4a_2b_3}}{2b_3} \\ b = \frac{a_1-b_1\rho}{(b_2+b_3(a+\rho)-a_3)} \end{cases}$$

where we need to keep only the root of a with absolute value smaller than 1. We can solve the numbers with computer (see attached code). For $\gamma = 1$, we have

$$\begin{cases} \widehat{k}_{t+1} = 0.9798\widehat{k}_t + 3.4127\widehat{z}_t \\ \widehat{z}_{t+1} = 0.95\widehat{z}_t + \epsilon_{t+1} \end{cases}$$

for $\gamma = 2$, we have

$$\begin{cases} \widehat{k}_{t+1} = 0.9868\widehat{k}_t + 3.1331\widehat{z}_t \\ \widehat{z}_{t+1} = 0.95\widehat{z}_t + \epsilon_{t+1} \end{cases}$$

(b) Using Matlab, compute impulse response functions for output, consumption, and investment in response to an innovation in the level of technology of size σ. That is, suppose that the economy is at the deterministic steady state and the innovation ϵ suddenly jumps in the current period to and then returns to 0 in all subsequent periods. Determine (numerically) the values of output, consumption, and investment in the current period and in the next, say, 10 periods. Express the impulse responses as percentage deviations from the (deterministic) steady-state values. Compare the impulse response functions for the two values of γ.

Run the Matlab code accompanying this answer.

2. **Consider a stochastic neoclassical growth model with the following structure:**

- **Each consumer has preferences of the form:**

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t + B\frac{(1-n_t)^{1-\nu}-1}{1-\nu}\right)^{1-\sigma} - 1}{1-\sigma}$$

where labor supply $n_t < 1$.

- **The economy's technology is described by:**

$$c_t + k_{t+1} - (1-\delta) k_t = z_t k_t^{\alpha} n_t^{1-\alpha}$$

where $\log(z_t)$ is stochastic and evolves according to a stationary AR(1) process:

$$z_{t+1} = \rho z_t^{\rho} \epsilon_{t+1}$$

with $\log(\epsilon_t)\tilde{}\, iidN\left(0, \sigma_\epsilon^2\right)$.

(a) **Derive the Euler equation for the savings decision.**

To solve this problem, we repeat the same steps as in the corresponding part in HW#4 (problem 2).

A recursive competitive equilibrium for the stochastic neoclassical growth model with valued leisure is a set of functions:

$$\begin{aligned}
price\ function &: r\left(\overline{k}, z\right), w\left(\overline{k}, z\right) \\
policy\ function &: k' = g_k\left(k, \overline{k}, z\right), n = g_n\left(k, \overline{k}, z\right) \\
value\ function &: v\left(k, \overline{k}, z\right) \\
aggregate\ state &: \overline{k}' = G\left(\overline{k}, z\right), z' = \rho z^{\rho} \epsilon
\end{aligned}$$

such that:

(1) Given the evolution law of aggregate state $\overline{k}' = G\left(\overline{k}, z\right)$ and $z' = \rho z^{\rho}\epsilon$, $k' = g_k\left(k, \overline{k}, z\right), n = g_n\left(k, \overline{k}, z\right)$ and $v\left(k, \overline{k}, z\right)$ solves consumer's problem:

$$\begin{aligned}
v\left(k, \overline{k}, z\right) &= \max_{n,k'} \frac{\left(c + B\frac{(1-n)^{1-\nu}-1}{1-\nu}\right)^{1-\sigma} - 1}{1-\sigma} + \beta E_z v\left(k', \overline{k}', z'\right) \\
& s.t. \\
& c + k' = r\left(\overline{k}, z\right) k + w\left(\overline{k}, z\right) n \\
& \overline{k}' = G\left(\overline{k}, z\right) \\
& z' = \rho z^{\rho} \epsilon,\ \log(\epsilon_t) \sim N\left(0, \sigma_\epsilon^2\right)
\end{aligned}$$

(2) Price is competitively determined:

$$\begin{aligned}
r\left(\overline{k}, z\right) &= F_1\left(\overline{k}, g_n\left(\overline{k}, \overline{k}, z\right)\right) = \alpha z \left(\frac{\overline{k}}{g_n\left(\overline{k}, \overline{k}, z\right)}\right)^{\alpha-1} + 1 - \delta \\
w\left(\overline{k}, z\right) &= F_2\left(\overline{k}, g_n\left(\overline{k}, \overline{k}, z\right)\right) = (1-\alpha) z \left(\frac{\overline{k}}{g_n\left(\overline{k}, \overline{k}, z\right)}\right)^{\alpha}
\end{aligned}$$

(3) Consistency:

$$G\left(\overline{k},z\right)=g_k\left(\overline{k},\overline{k},z\right)$$

The F.O.C. for k' is

$$\{k'\}:U_1\left(c,n\right)=\beta E\left(v_1\left(k',\overline{k}',z'\right)\right)$$

Use the envelope condition

$$v_1\left(k,\overline{k},z\right)=U_1\left(c,n\right)r\left(\overline{k},z\right)$$

we get the Euler equation for savings decision:

$$\begin{aligned}
&U_1\left(c_t,n_t\right)=\beta E\left[U_1\left(c_{t+1},n_{t+1}\right)r\left(\overline{k}_{t+1},z_{t+1}\right)\right]\\
\Rightarrow\quad&\left(c_t+B\frac{\left(1-n_t\right)^{1-\nu}-1}{1-\nu}\right)^{-\sigma}=\beta E\left[\left(c_{t+1}+B\frac{\left(1-n_{t+1}\right)^{1-\nu}-1}{1-\nu}\right)^{-\sigma}r\left(\overline{k}_{t+1},z_{t+1}\right)\right]
\end{aligned}$$

where $c_t=r\left(\overline{k}_t,z_t\right)k_t+w\left(\overline{k}_t,z_t\right)n_t$.

(b) **Derive the first-order condition for the labor-leisure decision.**
The F.O.C. for n is

$$\begin{aligned}
&U_1\left(c,n\right)w\left(\overline{k},z\right)+U_2\left(c,n\right)=0\\
\Rightarrow\quad&\left(c+B\frac{\left(1-n\right)^{1-\nu}-1}{1-\nu}\right)^{-\sigma}w\left(\overline{k},z\right)=\left(c+B\frac{\left(1-n\right)^{1-\nu}-1}{1-\nu}\right)^{-\sigma}B\left(1-n\right)^{-\nu}\\
\Rightarrow\quad&w\left(\overline{k},z\right)=B\left(1-n\right)^{-\nu}
\end{aligned}$$

(c) **Show that the first-order condition for the labor-leisure decision can be written as a simple function relating $\log(1-n)$ to $\log(w)$, where w is the wage rate.**
From the result in part (b), it is easy to calculate that

$$\begin{aligned}
&\log\left(w\left(\overline{k},z\right)\right)=\log B-v\log\left(1-n\right)\\
\Rightarrow\quad&\log\left(1-n\right)=\frac{1}{v}\left(\log B-\log\left(w\left(\overline{k},z\right)\right)\right)
\end{aligned}$$

(d) **Use the following facts from the hypothetical economy Pekrland to calibrate all of the model's parameters except ρ and σ_ϵ^2:**
We calibrate every coefficient in order.

- **Experimental evidence on Pekrlanders' attitudes towards risk shows that they all have a coefficient of relative risk aversion equal to 2.**
  It means that $\sigma=2$.
- **Capital's share of income is one-third.**
  It means that $\alpha=\frac{1}{3}$ since $\frac{rk}{zk^\alpha n^{1-\alpha}}=\frac{\alpha zk^\alpha n^{1-\alpha}}{zk^\alpha n^{1-\alpha}}=\alpha$.

- **The average value of the investment-to-output ratio is** 0.2**.**

$$\frac{I}{Y} = 0.2 \Rightarrow \frac{\delta k}{Y} = 0.2 \Rightarrow \delta = \frac{0.2}{k/Y} = 0.1 \text{ (since } k/Y = 2)$$

- **The average value of the capital-output ratio (in annual terms) is** 2**.**
From Euler equation for savings problem, in deterministic steady state we have

$$\begin{aligned} & r\left(\overline{k}, \overline{z}\right) = \frac{1}{\beta} \\ \Rightarrow\ & \alpha\overline{z}\left(\frac{k^*}{n^*}\right)^{\alpha-1} + (1-\delta) = \frac{1}{\beta} \\ \Rightarrow\ & \alpha\frac{y^*}{k^*} + (1-\delta) = \frac{1}{\beta} \\ \Rightarrow\ & \beta = \frac{1}{\alpha\frac{y^*}{k^*} + (1-\delta)} \\ \Rightarrow\ & \beta = \frac{1}{\frac{1}{3}\times\frac{1}{2} + (1-0.1)} = \frac{15}{16} = 0.9375 \end{aligned}$$

- **Labor economists in Pekrland have found that, in a log-log regression of a typical Perklander's hours of leisure on the wage, the coefficient on the log of the wage is** -0.5**: if wages go up by** 1%**, a typical Pekrlander works** 0.5% **more hours.**
It means that $\frac{1}{v} = 0.5$, or $v = 2$.
- **Pekrlanders work (on average) one-half of their total available time.**
From F.O.C. for labor choice, in deterministic steady state we have

$$\begin{aligned} & w\left(k^*, \overline{z}\right) = B\left(1-n^*\right)^{-\nu} \\ \Rightarrow\ & (1-\alpha)\,\overline{z}\left(\frac{k^*}{n^*}\right)^{\alpha} = B\left(1-n^*\right)^{-\nu} \\ \Rightarrow\ & B = \left(1-n^*\right)^{\nu}(1-\alpha)\,\overline{z}\left(\frac{k^*}{n^*}\right)^{\alpha} \end{aligned}$$

So we need only to determine the capital-labor ratio. From the equation

$$\begin{aligned} & \frac{k^*}{y^*} = 2 \\ \Rightarrow\ & \frac{k^*}{\overline{z}k^{*\alpha}n^{*1-\alpha}} = 2 \\ \Rightarrow\ & \frac{k^*}{n^*} = (2\overline{z})^{\frac{1}{1-\alpha}} \end{aligned}$$

Therefore, we have

$$\begin{aligned} B &= (1-n^*)^{\nu}(1-\alpha)\,\overline{z}\left(\frac{k^*}{n^*}\right)^{\alpha} \\ &= (1-n^*)^{\nu}(1-\alpha)\,\overline{z}\,(2\overline{z})^{\frac{\alpha}{1-\alpha}} \\ &= \left(1-\frac{1}{2}\right)^{2}\left(1-\frac{1}{3}\right)\overline{z}\,(2\overline{z})^{\frac{1}{2}} \\ &= \frac{\sqrt{2}}{6}\overline{z}^{3/2} \end{aligned}$$

where $\overline{z} = \rho^{\frac{1}{1-\rho}}$.

You do not need to compute all of the parameters numerically, but you do need to describe how you would compute them.

3. **Consider again the growth model with an externality in production that you studied in problem 2 of Homework #3. In particular, let $\gamma = 1-\alpha$ and let the consumer's felicity function have constant elasticity of intertemporal substitution σ^{-1}.**

(a) **Show that, in competitive equilibrium, this economy behaves like an "AK" model. Express the competitive equilibrium growth rate in terms of primitives of technology and preferences.**

From the corresponding part in HW#3, the Euler equation for a recursive competitive equilibrium is:

$$\begin{aligned} & \frac{\beta u'(c_{t+1})}{u'(c_t)}\left(r\left(\overline{k}_{t+1}\right)+1-\delta\right) = 1 \\ \Rightarrow\ & \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}(\alpha A+1-\delta) = 1 \\ \Rightarrow\ & \frac{c_{t+1}}{c_t} = [\beta(\alpha A+1-\delta)]^{\frac{1}{\sigma}} \\ \Rightarrow\ & \frac{A\overline{k}_{t+1}+(1-\delta)\overline{k}_{t+1}-\overline{k}_{t+2}}{A\overline{k}_t+(1-\delta)\overline{k}_t-\overline{k}_{t+1}} = [\beta(\alpha A+1-\delta)]^{\frac{1}{\sigma}} \end{aligned}$$

Now conjecture that $\overline{k}_{t+1} = e^{g^c}\overline{k}_t$, plug into F.O.C. we get

$$\begin{aligned} & \frac{Ae^{g}\overline{k}_t+(1-\delta)e^{g}\overline{k}_t-e^{2g}\overline{k}_t}{A\overline{k}_t+(1-\delta)\overline{k}_t-e^{g}\overline{k}_t} = [\beta(\alpha A+1-\delta)]^{\frac{1}{\sigma}} \\ \Rightarrow\ & \frac{Ae^{g}+(1-\delta)e^{g}-e^{2g}}{A+(1-\delta)-e^{g}} = [\beta(\alpha A+1-\delta)]^{\frac{1}{\sigma}} \\ \Rightarrow\ & g^c = \sigma^{-1}[\log(\alpha A+1-\delta)+\log\beta] \end{aligned}$$

Depending on the value of $\sigma^{-1}[\log(\alpha A+1-\delta)+\log\beta]$, the economy can have constant positive (consumption, capital, and output) growth rate (if $[\beta(\alpha A+1-\delta)]^{\frac{1}{\sigma}} >$

1), constant negative growth rate (if $[\beta(\alpha A+1-\delta)]^{\frac{1}{\sigma}}<1$), or zero growth rate (if $[\beta(\alpha A+1-\delta)]^{\frac{1}{\sigma}}=1$). In this sense it behaves like an "AK" model, which leads to endogenous growth.

(b) Show that the growth rate that would be chosen by a social planner is larger than the competitive equilibrium growth rate.

From the corresponding part in HW#3, the Euler equation for the central planning problem is:

$$\begin{aligned}
&\frac{\beta u'(c_{t+1})}{u'(c_t)}(A+1-\delta)=1\\
\Rightarrow\ &\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}(A+1-\delta)=1\\
\Rightarrow\ &\frac{c_{t+1}}{c_t}=[\beta(A+1-\delta)]^{\frac{1}{\sigma}}
\end{aligned}$$

By the same steps as in part (a), we get

$$\overline{k}_{t+1}=e^{g^o}\overline{k}_t$$

with

$$g^o=\sigma^{-1}[\log(A+1-\delta)+\log\beta]$$

Since $[\beta(A+1-\delta)]^{\frac{1}{\sigma}}>[\beta(\alpha A+1-\delta)]^{\frac{1}{\sigma}}$, the growth rate that would be chosen by a social planner is larger than the competitive equilibrium growth rate.

(c) Find a tax policy that would induce the competitive equilibrium growth rate to coincide with the growth rate chosen by a social planner.

The answers in this part depends on the specification of the tax form. To be consistent with HW#3, here we analyze the case with investment subsidy. From the corresponding part in HW#3, the Euler equation is

$$\begin{aligned}
&\frac{\beta u'(c_{t+1})}{u'(c_t)}\frac{1}{1-\tau}\left(r\left(\overline{k}\right)+(1-\tau)(1-\delta)\right)=1\\
\Rightarrow\ &\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\frac{1}{1-\tau}(\alpha A+(1-\tau)(1-\delta))=1\\
\Rightarrow\ &\frac{c_{t+1}}{c_t}=\left[\beta\left(\frac{\alpha A}{1-\tau}+1-\delta\right)\right]^{\frac{1}{\sigma}}
\end{aligned}$$

Solve for the equation

$$\begin{aligned}
&\left[\beta\left(\frac{\alpha A}{1-\tau}+1-\delta\right)\right]^{\frac{1}{\sigma}}=[\beta(A+1-\delta)]^{\frac{1}{\sigma}}\\
\Rightarrow\ &\tau=1-\alpha
\end{aligned}$$