Business Cycles

The purpose of this section is to introduce the study of business cycles. By business cycles we mean fluctuations of output around its long term growth trend. In this sense, it complements growth theory to provide a thorough explanation of the behavior of economic aggregates: First, output grows secularly. Second, it fluctuates around its long term trend. The former phenomenon we have already analyzed. The latter is our topic now.

We will first overview the empirical facts that constitute our object of study, and the history of the attempts at explaining these facts. After that, we will study the theory that has developed. We could separate the evolution of this theory in three phases: i) Pre-Real Business Cycles; ii) Real Business Cycles; iii) Current trends.

1 Introduction

There are two basic questions that gave birth to this area of macroeconomics:

1. Why are there cycles?

2. How do they work?

Before the real business cycles revolution, keynesianism approached the understanding of the business cycle by postulating that investors were driven by "animal spirits". These non rational "feelings" of investors propelled them to frantically invest or gloomily refrain from doing so, according to the prevailing mood. This notion has not completely disappeared from economics, however elaborate the explanations of investor behavior have now come to be. The current version of "animal spirits" does not refer to the moods of those who make investment decisions, but of those who make consumption decision: It is the fashionable indicator of "consumer confidence". Apparently, people go to mall whenever they wake up feeling confident.

The keynesians and their intellectual heirs did not base their approach to the business cycles on micro-foundations of macro behavior. Quite on the contrary, they study the effects of the mentioned "moods" on aggregate variables such as output and employment. Since acting on moods is an irrational way to make decisions, the economy looses potential value due to this lack of rationality; hence the government is called upon to correct this behavior. The role of government is hence one of the main topics of interest for these traditions in macroeconomics.

However, Lucas' [] "critique" to the aggregative approach to macroeconomics; and, more importantly, his [] paper on "understanding business cycles", generated the real business cycles revolution. The main pioneers in this tradition were Kydland and Prescott [], and Long and Rebelo []. Besides its relevance and ability to explain the business cycle, this approach has had a very significant methodological impact on the practice of macroeconomics.

Under this view, the reason for cycles in the economy is that there are "technology shocks" that affect the productivity of factors. The source of the shock is real, and the propagation mechanism is real as well: it is a consequence of the inter-temporal substitution of labor that optimal decision makers choose whenever confronted to one such "technology shock".

The critique to this approach is that the definition of a "technology shock" is somewhat blurred. What is exactly such shock? Notwithstanding this weakness, the real business cycles tradition has data against which to contrast its hypotheses. Technology shocks can be measured through the de-trended "Solow residual" from actual data.

Finally, the reason why this tradition has focused on the "real" explanation to business cycles is rather accidental. When Prescott undertook the research program laid down by Lucas' [] "understanding business cycles" paper, the initial schedule was to start with a real source of the cycle (the technology shock) and the real propagation mechanism (the inter-temporal substitution), thereafter to increase the complexity of the model and allow for monetary mechanisms. However, on seeing that the real / real approach was providing a seemingly thorough explanation of the phenomenon, the course of the research program deviated towards increasing the complexity and richness of the real / real setup (such as introducing heterogeneity in agents, incomplete markets, etcetera).

Of course, the real business cycles tradition is not the only one claiming the ability to provide for an explanation of short run fluctuations of output around its growth trend. Among the main current contestants, the most relevant are:

(i) *New Keynesian models.* Opposed to the real / real approach, these take a monetary / monetary approach: The source of the cycles are monetary fluctuations, and the main propagation mechanism is also monetary: price "stickiness".

(ii) *Sunspot theories.* These are micro foundations models in which agents have full rationality, but are faced with models that have multiple equilibria. This allows for self-fulfilling, rational expectations that may cause fluctuations of output, even in spite of the absence of an underlying change in the production or utility fundamentals in the economy. This can be interpreted as a coordination problem, in which agents fail to "choose" the "best" equilibrium out of the available ones. Notice that to some

extent, the "animal spirits" (or "consumer confidence") concept can be accommodated to explain why agents simultaneously believe that a given equilibrium will be realized, and act accordingly.

Before embarking on our topic of study, let us make a final comment on the current state of the art, in particular of the "real / real" approach. Most of the research has modelled typical, complete markets, usually operating under perfect competition. This rules out the possibility of the structure of markets itself playing a role in the business cycle. Notice that in the case of the New Keynesians, this is quite the opposite: it is the structure of a single market (the money market) which generates and propagates the fluctuations. Without taking this rather extreme view, the "real / real" approach could be enriched by allowing the structure of markets to have its share on the account of the cycle phenomenon. The new literature is exploring this by introducing the need to "search" in the behavior of decision makers. Information on prices and employment opportunities are not immediately available, as in the typical "decentralized" version of the planner's problem, as we have studied it. Introducing the possibility of informational frictions in markets can account for the existence of unemployment, and give a role to money in the business cycle.

2 Facts before theory

In this section we are interested in presenting the main "facts" that business cycle theory seeks to explain. We take a rather epistemological definition of the word: By "Facts" we mean not exactly data, but rather what the economics profession regards as the acceptable indicators to be drawn from that data, and what the meaning is. The history of business cycles research has a very significant "dialectical" dimension: What are the "facts" to be explained? How should these be presented? What is the correct methodology to transform "raw data" into "acceptable facts"? All these questions are more than just methodological: they also reveal the different underlying visions of the economic phenomenon that are sustained by different schools in the profession. In the (extremely) brief overview of the history of "facts" that follows, this underlying debate can be seen to take the form of a methodological discussion.

2.1 Early Facts

The first intellectual precedent in the study of short run fluctuations in output was Burns and Mitchell's [1946] "Measuring business cycles" paper. Their purpose was to obtain stylized facts, à la Kaldor's growth facts. Their main findings were:

∗ Output in different sectors of the economy have positive covariance.

∗ Both investment and consumption of durable goods exhibit high variability.

∗ Company profits are very pro-cyclical and variable.

∗ Prices are pro-cyclical as well

   (This is not true for the post-war years, but it was for the sample analyzed by Burns and Mitchell).

∗ The amount of outstanding money balances and the velocity of circulation are pro-cyclical.

∗ Short term interest rates are pro-cyclical.

∗ Long term interest rates are <u>not</u> pro-cyclical.

∗ Business cycles are "all alike" across countries and across time.

Burns and Mitchell's work was harshly criticized by Koopmans in his [] "Measurement without theory" paper. This critique was rather of a statistical, methodological nature. The main weaknesses highligthed in Burns and Mitchell's research were that:

- The work was not carefully done, and was hard to replicate.

- There was no solid underlying statistical theory. Relevant issues were not addressed altogether, such as the statistical significance of the assertions.

Koopmans' counter-argument discredited Burns and Mitchell's approach to the extent that no literature developed to improve and further their view. Instead of this, the leading study of business cycles was that undertaken in Yale's Cowles commission, which consisted of studying huge econometric models of macroeconomic variations. This was called the "macroeconometrics" approach. The main authors in this tradition were Klein (Nobel prize due to this research) and Goldberg.

However, little has been left behind by this methodology, which ended up consisting of building up large scale macroeconomic models, making them bigger and bigger variable-wise until the regressions explained something. Finally, Lucas' [] critique, that found widespread agreement through the economic profession, put an end to this research program.

As a result, research found itself on the need of new stylized facts to explain; since regressions were no longer regarded as a valid phenomenological source. The task to provide for "credible" (maybe just properly presented !) stylized facts, and then a suitable theoretical framework to explain them, was undertaken by Kydland and Prescott.

4

2.2 Kydland and Prescott: How to convert raw data into facts

Kydland and Prescott's work is the founding stone of the current consensus on what "facts" are. These authors went back to the Burns and Mitchell tradition of stylized facts. Unlike their predecessors, they succeeded because they were able to provide a solid methodological foundation to their approach (scientific debate is rational).

In the first place, since the phenomenon to be studied is the short run fluctuation of output around its long term growth, this fluctuation needs to be pinned down with precision. Raw data needs to be "cleaned up" of the secular growth component before the cycle can be identified. This is done by "filtering" data, using the method developed by Hedrick and Prescott (the "HP filter").

The HP filter works in the following way. Given a time series y_t, the purpose is to find out the trend component $\overline{y}_t$, and with this calculate the value of the residual $y_t - \overline{y}_t$. This residual will be the data from which "facts" will be drawn.

The HP filter procedure to de-trend data is to solve the following minimization problem:

$$
\max_{\{\overline{y}_t\}_{t=1}^T} \left\{ \sum_{t=1}^T (y_t - \overline{y}_t)^2 \right\}
$$
$$
s.t. \quad \sum_{t=2}^{T-1} \left[\left(\overline{y}_{t+1} - \overline{y}_t \right) - \left(\overline{y}_t - \overline{y}_{t-1} \right) \right]^2 \leq K
$$

In practice, K is set equal to 0, and this leads to the following Lagrangian:

$$
L = \sum_{t=1}^T (y_t - \overline{y}_t)^2 - \mu \cdot \left[\left(\overline{y}_{t+1} - \overline{y}_t \right) - \left(\overline{y}_t - \overline{y}_{t-1} \right) \right]^2
$$

Hedrick and Prescott choose $\mu = 1600$ to de-trend quarterly data, and $\mu = 400$ for annual data. Once the problem is solved, the object of study is the resulting $\{y_t - \overline{y}_t\}_{t=1}^T$ sequence. With this in hand, "facts" in business cycles research are a series of relevant statistics computed from de-trended data.

2.3 Kydland and Prescott's facts

1- Volatilities

Given a variable x, we define its percentage standard deviation as:

$$
\sigma_x \equiv \frac{(Var\,[x])^{1/2}}{\mu_x}
$$

where μ_x denotes the mean value of x.

Then we have the following findings:

- $\sigma_c < \sigma_y$

    $c \equiv$ consumption

    $y \equiv$ output

    What's behind this fact? Why is consumption less volatile than output? This can be interpreted as evidence for consumption smoothing behavior by agents.

- $\sigma_{c_o} > \sigma_y$

    $c_o \equiv$ consumer durables

- $\sigma_i \sim 3 \times \sigma_y$

    $i \equiv$ investment

- $\sigma_{TB} > \sigma_y$

    $TB \equiv$ trade balance

- $\sigma_N \sim \sigma_y$

    $N \equiv$ total hours worked

- $\sigma_E \sim \sigma_y$

    $E \equiv$ employment

- $\sigma_{N\,/\,week} < \sigma_y$

    $N\,/\,week \equiv$ hours per week

- $\sigma_K << \sigma_y$

    $K \equiv$ capital stock

    In short term periods, the stock of capital exhibits little variation.

- $\sigma_w < \sigma_{y\,/\,N}$

    $w \equiv$ real wage $=$ marginal product of labor

    $y\,/\,N \equiv$ output per worked hour - labor productivity

    The implication of this finding is that real wages are "sticky" - there is some smoothing of real wage fluctuations.

2- Correlations

- $\rho\left(\dfrac{y}{N}, y\right) > 0$

- $\rho\left(w, y\right) \sim 0$

    Recall that y/N is the average product of labor, and w is the marginal product.

- $\rho\left(K, y\right) \sim 0$

- $\rho\left(P, y\right) < 0$ (postwar period)

    $P \equiv$ Price level

<u>3- Persistence</u>

- $\rho\left[\left(y_t - \overline{y}_t\right), \left(y_{t-1} - \overline{y}_{t-1}\right)\right] \sim 0.9$ (from quarterly data)

<u>4- Leads and Lags</u>

This addresses questions such as whether consumption leads output or investment leads output. No strong patterns were found on this regard by Kydland and Prescott.

<u>5- The Business Cycle</u>

Finally, to top off the paradigm debate that underlies the methodological discussion, we must mention that the word "cycles" is not really in Kydland and Prescott's vocabulary. What the authors seek to study is volatilities and correlations in economic variables; not "cycles". Nevertheless, the NBER has a cycle-measuring methodology that assigns beginning and ending dates to business cycles.

3 Theory after facts

3.1 Basic methodology: Introduction to calibration

Once a definition of "facts" is at hand, a theory to account for them can be developed. The research on this was initiated by Kydland and Prescott's [1982]

and Long and Plosser's [1983] papers. The framework is the stochastic neoclassical growth model. And, remember: this project is quantitative. Everything is in the numbers. The success of a real business cycles model is measured by its ability to numerically replicate the "facts".

The basic model is the central planner's problem to optimize the use of resources according to a time-additive preference relation that admits a utility function representation. For example, if production is affected by a shock on total factor productivity that follows an AR(1) process, the problem is:

$$\max_{\{c_t,\, n_t,\, l_t,\, K_{t+1}\}_{t=0}^{\infty}} \left\{ E_0 \left[\sum_{t=0}^{\infty} \beta^t \cdot u\left(c_t,\, l_t\right) \right] \right\}$$
$$s.t. \quad \begin{array}{l} c_t + x_t = z_t \cdot F\left(K_t,\, n_t\right) \\ K_{t+1} = (1 - \delta) \cdot K_t + x_t \\ l_t + n_t = 1 \\ z_{t+1} = \rho \cdot z_t + \varepsilon_{t+1} \end{array}$$

The central methodological issue is how to pick the parameters in the utility and production functions. In this sense, the work of Kydland and Prescott has also a "dialectical" dimension. The authors are advocates of the technique known as "calibration". This is more than merely how to pick values for parameters to solve a numerical problem. It is a way of contrasting models against data, opposed to traditional econometrics.

Calibration, sometimes also called "back-of-the-envelope calculations", requires that values for parameters be picked from sources independent of the phenomenon under study. The discipline advocated by Kydland and Prescott bans "curve fitting" practices. For example, admissible sources of parameter values are:

- Household data on consumption, hours worked, and other microeconomic evidence, for individual preference parameters.

- Long run trend data for the factor shares in production (namely α in the Cobb-Douglass case).

3.2   Measurement: Solow growth accounting

The hypothesis of Kydland and Prescott [1982] and Long and Plosser [1983] is that the source of the observed volatilities and correlations in de-trended variables is a "technology shock". This is an aggregate stochastic shock that affects production, for example through total factor productivity as laid down above. There might be other types of stochastic shocks, however, such as changes in

individuals' preferences, or government-related shocks like wars. Nevertheless, we will abide by the technology shock in what follows:

$$GDP_t \equiv y_t = F_t\left(\cdot\right)$$

Where F is some function of production factors.

In order to measure z_t, we will borrow on the growth accounting technique developed by Solow. In this framework, there are two inputs, capital and labor, and a total productivity shock; hence the previous expression takes the form:

$$y_t = F_t\left(K_t,\, n_t\right) = z_t \cdot F\left(K_t,\, n_t\right)$$

The issue that we have to address is what z is. Or, more precisely, what is the counterpart in the data to the theoretical variable $\dfrac{z_{t+1}}{z_t} - 1$ (the "Solow residual"). To this effect, we will assume that time is continuous, and differentiate the production function:

$$dy_t = F\left(K_t,\, n_t\right) \cdot dz_t + z_t \cdot F_K\left(K_t,\, n_t\right) \cdot dK_t + z_t \cdot F_n\left(K_t,\, n_t\right) \cdot dn_t$$

We multiply and divide through by each component on the right hand side, so as to have percentage changes:

$$dy_t = \underbrace{z_t \cdot F\left(K_t,\, n_t\right)}_{y_t} \cdot \frac{dz_t}{z_t} + z_t \cdot F_K\left(K_t,\, n_t\right) \cdot K_t \cdot \frac{dK_t}{K_t} + z_t \cdot F_n\left(K_t,\, n_t\right) \cdot n_t \cdot \frac{dn_t}{n_t}$$

And next we divide both side by total output y_t:

$$\frac{dy_t}{y_t} = \frac{dz_t}{z_t} + \frac{z_t \cdot F_K\left(K_t,\, n_t\right) \cdot K_t}{y_t} \cdot \frac{dK_t}{K_t} + \frac{z_t \cdot F_n\left(K_t,\, n_t\right) \cdot n_t}{y_t} \cdot \frac{dn_t}{n_t} \qquad \text{(I)}$$

With this at hand, the next task is to find the data counterparts of the fractions $\dfrac{z_t \cdot F_K\left(K_t,\, n_t\right) \cdot K_t}{y_t}$ and $\dfrac{z_t \cdot F_n\left(K_t,\, n_t\right) \cdot n_t}{y_t}$ involved in (I). To do this, we need to introduce two additional assumptions:

[A1] The production technology exhibits constant returns to scale.

[A2] Markets operate under perfect competition.

Assumption [A1] allows an application of the Euler Theorem:

$$F_K\left(K_t,\, n_t\right) \cdot K_t + F_n\left(K_t,\, n_t\right) \cdot n_t = F\left(K_t,\, n_t\right)$$

Hence each of the fractions $\dfrac{z_t \cdot F_K\left(K_t,\, n_t\right) \cdot K_t}{y_t}$ and $\dfrac{z_t \cdot F_n\left(K_t,\, n_t\right) \cdot n_t}{y_t}$ are just shares of output.

9

Assumption [A2] provides the data counterpart for the derivatives. F_K and F_n. Perfect competition implies that

$$
\begin{aligned}
R_t &= z_t \cdot F_K\left(K_t,\, n_t\right) \\
w_t &= z_t \cdot F_n\left(K_t,\, n_t\right)
\end{aligned}
$$

And these factor remunerations can be measured from data. Replacing in expression (I),

$$
\frac{dy_t}{y_t} = \frac{dz_t}{z_t} + \underbrace{\frac{R_t \cdot K_t}{y_t}}_{\text{capital's share in income}} \cdot \frac{dK_t}{K_t} + \underbrace{\frac{w_t \cdot n_t}{y_t}}_{\text{labor's share in income}} \cdot \frac{dn_t}{n_t} \tag{II}
$$

Or, if we denote by α_t the share of capital in income at date t, we may rewrite (II) as:

$$
\frac{dy_t}{y_t} = \frac{dz_t}{z_t} + \frac{R_t \cdot K_t}{y_t} \cdot \frac{dK_t}{K_t} + \frac{w_t \cdot n_t}{y_t} \cdot \frac{dn_t}{n_t} \tag{II}
$$

Even though we have pinned down our empirical unknowns, measuring these is still an endeavored tasks. Some payments are not easily classified as labor or capital income; the treatment of government and foreign trade is unclear from this expression. For further discussion on this, see Cooley's [] book, Chapter 1. But, notwithstanding the methodological difficulties, everything in expression (II) can be directly "found" in the data, except for the Solow residual $\dfrac{dz_t}{z_t}$, which must be solved for.

Denoting by α_t the share of capital in income at date t, the Solow residual can be solved for from:

$$
\frac{dz_t}{z_t} = \frac{dy_t}{y_t} - \alpha_t \cdot \frac{dK_t}{K_t} - (1 - \alpha_t) \cdot \frac{dn_t}{n_t}
$$

In discrete time, we may approximate

$$
\alpha \sim \frac{\alpha_t + \alpha_{t+1}}{2}
$$

In the case of the US, $\alpha \sim .3$.

Finally, given the sequence $\{z_t\}_{t=1950}^{1990}$, we could fit a process to this data, such as an AR(1):

$$
z_{t+1} = \rho \cdot z_t + \varepsilon_{t+1}
$$

The data shows that $\widetilde{\rho} \sim .95$ is a good fit.

Some critiques to this approach:

1. z may be not technology, but just poor measurement (Jorgenson - Griliches' argument).

2. z exhibits a high variation - then what are these shocks? It should be possible to identify them. Furthermore, what is the meaning of a "negative" technological shock? Can technology somehow worsen from one period to the other?

3. The story of stochastic productivity shocks may be acceptable on an industry, or firm level. But the notion of "aggregate" technological shocks seems more dubious. An aggregation argument of individual, independent shocks can not work either, since by the law of large numbers this should lead to no variation at all. Some kind of aggregate component is needed (correlation of individual shocks is tantamount to an aggregate effect).

Comments on measurement:

Could the variation in z be just the product of measurement errors? It is clearly true that the variables on which the facts are observed are subject to these types of errors. In particular, the following are some of the sources of inaccuracy in measurement for some relevant variables:

∗ Total output (y):

- Quality improvements (especially in services and in government).
- Output that is not measured:
    Learning
    Human capital accumulation
    Research and development

∗ Physical capital (K):

- Scrapping is not observed.
    In the national accounts, K is measured indirectly. Data on investment is available; hence this is used to "update" the registered level of capital using the accumulation equation:

$$K' = (1 - \delta) \cdot K + i$$

- Utilization rates are not known.

∗ Labor input into production (n):

- There is little information on the phenomenon known as "labor hoarding": personnel that is kept in their posts doing unproductive tasks.

3.3 Real business cycles models: Brief cookbook procedure

The purpose of this section is to lay down the basic steps of the real business cycles research methodology. We will illustrate through an example one of the most crucial steps: the calibration.

Steps to follow in real business cycles research:

1. Specify of a model, including functional forms and parameters.

2. Pick of parameters through calibration.

3. Solve the model numerically.

   Most often, this will need to be done using linearization methods. Recall that in order to do this, given an AR(1) process for the stochastic shock:

   $$z' = \rho \cdot z + \varepsilon$$

   the policy rule guesses were linear in the state variables (K, z):

   $$
   \begin{aligned}
   K' &= a_K + b_K \cdot K + c_K \cdot z \\
   n &= a_n + b_n \cdot K + c_n \cdot z
   \end{aligned}
   $$

   And the task is to solve for the parameters a_K, a_n, b_K, b_n, c_K, c_n.

4. Simulate the model and analyze the outcome.

   A random number generator is used to simulate a realization of the stochastic shock. This gives place to a time series in each of the variables. These sequences are the researcher's "data set". Sample moments of the variables (in general, second moments) are computed and compared to actual data.

In what follows, we will illustrate the calibration of a real business cycles model through an example. We will assume that the stochastic shock to total factor productivity follows an AR(1) process; the statistics $\widetilde{\rho}$ and $\widetilde{\sigma}^2$ need to be computed from the de-trended (HP-filtered) data.

We will assume that preferences are represented by the utility function:

$$u(c, l) = \frac{\left(c^{1-\theta} \cdot l^{\theta}\right)^{1-\sigma} - 1}{1 - \sigma}$$

The economy is populated by a number of identical households. Each household derives "utility" from the consumption of goods and leisure of its representative member. The size of household's population grows at rate η. The centralized formulation of the utility maximization problem is:

$$\max_{\{c_t, l_t\}_{t=0}^\infty} \left\{ \sum_{t=0}^\infty \beta^t \cdot (1-\eta)^t \cdot u\left(c_t, l_t\right) \right\} \qquad (C1)$$

The central planner faces an aggregate resource constraint:

$$C_t + X_t = A \cdot (1+\gamma)^{t\cdot(1-\alpha)} \cdot K_t^\alpha \cdot N_t^{1-\alpha}$$

Where C_t, X_t, K_t, N_t denote aggregate variables. Production technology is subject to a labor-augmenting (deterministic) change process, with growth rate γ.

Let P_t denote the population size at t (that grows at rate η), and divide the resource constraint by this population size:

$$\frac{C_t}{P_t} + \frac{X_t}{P_t} = A \cdot (1+\gamma)^{t\cdot(1-\alpha)} \cdot \left(\frac{K_t}{P_t}\right)^\alpha \cdot \left(\frac{N_t}{P_t}\right)^{1-\alpha}$$
$$c_t + x_t = A \cdot (1+\gamma)^{t\cdot(1-\alpha)} \cdot k_t^\alpha \cdot n_t^{1-\alpha} \qquad (C2)$$

In addition, individuals' time endowment is limited, so:

$$l_t + n_t = 1 \qquad (C3)$$

The accumulation equation for capital is the usual:

$$K_{t+1} = (1-\delta) \cdot K_t + X_t$$

Dividing through by population at t, to obtain per capita terms:

$$(1+\eta) \cdot \frac{K_{t+1}}{P_t \cdot (1+\eta)} = (1-\delta) \cdot \frac{K_t}{P_t} + \frac{X_t}{P_t}$$
$$(1+\eta) \cdot k_{t+1} = (1-\delta) \cdot k_t + x_t \qquad (C4)$$

Equations (C1), (C2), (C3) and (C4) constitute our problem. In order to solve it, we first transform the growth model into a stationary one. Using our previous knowledge that in this framework all variables will grow at rate γ, we define the de-trended variables:

$$\widetilde{c}_t = \frac{c_t}{(1+\gamma)^t}$$
$$\widetilde{x}_t = \frac{x_t}{(1+\gamma)^t}$$
$$\widetilde{k}_t = \frac{k_t}{(1+\gamma)^t}$$

And we specify $\sigma = 1$ in the utility function; which leads to logarithmic utility. Notice that

$$\log c = \log \widetilde{c} + \log (1 + \gamma)^{-t}$$

but the term $\log (1 + \gamma)^{-t}$ is a constant, and hence is irrelevant for choosing a utility maximizing consumption-leisure sequence. We ignore this term, and hence the transformed problem is:

$$\max_{\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t \cdot (1 + \eta)^t \cdot \left[\log \widetilde{c}_t + \frac{\theta}{1 - \theta} \cdot \log l_t \right] \right\}$$
$$s.t. \ \widetilde{c}_t + (1 + \gamma) \cdot (1 + \eta) \cdot \widetilde{k}_{t+1} = A \cdot \widetilde{k}_t^{\alpha} \cdot (1 - l_t)^{1-\alpha} + (1 - \delta) \cdot \widetilde{k}_t$$

We need to pick values for the parameters involved. We begin with the ones that are immediately available:

α: .4 (capital share of output - constant)

γ: .02 (average long run growth rate)

η: .01 (average long run population growth rate)

A: Scaling factor. It is irrelevant.

δ: In the steady state of the transformed model (that is, in the balanced growth path), we have that

$$\widetilde{k}_{t+1} = \widetilde{k}_t = \widetilde{k}^*$$

Recall the capital accumulation equation:

$$(1 + \gamma) \cdot (1 + \eta) \cdot \widetilde{k}^* = (1 - \delta) \cdot \widetilde{k}^* + \widetilde{x}^*$$

Dividing both sides by $\widetilde{k}^*$,

$$(1 + \gamma) \cdot (1 + \eta) = 1 - \delta + \frac{\widetilde{x}^*}{\widetilde{k}^*}$$
$$= 1 - \delta + \frac{X^*}{K^*}$$

In this equation, γ, η and the ratio of investment to capital stock, $\frac{X^*}{K^*}$, are known. From the data,

$$\frac{X^{US}}{K^{US}} = .076$$

Hence δ can be solved for, and the result is $\delta = .0458$. (*this is a suspicious result*)

14

Next we look at the parameters in the utility function: β and θ. For these we need to take first order conditions of the problem. From the derivative with respect to next period's capital k_{t+1}:

$$\beta \cdot (1 + \eta) = \frac{1}{(1 + \gamma) \cdot (1 + \eta)} \cdot \left[\alpha \cdot A \cdot \left(\frac{\widetilde{k}_t}{1 - l_t} \right)^{\alpha - 1} + 1 - \delta \right]$$

We can observe that

$$\alpha \cdot A \cdot \left(\frac{\widetilde{k}_t}{1 - l_t} \right)^{\alpha - 1} = \alpha \cdot \frac{\widetilde{y}_t}{\widetilde{k}_t} = \alpha \cdot \frac{Y_t}{K_t}$$

and $\dfrac{Y_t}{K_t}$ is available from actual data:

$$\frac{Y_t^{US}}{K_t^{US}} \sim 3.32^{-1} \quad \text{(annual output data)}$$

And with this, β can be solved for. The result is $\beta = 1.032849$ (*This is a very suspicious result. Using $\delta = .1$, the result is a more reasonable -though still high - $\beta = .980759$.)*

The parameter θ is more controversial. The remaining first order conditions are:

$$\widetilde{c}_t \quad : \quad \frac{1}{\widetilde{c}_t} = \lambda_t$$

$$l_t \quad : \quad \frac{\theta}{1 - \theta} \cdot \frac{1}{l_t} = \lambda_t \cdot (1 - \theta) \cdot A \cdot \widetilde{k}_t^{\alpha} \cdot (1 - l_t)^{-\alpha}$$

Then the Euler equation determining labor supply is:

$$\frac{\theta}{1 - \theta} \cdot \frac{1 - l_t}{l_t} = (1 - \theta) \cdot \frac{\widetilde{y}_t}{\widetilde{c}_t}$$
$$= (1 - \theta) \cdot \frac{Y_t}{C_t}$$

We have that

$$\frac{Y_t}{C_t} = \frac{Y_t}{K_t} \cdot \frac{K_t}{C_t}$$

and

$$X_t + C_t = Y_t$$
$$\Rightarrow \frac{X_t}{K_t} + \frac{C_t}{K_t} = \frac{Y_t}{K_t}$$
$$\Rightarrow \frac{C_t}{K_t} = \frac{Y_t}{K_t} - \frac{X_t}{K_t}$$

We know the values of $\dfrac{Y_t}{K_t}$ and $\dfrac{X_t}{K_t}$ in actual data.

15

The next issue is what is a reasonable estimation of l_t. In this case, we must borrow from microeconomic studies. We can see that out of the total 24 hours daily endowment, 8 hours are used for sleeping, 8 for work, and the remaining 8 for leisure. Then we can use $l_t \sim \dfrac{2}{3}$.

Using this, we can solve for θ. The two roots of the second degree polynomial in θ are $\theta_1 = .547562$ and $\theta_2 = 1.826278$.

Besides the indeterminacy between the two possible values of θ, the methodology used is controversial from a microeconomic point of view, due to the response to wage changes that it implies. In the decentralized problem, the first order condition for leisure is:

$$\frac{\theta}{1-\theta} \cdot \frac{1}{1-n_t} = \frac{1}{\widetilde{c}_t} \cdot \widetilde{w}_t$$

Then (de-trending the variables)

$$(1 - n_t) \cdot w_t = \frac{\theta}{1-\theta} \cdot c_t$$

We want to examine the change in labor supply n_t arising from a change in the wage w_t. There are several ways to measure this elasticity, but let us use:

$$\left. \frac{d \log n_t}{d \log w_t} \right|_{c_t \text{ constant}} \equiv \xi$$

This is called the "λ-constant" labor supply elasticity. From the first order conditions, we have

$$
\begin{aligned}
n_t &= 1 - \frac{\theta}{1-\theta} \cdot \frac{c_t}{w_t} \\
\log n_t &= \log \left(1 - \frac{\theta}{1-\theta} \cdot \frac{c_t}{w_t} \right) \\
\xi &= \frac{\dfrac{\theta}{1-\theta} \cdot \dfrac{c_t}{w_t}}{1 - \dfrac{\theta}{1-\theta} \cdot \dfrac{c_t}{w_t}} \\
&= \frac{1 - n_t}{n_t}
\end{aligned}
$$

So inserting the value of $n_t = 1/3$ used above,

$$\xi = 2$$

But this figure is wildly contradicted by data. Labor economists have found that this elasticity is approximately 0 - <u>hours</u> worked are very little sensitive to

16

wage changes. This is a serious drawback for the calibration used: parameter values that are not consistent with microeconomic data.

Summary

In conclusion, parameter values have been determined as follows:

σ - microeconomic data

η - long run demographic data

γ - long run growth data

α - long run macroeconomic data

A - irrelevant parameter

δ - using average investment-to-capital data X/K

β - using Euler equation for capital accumulation and average capital-to-output ratio K/Y (alternatively, we could have used an interest rate - around 6%)

l - $2/3$ from "micro" data

θ - using Euler equation for leisure

3.4 Real business cycles in action: A model example

The purpose of this section is to briefly give an example of an actual model. This is a model developed by Hansen and Rogerson [] to introduce equilibrium unemployment, the lack of which is a shortfall in the usual real business cycles models.

Hansen and Rogerson [] model an economy populated by many (a measure 1 of) agents, and in which there is some sort of fixed cost to working. As a consequence, the employment decision is a discrete variable: n_t can only take the values $\overline{n}$ or 0. Hence, leisure can only be either $1 - \overline{n}(\equiv \overline{l})$ or 1.

The main problem with this assumption is that a competitive equilibrium may not exist. In order to overcome this difficulty, the authors introduce an "employment lottery", whereby individuals become employed in a "full time" job with probability $1 - \mu$, and unemployed otherwise. This plays the role of

"convexifying" the decision making environment, and lead to the applicability of the usual existence and welfare theorems.

The central planner maximizes the following function:

$$E[(1 - \mu) \cdot (\log c_e + \frac{\theta}{1 - \theta} \cdot \log \bar{l}) + \mu \cdot (\log c_u + \frac{\theta}{1 - \theta} \cdot \underbrace{\log 1}_{0})]$$

Where the expectation is taken across agents - hence it is in fact a weighted sum of utilities. The outcome from solving the maximization problem will be a point in the Pareto frontier of the economy (recall the Neguishi characterization). The term between brackets is an individual agent's expected utility for a given period. c_e is the consumption level that the planner assigns to working individuals, and c_u to the unemployed.

The resource constraint that the planner faces is:

$$(1 - \mu) \cdot c_e + \mu \cdot c_u + \bar{i} = z \cdot K^\alpha \cdot [(1 - \mu) \cdot \bar{n}]^{1-\alpha}$$

Where the law of large numbers was used to assert that μ is the fraction of the population that will be unemployed in a given period. The choice variables for the planner are the consumption levels c_e and c_u, the probability of unemployment μ, and the aggregate investment $\bar{i}$.

We take $\bar{i}$ as given and solve the resulting "sub-optimization" problem to find c_e, c_u and μ. The first order conditions are:

$$c_e \quad : \quad \frac{1 - \mu}{c_e} = \lambda \cdot (1 - \mu)$$

$$c_u \quad : \quad \frac{\mu}{c_u} = \lambda \cdot \mu$$

Which leads to

$$c_e = c_u$$

This is a complete markets result: complete risk sharing. The conclusion is that in this model, the employed individuals are the unlucky ones. We can use the result $c_e = c_u = c$ to reformulate the problem for μ:

$$\max_{\mu} \left\{ \log c + (1 - \mu) \cdot \frac{\theta}{1 - \theta} \cdot \log \bar{l} \right\}$$
$$s.t. \ c + \bar{i} = z \cdot K^\alpha \cdot [(1 - \mu) \cdot \bar{n}]^{1-\alpha}$$

Then $(1 - \mu)$ can be viewed as total hours worked ($\bar{n}$ is just a normalization). So the result is that we have modified preferences, that evolved from being logarithmic in leisure to being actually linear in leisure (notice that since $\bar{l} < 1$, an increase in μ leads to an increase in utility). As a result of this linearity,

labor supply will be very elastic IN THE AGGREGATE, in spite of being TOTALLY INELASTIC at the individual level.

Some intuition on the model mechanics

Assume 100% depreciation (so that the model can be solved analytically). The decision rule will be

$$c_t = (1 - \alpha \cdot \beta) \cdot y_t$$

Then what happens when there is an exogenous shock to productivity? The labor demand equation is:

$$w = (1 - \alpha) \cdot z \cdot K^\alpha \cdot \left(n^d\right)^{-\alpha}$$

While the labor supply comes from the consumers' first order condition:

$$w = \frac{\theta}{1 - \theta} \cdot \frac{c}{1 - n^s}$$

Substitution effect will dominate income effect if c is held constant. But recall that c depends on n - we have $c(n)$. This is problematic, and should be a remainder to you of the perils of performing static analyzes of dynamic problems.

So suppose that there is a shock to productivity. Labor supply will increase because $c(n)$ is a function of the shock. Since both consumption and leisure are normal goods, a positive shock will induce an increase in the intake of both of them.

How to find the new equilibrium? We can try a "smart" guess and verify. The Cobb-Douglass case with full depreciation is very friendly for this type of riddles. Guess that n is unchanged - then output increases by exactly the percentage increase in z. The same happens to the wage w. But this implies that the guess is verified: n need not change for a new equilibrium to be reached.

A similar analysis is valid for the capital market. The supply of capital is fixed; we know that hours worked need not change; hence the interest rate needs to increase by the same percentage as z for equilibrium to be restored.

We can also go a step further. Suppose that z is constant, but that the news of next period's realization, z', is known in advance. Imagine that an increase in next period's z level is realized (and becomes known in the current period). In this case, it is no longer true that the supply of capital is fixed. However, it is true that the supply <u>function</u> is not affected. The reason is that this function is the result of the optimal savings decision derived from the first order conditions.

19

Finally, assume that the stochastic process follows an AR(1) process:

$$z' = \rho \cdot z + \varepsilon$$

Then if z increases in the current period, it will be expected to be higher in the next period. Hence there is a combination of the two effects on the market for capital analyzed above. The analysis should also consist of a combination of the two previous analyses.

Final remark: The equation $w = \dfrac{\theta}{1-\theta} \cdot \dfrac{c(n)}{1-n^s}$ on which part of the previous analysis rests, is wildly contradicted by the data on microeconomic behavior.

4   Sunspot models of the business cycle

The main advocates of these models are Farmer and Azariadis. The main characteristics of their approach are:

- There are no <u>fundamental</u> (or <u>intrinsic</u>) shocks.

- The competitive equilibrium anyway. Aggregate variables (c, y) will fluctuate randomly.

Q: Could this happen in an environment where the equilibrium is Pareto optimal?

A: No (assuming strict risk aversion). If consumption fluctuates but could be constant, this can not be an optimal result.

These are models that show either or both of:

- Distortions
- Externalities

In real-business-cycles-like macroeconomic models, it is possible to prove that sunspot equilibria exist whenever the equilibrium is indeterminate. Recall the 2nd order differential equations arising from the linearization of the neoclassical growth model. If the two roots had $|\lambda_1| < 1$, $|\lambda_2| > 1$, then we were able to rule out an exploding path. However, if both λ_1 and λ_2 resulted smaller than 1 in absolute value, we run into "indeterminacy" - several convergent paths were equally admissible as a solution.

Then sunspot equilibria are the result of a randomization among the convergent paths. The procedure in this research area is to build real-business-cycles-like models and play with the parameters until a situation with $|\lambda_1| < 1$ and $|\lambda_2| < 1$ is reached. This is not an easy task, and demands a high level of distortions and externalities from the model.

5   References

Cooley, <u>Frontiers of business cycles research</u>,   , Chapter 1

Hansen and Rogerson,

Kydland and Prescott, _, *Econometrica*, 1982.

Long and Plosser, _, *Journal of Political Economy*, 1983.

Prescott, Edward C., <u>Theory ahead of business cycles measurement</u>, *Minneapolis Federal Reserve Quarterly Report*, 1986