Econ 510a (second half)
Yale University
Fall 2004
Prof. Tony Smith

FINAL EXAMINATION

This is a closed-book and closed-notes exam. You have three (3) hours to complete the exam. There are four (4) questions on the exam for a total of 100 points. The points allocated to each question are indicated below. Please put the answer to each question in a different blue book. *To receive full credit, you must provide convincing explanations to support your answers. Please write as neatly as possible.*

1. (25 points) Consider an exchange economy with two (types of) consumers (labelled A and B), three time periods (labelled 0, 1, and 2), and a single good in each period. Consumer A is endowed with 4 units of the good in each of the three time periods, while consumer B is endowed with 0 units of the good in period 0, 12 units of the good in period 1, and 0 units of the good in period 2. The consumer's endowments are not storable. The two consumers value consumption streams according to: $\sum_{t=0}^{2} \beta^t \, u(c_t^i)$, where c_t^i is the consumption of consumer i in period t and $\beta \in (0,1)$. The felicity function u is strictly increasing and strictly concave.

(a) Carefully define a competitive equilibrium with date-0 trading (i.e., an Arrow-Debreu equilibrium) for this economy.

(b) Let $u(c) = \log(c)$ and $\beta = 1/2$. Solve for the competitive equilibrium allocations and prices.

(c) Carefully define a competitive equilibrium with sequential trading for this economy, assuming that each consumer is endowed with zero assets in period 0. Under the assumptions in part (b) determine the equilibrium rates of return on one-period loans issued in periods 0 and 1, respectively.

2. (30 points) Consider a neoclassical growth model in which the production of output creates pollution as a byproduct. The aggregate amount of pollution $\bar{s}_t$ created in period t is proportional to aggregate output in period t:

$$\bar{s}_t = \theta f(\bar{k}_t, \bar{n}_t),$$

where $\bar{k}_t$ is the aggregate stock of capital, $\bar{n}_t$ is aggregate labor supply, and the aggregate production f exhibits constant returns to scale. Capital depreciates at rate δ. There is a continuum of identical consumers whose preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, \bar{s}_t),$$

where c_t is a typical consumer's level of consumption in period t. The felicity function u is strictly increasing and strictly concave in its first argument and strictly decreasing in its second argument. Consumers do not value leisure, so without loss of generality set $\bar{n}_t = 1$ for all t.

(a) Carefully define a recursive competitive equilibrium for this economy (assuming that consumers own the factors of production).

(b) Does the steady-state aggregate capital stock depend on θ, and if so, how?

(c) Explain how to calculate the speed of convergence of the aggregate capital stock to its steady-state value (in a neighborhood of the steady state). You do not have to compute the speed of adjustment explicitly, but you should describe the procedure in sufficient detail so that someone who can do algebra—but who knows nothing about economics—could implement it.

(d) State the social planning problem for this economy. Do the social planning and competitive equilibrium allocations coincide in this economy? Explain as fully as you can.

3. (25 points) Consider the following model of growth: there is one consumption good per period, one (type of) consumer who lives forever, and there are two sectors, a consumption-goods sector and an investment-goods sector. Production in the consumption-goods sector satisfies:

$$c = Ak_c^{\alpha} n^{1-\alpha},$$

where k_c is the amount of capital used in this sector, n is the amount of labor used in this sector, and $0 < \alpha < 1$. Production in the investment-goods sector satisfies:

$$i = Bk_i,$$

where $B > 0$ and k_i is the amount of capital used in this sector. The total stock of capital accumulates according to:

$$k' = (1 - \delta)k + i,$$

and the stock of capital in any period can be freely allocated across the two sectors subject to:

$$k = k_c + k_i.$$

The consumer has one unit of labor in every period but does not value leisure; his preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t \log(c_t).$$

(a) State (in recursive form) the social planning problem for this economy. Identify clearly the state variables and the control (choice) variables.

(b) Show that the solution to the planning problem features exact balanced growth: that is, the economy is always on its balanced growth path. Express the growth rate in terms of primitives.

4. (20 points) Consider an economy with overlapping generations of consumers who live for two periods. Young consumers are born with zero assets and are endowed with one unit of labor. Old consumers are endowed with λ units of labor, where $0 \leq \lambda \leq 1$. Output y is produced by a competitive firm according to: $y = k^{\alpha} n^{1-\alpha}$, where k is the aggregate capital stock and n is the aggregate supply of labor. Capital depreciates fully in one period. Consumers do not value leisure; the preferences of a young consumer born in period t are given by:

$$\log(c_{1t}) + \beta \log(c_{2,t+1}),$$

where c_{1t} is his consumption when young and $c_{2,t+1}$ is his consumption when old.

(a) Find the equilibrium law of motion of the aggregate capital stock.

(b) Show that the steady-state of this economy is dynamically efficient (for all α and β) provided that λ is greater than some "cutoff" value $\bar{\lambda} \in (0, 1)$. (You do not need to find an explicit expression for $\bar{\lambda}$.) Give an intuitive explanation for this result.