Econ 510a (second half)
Yale University
Fall 2005
Prof. Tony Smith

FINAL EXAMINATION

This is a closed-book and closed-notes exam. You have three (3) hours to complete the exam. There are four (4) questions on the exam for a total of 100 points. The points allocated to each part of each question are indicated below. Please put the answer to each question in a different blue book. *To receive full credit, you must provide convincing explanations to support your answers. Please write as neatly as possible.*

1. Consider an exchange economy with two types of consumers, each of which comprises one-half of the economy's population. The economy lasts for two time periods (labelled 0 and 1) and there is one nonstorable consumption good in each period. Each type-1 consumer is endowed with one (1) unit of the consumption good in each time period. Each type-2 consumer is endowed with one (1) unit of the consumption good in the *first* period. In the *second* period, however, each type-2 consumer has a stochastic endowment: with probability one-half he is endowed with two (2) units of the consumption, and with probability one-half he is endowed with zero (0) units of the consumption good. Each consumer has the same preferences over consumption goods: $\log(c_0) + E[\log(c_1)]$, where c_t is consumption in period t.

(a) **(8 points)** Assume that markets are complete. Carefully define a sequential competitive equilibrium for this economy (i.e., one in which consumers trade Arrow securities in period 0).

(b) **(8 points)** Compute the competitive equilibrium allocations and prices (i.e., the prices of the Arrow securities). Interpret your answers.

(c) **(4 points)** Use your answer from part (c) to find the price (in terms of period-0 consumption goods) of a riskfree bond that delivers one unit of the consumption good in all states of the world in period 1.

(d) **(8 points)** Suppose now that markets are not complete: consumers are allowed to trade *only* a riskfree bond in period 0. Define a competitive equilibrium and find an equation that determines the price of the riskfree bond (you do not have to solve explicitly for this price).

2. Consider a (deterministic) neoclassical growth model in which the government taxes capital income at a proportional rate $\tau > 0$ and returns the proceeds as a lump-sum transfer to consumers. Consumers have identical preferences $\sum_{t=0}^{\infty} \beta^t u(c_t)$, where c_t is period-t consumption. Each consumer is endowed with k_0 units of capital in period 0 and with one unit of time in each period. A typical consumer's budget constraint in period t reads:

$$(1-\tau)r_t k_t + w_t + S_t = c_t + k_{t+1} - (1-\delta)k_t,$$

where k_t is period-t holdings of capital, S_t is the lump-sum transfer, and r_t and w_t are the rental price of capital and the wage rate, respectively. Profit-maximizing firms operate a Cobb-Douglas production technology $F(k,n) = k^\alpha n^{1-\alpha}$, where n is labor supply. The government balances its budget in every period.

(a) **(9 points)** Carefully define a sequential competitive equilibrium for this economy.

(b) **(9 points)** Find an algebraic expression for the steady-state capital stock in competitive equilibrium.

(c) **(9 points)** Is the steady-state capital stock in part (b) higher, lower, or the same as the one that would be chosen by a social planner? Explain fully.

3. Consider a (deterministic) neoclassical growth model in which the representative consumer's preferences are given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t, \ell_{t-1}).$$

That is, current-period utility depends on the current amount of consumption c_t, the current amount of leisure ℓ_t, and the *lagged* amount of leisure ℓ_{t-1}. Profit-maximizing firms operate a Cobb-Douglas production technology: $y = k^\alpha n^{1-\alpha}$, where y is output, k is capital, and $n = 1 - \ell$ is labor supply. Capital accumulates according to: $k' = (1-\delta)k + i$, where i is investment.

(a) **(9 points)** Carefully define a recursive competitive equilibrium for this economy, assuming that consumers own the factors of production.

(b) **(9 points)** Display the Bellman equation for the social planning problem in this economy.

(c) **(9 points)** Find a pair of difference equations that determine, together with the transversality condition, the competitive equilibrium behavior of capital and labor in this economy. (You do not need to display the transversality condition.)

4. Consider a version of the Lucas "tree" model in which trees not only yield fruit (or dividends) but also enter the utility function directly. In particular, the representative consumer's preferences are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + A \log(s_t)\right],$$

where c_t is period-t consumption, s_t is the number of trees held in period t, and A is positive. Thus owning more trees leads to higher utility: trees are considered "beautiful" and consumers value beauty. The tree yields a stochastic dividend stream $\{d_t\}_{t=0}^{\infty}$. Assume that d_t is independent and identically distributed (so that its realization today is statistically independent of its past realizations) and assume that $E(d_t^i) = m_i$ for all nonzero integers i. Dividends are the only source of consumption goods in this economy and they are not storable. Each consumer is endowed initially with one tree. Consumers can buy and sell trees in a competitive market.

(a) **(9 points)** Derive the Euler equation of a typical consumer.

(b) **(9 points)** Use your answer from part (a) to find the equilibrium price of a tree as a function of the current dividend. (Hint: Guess that the price is equal to a constant B times the current dividend, and then solve for B in terms of parameters.)