Econ 510a (second half)
Yale University
Fall 2006
Prof. Tony Smith

FINAL EXAMINATION

This is a closed-book and closed-notes exam. You have three (3) hours to complete the exam. There are four (4) questions on the exam for a total of 100 points. The points allocated to each part of each question are indicated below. Please put the answer to each question in a different blue book. *To receive full credit, you must provide convincing explanations to support your answers. Please write as neatly as possible.*

1. A farmer in Italy produces bread and wine for his own consumption. The farmer is endowed with T hours of time in each period. He allocates his time between two activities: baking bread and pressing grapes to make grape juice. The farmer does not value leisure. The only input required to produce bread and grape juice is labor. Moreover, the production technology is linear: each unit of time devoted to baking bread produces one unit of bread and each unit of time devoted to pressing grapes produces one unit of grape juice. The farmer does not consume grape juice, but each unit of grape juice becomes (via the process of fermentation) one unit of wine (which he does consume) in the next period. Both bread and wine are perishable (i.e., non-storable) goods.

The farmer allocates his time between baking bread and pressing grapes in order to maximize the lifetime utility of his consumption of bread and wine; this utility is given by $\sum_{t=0}^{\infty} \beta^t u(b_t, w_t)$, where b_t is the amount of bread consumed in period t, w_t is the amount of wine consumed in period t, and $\beta \in (0, 1)$. The farmer is endowed with $w_0 > 0$ units of wine in period 0.

(a) [**8 points**] Formulate the farmer's optimization problem as a dynamic programming problem (i.e., display the Bellman equation for the farmer's problem). Identify clearly the state and control (or choice) variables.

(b) [**6 points**] Find the farmer's Euler equation.

(c) [**6 points**] Explain how to compute the slope of the farmer's decision rule at the steady state. You do not need to calculate the slope, but you need to describe a procedure for doing so in sufficient detail so that someone with no knowledge of economics could implement it.

2. Consider an exchange economy with two types of infinitely-lived consumers, each of which comprises one-half of the economy's population. The two types of consumers have identical preferences over consumption streams given by:

$$E\left(\sum_{t=0}^{\infty}\beta^t \log(c_t)\right),$$

where $\beta \in (0,1)$. The two types of consumers differ in their endowments of the (non-storable) consumption good. Type-1 consumers have a constant endowment of 1 in every period. Type-2 consumers have a stochastic endowment: in each period, it is 1 with probability π and 0 with probability $1-\pi$, where the probability π does not depend on time or on the previous realization shocks. Markets are complete.

(a) [**9 points**] Carefully define a competitive equilibrium with date-0 trading for this economy.

(b) [**9 points**] Find the prices of the Arrow securities in this economy. Show your work.

(c) [**9 points**] A *two-period risk-free bond* is a sure claim to one unit of the consumption good two periods from now. Express the price of a two-period risk-free bond (relative to the price of the current consumption good) as an explicit function of the Arrow security prices. Your answer should depend on the current state of the economy. (Note: You can answer this question even if you were unable to answer the question in part (b).)

3. Consider an exchange economy with two types of infinitely-lived consumers, each of which represents half of the economy's population. Each consumer has the same endowment stream $\{\omega_t\}_{t=0}^{\infty}$. A consumer of type i, $i = A, B$, has preferences over consumption streams of the form

$$\sum_{t=0}^{\infty}\beta^t \frac{c_t^{1-\sigma_i}-1}{1-\sigma_i},$$

where $\beta \in (0,1)$. Assume that $\sigma_A^{-1} > \sigma_B^{-1} > 0$: the elasticity of intertemporal substitution of type-A consumers is larger than that of type-B consumers. Consumers trade a risk-free bond in each period (i.e., a sure claim to one unit of the consumption good in the next period). There is no restriction on borrowing except for the no-Ponzi-game restriction. Each consumer begins with zero asset holdings.

(a) [**10 points**] Carefully define a sequential competitive equilibrium for this economy.

(b) [**10 points**] Suppose that $\omega_t = \bar{\omega}$ for all t. Show that, in this case, the economy has a steady state. Express the steady-state interest rate in terms of primitives.

(c) [**7 points**] Now suppose that the endowment grows over time: $\omega_{t+1} = (1+g)\,\omega_t$, where $g > 0$. In this case, does there exist a steady state, that is, an equilibrium in which the consumption of a type-A consumer grows at the same rate as the consumption of a type-B consumer? Explain why or why not.

4. Consider a neoclassical growth model in which the government has government expenditures equal to g in every period. The government finances these expenditures by taxing labor income at a proportional rate τ_t in period t. The government balances its budget in every period: for all t, the tax rate τ_t is chosen so that period-t expenditures (i.e., g) are exactly equal to the revenues raised by the labor income tax.

The economy is populated by a continuum (of measure one) of identical consumers. Each consumer has preferences $\sum_{t=0}^{\infty} \beta^t \left(\log(c_t) + A\log(\ell_t)\right)$, where c_t is period-t consumption, ℓ_t is period-t leisure, and $\beta \in (0,1)$.

Consumers do not value government expenditures, so that government expenditures are a pure drain on output in this economy: the aggregate resource constraint is

$$\bar{y}_t = \bar{c}_t + \bar{x}_t + g,$$

where $\bar{y}_t$ is aggregate output in period t, $\bar{c}_t$ is aggregate consumption in period t, and $\bar{x}_t$ is aggregate investment in period t.

Each consumer is endowed with k_0 units of capital in period 0 and with one unit of time in each period. In each period, consumers rent the services of capital and labor to firms in competitive markets. Firms maximize profits and produce output according to: $y = k^{\alpha} n^{1-\alpha}$, where y is output, k is capital, and n is labor supply; the parameter $\alpha \in (0,1)$. Finally, capital accumulates according to $k_{t+1} = (1-\delta)k_t + x_t$, where $\delta \in (0,1]$.

(a) [**10 points**] Carefully define a recursive competitive equilibrium for this economy.

(b) [**10 points**] Show that the steady-state capital-to-labor ratio does not depend on government expenditures g.

(c) [**6 points**] How do changes in g affect aggregate labor supply in the steady state? If you cannot obtain a complete answer, then display an equation that relates steady-state labor supply to g and explain how you could use this equation to answer the question.