Econ 510a (second half)
Yale University
Fall 2007
Prof. Tony Smith

FINAL EXAMINATION

This is a closed-book and closed-notes exam. You have three (3) hours to complete the exam. There are four (4) questions on the exam for a total of 100 points. The points allocated to each part of each question are indicated below. Please put the answer to each question in a different blue book. *To receive full credit, you must provide convincing explanations to support your answers. Please write as neatly as possible.*

1. A consumer seeks to maximize her lifetime utility of consumption and leisure, which is given by: $\sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t)$, where c_t is consumption in period t and ℓ_t is leisure in period t. The felicity function u is strictly increasing and strictly concave in both of its arguments.

   The consumer is endowed with one unit of time in each period, which she allocates between leisure and work. The consumer's labor income in period t equals wh_tn_t, where w is the (time-invariant) wage per unit of human capital, h_t is the the consumer's level of human capital in period t, and n_t is the amount of time that the consumer spends working in period t. Human capital accumulates over time according to:

   $$h_{t+1} = (1 - \delta)h_t + f(h_t, n_t),$$

   where f is strictly increasing in both arguments, strictly concave in its first argument, and satisfies: $f(0, n) = f(h, 0) = f(0, 0) = 0$. In other words, human capital depreciates at rate δ, but the consumer can invest in human capital by working. The consumer has human capital equal to $h_0 > 0$ in period 0. To keep things simple, suppose that the consumer does not participate in asset markets and instead simply consumes her entire labor income in every period.

   (a) [8 points] Formulate the consumer's dynamic optimization problem as a dynamic programming problem. That is, display the consumer's Bellman equation and identify clearly the control (or choice) variable(s) and the state variable(s).

   (b) [9 points] Find an equation that determines (implicitly) the steady-state level of human capital.

(c) [5 points] Suppose instead that f is linear: $f(h_t, n_t) = g(n_t)h_t$, where g is strictly increasing and satisfies $g(0) = 0$. Do you think that the consumer's optimal path for human capital converges to a steady state in this case? Explain.

2. Consider a version of a "Lucas tree" economy in which there are two types of trees. Both types of trees are perfectly durable; a type-i tree ($i = 1, 2$) yields a random amount of dividends equal to d_{it} in period t. Assume that $\{d_{1t}\}_{t=0}^{\infty}$ and $\{d_{2t}\}_{t=0}^{\infty}$ are i.i.d. sequences of random variables and that d_{1t} and d_{2s} are statistically independent for all t and s. In addition, for $i = 1, 2$, assume that d_{it} equals d_L with probability π_i and equals $d_H > d_L$ with probability $1 - \pi_i$.

   The economy is populated by a continuum (of measure one) of identical consumers with preferences over consumption streams given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t),$$

   where c_t is consumption in period t. In period 0, each consumer owns one tree of each type. Dividends are non-storable and are the only source of consumption goods. There are competitive markets in which consumers can buy and sell both types of trees.

   (a) [5 points] Define a sequential competitive equilibrium in which the only assets that consumers trade are the two (types of) trees.

   (b) [9 points] Find an algebraic expression for the equilibrium price of a type-1 tree (measured in terms of today's consumption goods), assuming that the dividends of both types of trees are equal to d_L today. Your expression should depend only on primitives (i.e., on the parameters describing preferences and technology).

   (c) [9 points] How many Arrow securities are there in this economy? Express the prices of these securities in terms of primitives.

   (d) [5 points] Use your answer from part (c) to find the price (expressed in terms of today's consumption goods) of an asset that pays one unit of the consumption good in the next period if the dividends of the two trees (in the next period) are not equal to each other and pays zero otherwise.

3. Consider a Diamond overlapping-generations economy in which there are two types of consumers. In each period, a continuum of measure one of each type of consumer is born. Each type of consumer lives for two periods and supplies one unit of labor inelastically when young and zero units of labor when old. Each type of consumer is born with zero assets. Type-i consumers, $i = A, B$, who are born in period t have preferences given by: $u_i(c_{1t}) + \beta u_i(c_{2,t+1})$, where c_{1t} is consumption when young and $c_{2,t+1}$ is consumption when old. Let $u_A(c) = \log(c)$ and $u_B(c) = (1-\sigma)^{-1} c^{1-\sigma}$, where $\sigma \neq 1$. Let the economy's aggregate production function take the form: $y = k^{\alpha} n^{1-\alpha}$, where y is aggregate output, k is the aggregate capital stock, and n is aggregate labor supply. Assume that capital depreciates fully in one period.

   (a) [5 points] Find the savings decision rule of a young consumer of each type.

   (b) [9 points] Use your answer from part (a) to find an equation that determines (implicitly) the equilibrium law of motion of the aggregate capital stock.

   (c) [8 points] Under what conditions is the steady-state of this economy dynamically inefficient? Explain.

4. Consider a neoclassical growth model populated by identical consumers with preferences over consumption streams given by: $\sum_{t=0}^{\infty} \beta^t u(c_t)$, where c_t is consumption in period t. Consumers do not value leisure and supply one unit of labor inelastically in every period. Firms have identical constant-returns-to-scale production functions. In every period, the government taxes labor income in order to subsidize investment expenditures. That is, the period-t budget constraint of a typical consumer reads:

$$c_t + (1+\tau)x_t = r_t k_t + (1-\theta_t)w_t,$$

   where r_t is the period-t rental rate of capital, w_t is the period-t wage rage, k_t is the consumer's capital stock at the beginning of period t, $x_t \equiv k_{t+1} - (1-\delta)k_t$ is the consumer's period-t investment expenditures, $\tau < 0$ is the (time-invariant) subsidy rate on investment expenditures, and $\theta_t > 0$ is the period-t tax on labor income. The government balances its budget in every period.

   (a) [9 points] Carefully define a recursive competitive equilibrium for this economy.

   (b) [7 points] Find a second-order difference equation that governs the evolution of the aggregate capital stock in equilibrium.

(c) [6 points] Is the steady-state aggregate capital stock higher or lower than it would be if $\tau = 0$? Explain.

(d) [6 points] Is the competitive equilibrium allocation Pareto efficient when $\tau < 0$? Explain.