Econ 511b (Part I)
Yale University
Spring 2004
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MIDTERM EXAMINATION

This is a closed book and closed notes exam. You have two-and-a-half (2.5) hours to complete the exam. There are three questions of equal value on the exam. To receive full credit, you must provide convincing explanations to support your answers. Please write as neatly as possible.

1. Consider a competitive equilibrium neoclassical growth model populated by identical consumers whose preferences over consumption streams are given by $\sum_{t=0}^{\infty} \beta^t u(c_t)$, where u is strictly increasing and strictly concave. Consumers do not value leisure and are endowed with k_0 units of capital in period 0 and with one unit of labor in every period. Consumers rent the services of capital and labor in competitive markets to profit-maximizing firms with identical constant-returns-to-scale production functions. Capital depreciates fully in one period. The government, which balances its budget in every period, taxes capital income at a (time-invariant) proportional rate τ and returns the proceeds to consumers in the form of a lump-sum subsidy to income.

(a) Carefully define a recursive competitive equilibrium for this economy.

(b) Show that the competitive equilibrium allocation for this economy is identical to the allocation chosen by a social planner whose preferences over consumption streams are given by $\sum_{t=0}^{\infty} \tilde{\beta}^t u(c_t)$, where $\tilde{\beta}$ is a "distorted" discount rate that differs from the discount rate of a typical consumer. Express the distorted discount rate in terms of τ and β.

(c) Does the result from part (b) continue to hold if consumers value leisure and the government taxes both labor income and capital income at a proportional rate τ? Explain why or why not.

2. Consider a real-business-cycle model with variable capital utilization. There is a representative consumer whose preferences are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where u is strictly increasing and strictly concave. The aggregate resource constraint reads: $c_t + x_t = f(k_t, n_t, z_t, h_t) \equiv \exp(z_t)(h_t k_t)^\alpha n_t^{1-\alpha}$. The variable $h_t \geq 0$ measures the "utilization level" of machines, and it is a choice variable at all points in time. Capital accumulates according to:

$$k_{t+1} = (1 - \delta(h_t))k_t + x_t,$$

where the function $\delta(h_t)$ is given by:

$$\delta(h_t) = \delta_0 + \delta_1 \frac{h_t^\omega}{\omega}.$$

The parameters δ_0 and δ_1 are positive and the parameter ω is greater than 1. Thus, the depreciation rate of capital in period t is an increasing and convex function of the utilization level h_t.

Individuals do not value leisure and supply labor inelastically. Without loss of generality, set labor resources n_t equal to 1. The productivity variable z_t is stochastic and evolves according to:

$$z_{t+1} = \rho z_t + \epsilon_{t+1},$$

where $\{\epsilon_{t+1}\}_{t=0}^{\infty}$ is an independent and identically distributed sequence of shocks drawn from a $N(0, \sigma_\epsilon^2)$ distribution and $|\rho| < 1$.

(a) Carefully define a recursive competitive equilibrium for this economy. Assume that consumers own the factors of production and rent their services to firms in every period in competitive markets. (Hint: Let the firm's production function be $F(k_t, n_t, z_t, h_t) \equiv f(k_t, n_t, z_t, h_t) + (1 - \delta(h_t))k_t$.)

(b) What is the deterministic steady-state value of the aggregate capital stock in the competitive equilibrium of this economy? (You need to find an equation that determines the steady-state capital stock in terms of primitives, but you do not have to solve it.)

(c) Explain how to use linearization methods to obtain an approximation to the stochastic behavior of the competitive equilibrium. You do *not* have to carry out any explicit computations, but you should provide a careful, detailed description of how to perform the required computations.

3. Consider an exchange economy with two infinitely-lived consumers with identical preferences given by:

$$E\left(\sum_{t=0}^{\infty} \beta^t \log(c_t)\right).$$

Both of the consumers have random endowments that depend on an (exogenous) sequence of state variables $\{s_t\}_{t=0}^{\infty}$. The s_t's are statistically independent random variables with identical probability distributions. Specifically, for each t, $s_t = H$ with probability π and $s_t = L$ with probability $1 - \pi$, where π does not depend on time or on the previous realization of states. If $s_t = H$, then the first consumer's endowment is 2 and the second consumer's endowment is 1; if $s_t = L$, then the first consumer's endowment is 1 and the second consumer's endowment is 0. Markets are complete.

(a) Carefully define a competitive equilibrium with date-0 trading for this economy. (Assume that consumers make decisions before observing the realization of the state in period 0.)

(b) Determine the competitive equilibrium allocation in terms of primitives.

(c) Determine the prices of the Arrow securities in terms of primitives.

(d) Use your answer from part (c) to determine the average rate of return on a (one-period) riskless bond in this economy.