Finn Kydland and Edward Prescott’s Contribution to Dynamic Macroeconomics: The Time Consistency of Economic Policy and the Driving Forces Behind Business Cycles
1 Introduction

Finn Kydland and Edward Prescott have been awarded the 2004 Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel for their fundamental contributions to two closely related areas of macroeconomic research. The first concerns the design of macroeconomic policy. Kydland and Prescott uncovered inherent imperfections—credibility problems—in the ability of governments to implement desirable economic policies. The second area concerns business cycle fluctuations. Kydland and Prescott demonstrated how variations in technological development—the main source of long-run economic growth—can lead to short-run fluctuations. In so doing, they offered a new and operational paradigm for macroeconomic analysis based on microeconomic foundations. Kydland and Prescott’s work has transformed academic research in economics, as well as the practice of macroeconomic analysis and policymaking.

1.1 General background

During the early postwar period, macroeconomic analysis was dominated by the view marshalled by Keynes (1936). In this view, short-run fluctuations in output and employment are mainly due to variations in aggregate demand, i.e., in investors’ willingness to invest and consumers’ willingness to consume. Moreover, macroeconomic stabilization policy can, and should, systematically control aggregate demand so as to avoid recurring fluctuations in output. These ideas largely reflected the experience from the Great Depression, when a deep protracted trough in aggregate output, along with falling employment and capital utilization, were observed throughout the western world. Keynesian macroeconomic analysis interpreted these phenomena as failures of the market system to coordinate demand and supply, which provided an obvious motive for government intervention.

Until the mid-1970s, the dominating Keynesian paradigm seemed quite successful in explaining macroeconomic fluctuations. But real-world developments in the late 1970s revealed serious shortcomings of the earlier analysis. It could not explain the new phenomenon of simultaneous inflation and unemployment. This so-called stagflation seemed closely related to shocks on the supply side of the economy: oil price hikes and the worldwide slowdown of productivity growth. Such supply shocks had played only a subordinate role in the Keynesian framework. Moreover, conventional macroeconomic policy, based on existing theory, was unable to cope with the new problems. Rather, monetary and fiscal policy appeared to make matters worse in many countries by accommodating private-sector expectations of high price and wage increases. This occurred despite the stated objective of governments and central banks to maintain low and stable inflation.

Keynesian models were also criticized on methodological grounds. Models used in applied work built on broad theoretical and empirical generalizations (“reduced forms”) summarizing the relationships governing the main macroeconomic variables, such as output, inflation, unemployment, and consumption. Robert Lucas’s research in the early and mid-1970s (Lucas 1972, 1973, 1976) pointed to the drawbacks of this approach, in particular that the relationships between macroeconomic variables are likely to be influenced by economic policy itself. As a result, policy analysis based on these relationships might turn out to be
erroneous. Lucas concluded that the effects of macroeconomic policy could not be properly analyzed without explicit microeconomic foundations. Only by carefully modeling the behavior of individual economic agents, such as consumers and firms, would it be possible to derive robust conclusions regarding private-sector responses to economic policy. The building blocks of such an analysis—e.g., consumers’ preferences, firms’ technologies, and market structures—are likely to be robust to changes in economic policy.

As the Lucas critique rapidly gained wide acceptance, development of an alternative and operational macroeconomic framework was called for. This was a daunting task, however. Such a new framework had to be based on solid microeconomic foundations. It also had to give an integral role to economic policy and economic agents’ perceptions of how policy is determined. The award-winning contributions by Kydland and Prescott appeared in two joint articles, which took decisive steps forward in these respects.

1.2 The contributions in brief

“Rules Rather than Discretion: The Inconsistency of Optimal Plans”, from 1977, studies the sequential choice of policies, such as tax rates or monetary policy instruments. The key insight is that many policy decisions are subject to a fundamental time consistency problem. Consider a rational and forward-looking government that chooses a time plan for policy in order to maximize the well-being of its citizens. Kydland and Prescott show that if given an opportunity to re-optimize and change its plan at a later date, the government will generally do so. What is striking about this result is that it is not rooted in conflicting objectives between the government and its citizens, nor is it due to the ability of unrestricted policymakers to react to unforeseen shocks. The result, instead, is simply a problematic logical implication of rational dynamic policymaking when private-sector expectations place restrictions on the policy decisions.

A significant upshot is that governments unable to make binding commitments regarding future policies will encounter a credibility problem. Specifically, the public will realize that future government policy will not necessarily coincide with the announced policy, unless the plan already encompasses the incentives for future policy change. In other words, sequential policymaking faces a credibility constraint. In mathematical terms, optimal policy decisions cannot be analyzed solely by means of control theory (i.e., dynamic optimization theory). Instead they should be studied as the outcome of a game, where current and future policymakers are modeled as distinct players. In this game, each player has to anticipate the reaction of future players to current play: rational expectations are required. Kydland and Prescott analyzed general policy games as well as specific games of monetary and fiscal policymaking. They showed that the outcome in a rational-expectations equilibrium where the government cannot commit to policy in advance—discretionary policymaking—results in lower welfare than the outcome in an equilibrium where the government can commit.

Kydland and Prescott’s 1977 article had a far-reaching impact not only on theoretical policy analysis. It also provided a new perspective on actual policy experience, such as the stagflation problem. The analysis showed that a sustained high rate of inflation may not be the consequence of irrational policy decisions; it might simply reflect an inability of policymakers to commit to monetary policy. This insight shifted the focus of policy analysis
from the study of individual policy decisions to the design of institutions that mitigate the
time consistency problem. Indeed, the reforms of central banks undertaken in many countries
as of the early 1990s have their roots in the research initiated by Kydland and Prescott.
Arguably, these reforms are an important factor underlying the recent period of low and
stable inflation. More broadly, the insight that time inconsistency is a general problem for
economic policymaking has shifted the focus not only towards normative research on the
optimal design of institutions such as central banks, but also towards positive research on
the interaction between economic decision-making and political institutions. It has inspired a
large cross-disciplinary literature at the intersection between economics and political science.

“Time to Build and Aggregate Fluctuations”, from 1982, proposed a theory of business
cycle fluctuations far from the Keynesian tradition. In this article, Kydland and Prescott
integrated the analyses of long-run economic growth and short-run macroeconomic fluctuations,
by maintaining that a crucial determinant of long-run living standards, i.e., growth
in technology, can also generate short-run cycles. Moreover, rather than emphasizing the
inability of markets to coordinate supply and demand, Kydland and Prescott’s business cy-
cle model relied on standard microeconomic mechanisms whereby prices, wages, and interest
rates enable markets to clear. They thus argued that periods of temporarily low output
growth need not be a result of market failures, but could simply follow from temporarily
slow improvements in production technologies.

Kydland and Prescott showed that many qualitative features of actual business cycles,
such as the co-movements of central macroeconomic variables and their relative variabili-
ties, can be generated by a model based on supply (technology) shocks. Using fluctuations
in technology growth of the same magnitude as those measured from data, Kydland and
Prescott also demonstrated that their simple model could generate quantitatively significant
cycles. It thus appeared that technology shocks should be taken seriously as a cause of
business cycles.

From a methodological point of view, Kydland and Prescott’s article answered Lucas’s
call for an alternative to the Keynesian paradigm. It was the first study to characterize the
general equilibrium of a full-fledged dynamic and stochastic macroeconomic model based on
microeconomic foundations. This required solving a set of interrelated dynamic optimization
problems, where consumers and firms make decisions based on current and expected future
paths for prices and policy variables, and where the equilibrium price sequences are such
that private-sector decisions are consistent with clearing markets at all points in time and
all states of the world. Kydland and Prescott showed that this challenging analysis could
be carried out in practice by extensive use of numerical methods. Their empirical approach
relied on model simulation, based on so-called “calibration”, and on comparing the synthetic
data from simulations with actual data. Such calibration can be regarded as a simple form of
estimation, where model parameters are assigned values so as to match the model’s long-run
macroeconomic features with those in the data and render the behavior of individual agents
in the model consistent with empirical microeconomic studies.

Kydland and Prescott’s work on business cycles initiated an extensive research program.
Successively more sophisticated dynamic models of business cycles have been formulated,
solved numerically, and compared to data using both calibration methods and econometric
estimation. Kydland and Prescott’s emphasis on supply shocks led researchers to reconsider the origins of business cycles and assess the relative importance of different shocks. Their results were established for well-functioning markets, while subsequent research considered various market imperfections and examined their implications. As a result of these efforts, current state-of-the-art business-cycle models give prominence to both supply and demand shocks. These models rely on explicit microeconomic foundations to a much larger extent than did earlier Keynesian models. For example, so-called “new-Keynesian” models, which have become a standard tool for analyzing monetary policy, have a core similar to Kydland and Prescott’s original model, but incorporate frictions in the form of “sticky” prices and wages.

Kydland and Prescott’s two articles have central themes in common. Both articles view the macroeconomy as a dynamic system, where agents—private agents and policymakers—make rational, forward-looking, and interrelated decisions. Both articles provide insights into postwar developments in the world economy, in terms of private-sector or government behavior. Both articles offer a new perspective on good macroeconomic policy, leading to a reconsideration of policymaking institutions and a different approach to stabilization policy. Separately, each of the articles spawned a large independent literature. In the following, we describe the contributions in more detail.

2 Time consistency of economic policy

In the late 1960s and early 1970s, macroeconomic research paid particular attention to the expectations held by private agents. A first step was to emphasize expectations as important determinants of economic outcomes. Friedman (1968) and Phelps (1967, 1968) based their natural-rate theories of unemployment on the expectations-augmented Phillips curve, where the relationship between actual inflation and unemployment depends on expected inflation. A second step was to study expectations formation in more depth. Lucas (1972, 1973) based his analysis on the rational-expectations hypothesis, according to which economic agents make the best possible forecasts of future economic events given available information, including knowledge of how the economy functions, and where the best possible forecast presumes that other agents act according to the same principle, now and in the future.

Kydland and Prescott’s analysis of economic policy design added a new dimension to expectations formation. Their model endogenized government decision-making by assuming that governments choose policy in order to maximize the welfare of their citizens. Kydland and Prescott followed Lucas in assuming that the private sector’s expectations about future government policy are rational; they also followed Friedman and Phelps in assuming that those expectations are important determinants of economic outcomes. Under these assumptions, Kydland and Prescott showed—by way of a general argument as well as specific examples—that government policymaking is subject to a time consistency problem.

Kydland and Prescott’s 1977 paper contained several related contributions, both methodological and substantive. First, they pointed to the general origin of the time consistency problem: without a commitment mechanism for future policy, the government faces an additional constraint in policymaking because its policy has to be credible. In other words,
if private expectations about future policy choices are rational, a certain set of economic outcomes are simply not attainable under discretionary policy. Second, they derived the policy outcomes that would result in this case if both private agents and policymakers act rationally. They characterized time-consistent equilibria without commitment and demonstrated that such equilibria involve lower welfare than those with government commitment. Third, they argued that more or less unalterable policy rules may be called for. This initiated a discussion about institutional design aimed at creating commitment mechanisms that enlarge the set of feasible economic outcomes and improve economic well-being. In all these ways, Kydland and Prescott’s contribution has fundamentally changed the way we think about economic policy.

2.1 The general idea

The following simple and abstract model with two time periods, \( t-1 \) and \( t \), suffices for describing the problem. In period \( t-1 \), a government wants to attain the best possible outcome for economic agents in period \( t \). Economic outcomes in period \( t \) depend not only on the policy undertaken in period \( t \), but also on private-sector decisions made in period \( t-1 \) (determining, for example, savings or wages in period \( t \)). Private-sector decisions in period \( t-1 \) depend on expectations about the period-\( t \) policy. These expectations are rationally formed. In period \( t-1 \), private-sector agents understand the determinants of government policy in period \( t \) and base their forecasts on this knowledge. There is no uncertainty in the model, so that rational expectations imply perfect foresight on the part of private-sector agents.

In the case with commitment, the government chooses its period-\( t \) policy in period \( t-1 \), without the ability to change this policy later on. Clearly then, the optimal choice of period-\( t \) policy has to take into account its effects on private-sector decisions in period \( t-1 \). Because the equilibrium period-\( t \) policy pins down the expectations in period \( t-1 \) as to what this policy will be, it influences private-sector decisions made in period \( t-1 \) that affect period-\( t \) economic outcomes.

In the more realistic case without commitment, i.e., with discretionary policy, in period \( t-1 \) the government cannot make binding decisions over policy in period \( t \) until period \( t \) arrives. In this case, by contrast, the period-\( t \) policy choice will not take into account how private-sector decisions are made in period \( t-1 \), because when this policy choice is undertaken, private-sector decisions in period \( t-1 \) have already been made and can no longer be influenced. This will generally lead to different period-\( t \) policy choices than in the commitment case. As a result, economic outcomes will lead to lower welfare than under commitment. This follows from the fact that with rational expectations, policy in period \( t \) is perfectly anticipated, but owing to the time sequencing of decision-making, there is no way for the government to influence these expectations. When deciding its policy in period \( t \), the government thus solves an optimization problem that does not consider all the effects of its policy choice.

To see how time inconsistency arises in this example, suppose a government that cannot make commitments in period \( t-1 \) announces an intention to adopt the same period-\( t \) policy as the optimal policy it would (hypothetically) have selected under commitment. This an-
ouncement would not be credible, because when period \( t \) arrives and the government makes its actual policy choice, it would discover that it is optimal to renege on its announcement.

2.2 Methods

Given the problem at hand, the government’s policymaking problem cannot be analyzed as an optimal-control problem, as in conventional macroeconomics. In an optimal control problem, the optimizer chooses a control-variable sequence that maximizes an objective function subject to constraints. But in Kydland and Prescott’s setup the dependence of private behavior on (rational) private expectations about policy makes the constraints endogenous. These constraints describe relations between current policy and current economic outcomes that—via private-sector expectations—are influenced by future policy choices. This necessitates the use of game-theoretic methods in order to determine equilibrium outcomes. Kydland and Prescott used different equilibrium concepts in the different models and examples appearing in their paper. In one, they used the equilibrium concept proposed by Nash (1950). Another model solution was based on sequential rationality, close to Selten’s (1965) subgame-perfect equilibrium, where the expectations of all agents—all private agents as well as the government—are consistent with future equilibrium behavior, regardless of the choices made today. In games with a finite time horizon, such a solution can be found by backwards induction. Kydland and Prescott also studied a model example with an infinite time horizon showing how to use recursive methods. Such methods are particularly useful for defining a special type of equilibria, so-called Markov-perfect equilibria, where current actions are time-invariant functions of payoff-relevant variables only.\(^1\)

2.3 Examples and applications

Kydland and Prescott dealt informally with several examples of time inconsistency. They pointed out that government assistance in protection against natural disasters, such as floods and earthquakes, may not be optimal ex ante, whereas it is optimal ex post. Suppose that an uninhabited area is likely to be affected by tropical storms, and that this risk is so high that it is not socially desirable from an ex-ante perspective for the population to settle there. The necessary protective assistance, which only the government can undertake, is too costly. The question then is what action the government would undertake if the area is in fact settled: either it assists settlers in constructing protective devices to limit losses in the event of a storm, or it refrains. When it is socially desirable to provide protection ex post, there is a time consistency problem. If the government can commit to not providing such assistance in the event the area is settled, the citizens will simply not settle there and the socially desirable outcome is attained. If, on the other hand, the government cannot commit, there will be settlement, since the citizens then know that they will receive assistance and protection, and a socially less desirable outcome is obtained.

\(^1\)In a stationary model with an infinite time horizon, a Markov-perfect equilibrium is formally found as a fixed point in function space: the equilibrium strategy (rule) has to represent optimal current behavior when expectations of future behavior are given by the same rule. Under an infinite time horizon, other types of equilibria are also possible. For a more detailed discussion, see Maskin and Tirole (2001).
Another example concerns the protection of patents for technological innovations. Suppose a government can commit regarding the extent to which it will protect patents in the future. It can then optimally balance the negative effects of monopoly power for the innovators receiving the patents, against the benefits of creating incentives for innovation so as to obtain ex-post monopoly power. If the government cannot commit, however, the relevant incentives for innovation will not be taken into consideration.

In addition to an abstract general model, Kydland and Prescott’s formal analysis treated two cases: tax policy and stabilization (inflation) policy. Here, we first consider a tax-policy problem, similar in spirit to Kydland and Prescott’s but simplified for expositional reasons. We then turn to Kydland and Prescott’s own example of stabilization policy.

2.3.1 Optimal taxation

A government taxes a large number of identical consumers in order to finance a given level of (per capita) expenditures, $G$. This government has access to two tax bases, capital income, $K$, and labor income, $L$, and takes into account the distortions caused by taxation. Each of the (per-capita) tax bases in period $t$ depends negatively on the tax rate that applies to it: $K(\theta_t)$ and $L(\tau_t)$, where $\theta_t$ and $\tau_t$ are the tax rates on capital and labor income, respectively, are both decreasing functions. The government budget constraint can thus be stated as

$$\theta_t K(\theta_t) + \tau_t L(\tau_t) = G.$$ 

Next, assume that private agents decide on the supply of capital in period $t-1$, whereas they decide on the supply of labor in period $t$. We consider two cases.

Suppose first that the government can set the tax rates $\theta_t$ and $\tau_t$ already in period $t-1$, i.e., with full commitment. Optimal behavior, provided that the government wants to maximize the representative consumer’s equilibrium utility, follows Ramsey’s well-known elasticity principle. In particular, the optimal tax rates can be solved from the government’s budget constraint and the equation

$$\frac{\theta_t}{1-\theta_t} \epsilon_K = \frac{\tau_t}{1-\tau_t} \epsilon_L,$$

where $\epsilon_x$ is the elasticity of $x$ with respect to its own (net-of-tax) price: $\epsilon_x \equiv (dx/x)/(dp_x/p_x)$. Intuitively, the government wants to equalize the distortion on the last unit of revenue raised across the two tax bases, which implies that the less elastic tax base is assigned a higher tax rate. What is particularly important here, however, is that the government’s choice of $\theta_t$ and $\tau_t$ takes into account how the supply of capital—from the consumer’s savings in period

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2. The presentation here is related to Fischer (1980) and is based on Persson and Tabellini (1990).

3. For simplicity, we set the prices of both capital and labor services to one.

4. The assumptions on $K$ and $L$ can be derived from first principles: suppose that the consumer’s (quasi-linear) utility function reads $u(c_{t-1}) + c_t + v(1-l_t)$. Here, $u$ and $v$ are strictly increasing and strictly concave functions, and $1-l_t$ is interpreted as leisure time, where 1 is the total time endowment. With the budget constraints $c_{t-1} + k_t = y_{t-1}$ and $c_t = (1-\theta_t)k_t + (1-\tau_t)l_t + y_t$, where $y_{t-1}$ and $y_t$ represent income from other sources in periods $t-1$ and $t$, respectively, utility maximization gives strictly decreasing capital and labor supply functions similar to those in the text.
depends on the choice of \( \theta_t \). Thus, the Ramsey formula for tax rates represents the optimal solution to the ex-ante taxation problem.

Suppose, by contrast, that the government cannot commit to \( \theta_t \) and \( \tau_t \) in advance. Given some amount of savings from period \( t - 1 \), how will the government set tax rates in period \( t \), once this period arrives? The taxation problem becomes trivial and Ramsey’s principle for taxation can be applied to the ex-post taxation problem. Since the supply of capital is entirely inelastic at this point—capital is predetermined—all the capital income should be taxed away before labor taxes are used! Ex post, this creates no distortions on the supply of capital and mitigates distortions on the supply of labor. Because income from capital has a different elasticity ex ante than ex post, the commitment solution is not time consistent. If the government were given a chance to re-optimize in period \( t \), it would change the ex-ante optimal plan. Thus, there is a credibility problem: in period \( t - 1 \), the government cannot just announce the tax rates that solve the problem under commitment and hope that the private sector will believe the announcement.

What is the time-consistent solution when both the government and private agents are rational? Applying the requirement of sequential rationality discussed in Section 2.2, no matter how much is saved in period \( t - 1 \), the decisions of the government and the private sector have to be optimal in period \( t \). Suppose that \( G \) is sufficiently large, i.e., that labor taxation is always necessary in order to finance the government’s expenditures. Then, rationality in period \( t \) dictates that the ex-post tax rate on capital is 100 percent. As a result, all consumers choose zero savings in period \( t - 1 \): rational expectations of government policy choice in \( t \) tells them that any savings are wasted from their perspective. It follows that, for any amount of private savings,

\[
0 + \tau_t L(\tau_t) = G,
\]

which can be used to solve for the tax rate on labor in period \( t \).

\[ \text{Clearly, this outcome is worse than the outcome under commitment (as both tax rates are higher, the utility of consumers must be lower).} \]

### 2.3.2 Optimal stabilization policy

Consider a monetary policymaker who faces a trade-off between inflation and unemployment. Private-sector behavior is represented by an expectations-augmented Phillips curve. Unemployment in period \( t \), \( u_t \), is given by

\[
u_t = u^* - \alpha(\pi_t - E(\pi_t)),\]

where \( u^* \) is the equilibrium (natural) rate of unemployment, \( \pi_t \) is the inflation rate between periods \( t - 1 \) and \( t \), \( E(\pi_t) \) is the inflation rate expected by the private sector in period \( t - 1 \), and \( \alpha \) is a positive exogenous parameter. This is a reduced-form relation which could be motivated, for example, by assuming that (i) the demand for labor depends negatively (and therefore unemployment depends positively) on the real wage in period \( t \); and (ii) nominal wage contracts are set in advance—in period \( t - 1 \)—based on expectations of period-\( t \) prices.

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\[ ^5 \text{I n general, (at least) two values of } \tau_t \text{ solve the equation } \tau_t L(\tau_t) = G. \text{ As the government optimizes, it will choose the lowest of these on the upward-sloping portion of the “Laffer curve”}. \]
Then, higher-than-expected inflation lowers the real wage, labor demand increases, and unemployment falls.

The policymaker’s objective is to maximize the function

\[ S(u_t, \pi_t), \]

where \( S \) is weakly decreasing and concave in each argument and has a maximum at the point where \( u_t < u^* \) and \( \pi_t = 0 \). The indifference curves representing equal values of \( S \) are shown in Figure 1 (taken from Kydland and Prescott, 1977). The literature has subsequently tended to use a particular quadratic form for \( S \):

\[ S = -\frac{1}{2}(u_t - ku^*)^2 - \frac{1}{2}\gamma \pi_t^2, \]

where \( \gamma \) represents the weight the policymaker assigns to inflation (relative to unemployment) and \( k < 1 \) represents some distortion that makes the government target an unemployment rate lower than the equilibrium rate.\(^6\)

The policymaker can use monetary policy in period \( t \) to control \( \pi_t \). Without uncertainty, as assumed here, rational expectations on the part of the private sector imply \( E(\pi_t) = \pi_t \). Therefore, actual unemployment always equals equilibrium unemployment, i.e., \( u_t = u^* \).

If, in period \( t - 1 \), the policymaker can commit to the inflation rate in period \( t \), then the optimal policy is obvious. Since \( u_t = u^* \) must hold, the policymaker will choose \( \pi_t = 0 \). The ex-ante optimal outcome \( (u^*, 0) \) is illustrated in Figure 1 as point \( O \).

The commitment outcome, however, is not time consistent. Ex post, when expectations have already been formed (nominal wages are predetermined when the government chooses period-\( t \) inflation), the government finds it optimal to choose an inflation rate higher than 0 if it is allowed to change its zero-inflation plan. An inflationary policy will reduce unemployment, which by assumption raises welfare.\(^8\) Ex post, optimal policy is dictated by the condition \( \alpha \cdot \partial S/\partial u = \partial S/\partial \pi \): the inflation rate is increased to the level at which the marginal gain from lower unemployment equals the marginal cost of higher inflation. This outcome, characterized by an "inflation bias", is labeled \( C \) in Figure 1.

Using the parametric form of the model, it is easily shown that equilibrium inflation under lack of commitment is given by\(^9\)

\[ \pi_t = \frac{\alpha(1 - k)u^*}{\gamma}. \]

Thus, inflation is higher when the difference between the policymaker’s unemployment target and the equilibrium rate is larger (\( k \) lower, or \( u^* \) higher), the weight it assigns to inflation is

\(^6\)Kydland and Prescott’s assumption that a zero inflation rate is optimal is not essential for the conclusions.

\(^7\)This formulation was first used by Barro and Gordon (1983a).

\(^8\)The key assumption required here is that the marginal welfare gain from lowering unemployment below \( u^* \) should exceed the marginal welfare loss from raising inflation above zero; for the functional form given, the former is strictly positive and the latter is zero (to the first order).

\(^9\)This expression is derived by using the first-order condition, when the government takes \( E(\pi_t) \) as given, i.e., \( \gamma \pi_t = \alpha(u_t - ku^*) \). Combining this condition with the Phillips curve \( u_t = u^* - \alpha(\pi_t - E(\pi_t)) \) and using the rational expectations condition \( \pi_t = E(\pi_t) \), one arrives at the expression in the text.
which used direct measures of expectations and found that forecast errors were systematically related to lagged sales, so we will do so. Responses to this finding are that there may be biases in their measurement of expectations, and these biases are related to lagged sales. This is not implausible, given the subtleness of the expectations concept and the imprecision of survey instruments. Further, even if there were a systematic forecast error in the past, now that the Hirsch and Lovell results are part of agents’ information sets, future forecast errors should not be subject to such biases.

To complete the model, a theory of policy selection is needed. Here it is assumed that there is some social objective function which rationalizes policy choice:

\[ S(x_t, u_t). \]

If the rationalization is not perfect, a random term must be introduced into the function. The consistent policy maximizes this function subject to the Phillips curve constraint (5).

Figure 1 depicts some Phillips curves and indifference curves. From (5) the Phillips curves are straight lines having slope \(-\dot{\lambda}^{-1}\) and intersecting the vertical axis at \(x^*_t\). For a consistent equilibrium, the indifference curve must be tangent to a Phillips curve at a point along the vertical axis—as at point \(C\). Only then are expectations rational and the policy selected.
lower (γ lower), or the responsiveness of unemployment to real wages is greater (α higher). Policymakers who care a great deal about unemployment, but little about inflation, thus end up without lower unemployment, but with higher inflation, than do policymakers with a stronger preference for low inflation relative to low unemployment. This striking result is entirely due to the lack of commitment. Under commitment, the ex-post temptation to inflate is absent, and rational expectations dictate the same outcome for any preference parameters.

2.4 Subsequent research

Kydland and Prescott’s contribution offered a politico-economic explanation as to why many countries found themselves in a process of self-generating inflation despite repeated promises to combat it. In particular, Barro and Gordon (1983a) expressed and elaborated further on this view. Based on Kydland and Prescott’s analysis, they formulated a positive theory of monetary policy and inflation. According to this theory, inflation is higher when equilibrium unemployment is higher relative to the unemployment target that politicians try, but are not able, to attain.

Barro and Gordon (1983a,b) also introduced supply shocks and stabilization policy into the model and showed that these extensions did not alter the basic conclusions. In the extended model, a prior commitment to adhere to a certain policy should be interpreted as commitment to a rule that makes policy respond optimally to future macroeconomic shocks. Such a (contingent) rule leads to better macroeconomic outcomes than discretionary policy: it delivers lower inflation and the same average unemployment rate over the cycle, as well as the same extent of macroeconomic stabilization.

A challenging issue in research as well as in practice was whether the time-consistency problem could be resolved. One possibility was offered by Barro and Gordon (1983b), Backus and Driffill (1985), and Tabellini (1985). These authors borrowed insights from the theory of repeated games to show that, under some conditions, equilibria with low inflation could also arise under discretionary policymaking. According to this theory, when monetary policymakers invest in a “good reputation” by combatting inflation, they influence private-sector expectations regarding future inflation rates.

Kydland and Prescott also pointed to the potential benefit of legislated rules that would introduce a time lag between policy decisions and their implementation (similar to the lag often applied to constitutional change). An apparent drawback of such rules is that, in practice, they would have to be quite simple; this would make it difficult to react to unforeseen macroeconomic events that required a policy response. Thus, simple, non-contingent rules for monetary policy, such as rules prescribing a fixed rate of inflation or a fixed exchange rate, may be less desirable than discretionary policy because the fluctuations in output under the simple rule would become too large. These problems prompted other researchers to focus on institutional reforms that would improve the performance of discretionary policymaking.

In the context of monetary policy, Rogoff (1985) demonstrated that a proper balance between credibility (of low-inflation policy) and flexibility (stabilization) could be achieved through delegation of monetary policy to an independent central bank. In particular, according to Rogoff’s analysis, if an independent central bank is managed by a “conservative”
central banker—i.e., an agent who is more inflation averse than citizens in general—then better welfare outcomes can be achieved. In the language of the stabilization policy example above, an agent with a higher $\gamma$ than in the social welfare function of society should be selected to head and independently run the central bank. This finding has been used in the practical policy debate in many countries to recommend the independence of central banks from direct government influence. The theoretical presumption—that it should be possible to reduce inflation through such institutional reform without any cost in terms of increased average unemployment over the business cycle—has received support from empirical research examining the effects of monetary policy institutions on macroeconomic outcomes both across countries and over time.¹⁰

Rogoff’s idea has since been developed and refined substantially, for example in analyses of monetary regimes with explicit inflation targets and incentive contracts for central bankers (e.g., Walsh, 1995 and Svensson, 1997). The literature has also considered a number of extensions of the economic environment from a simple Phillips curve to dynamic models of the macroeconomy. Much of the recent literature has focused on time consistency problems associated with efforts to use monetary policy for the purpose of stabilizing inflation and unemployment around their target values. Such problems also arise when monetary policy is not characterized by a general inflation bias. Some of these models have analyzed the potential for counteracting temporary increases in inflation with lower costs in terms of lost output and employment if future low-inflation policies are credible (see, e.g., Clarida, Gali, and Gertler, 1999, for a survey of this research). Many of these applications are based on Kydland and Prescott’s methods for characterizing equilibria. Other aspects of the design of monetary institutions, such as the transparency and accountability properties of alternative institutions, have also been analyzed from the stepping stone of Kydland and Prescott’s insights.

Time consistency problems have also been examined in other areas of government policymaking. Important early examples in the literature on fiscal policy are Fischer’s (1980) analysis of the taxation of capital and labor as well as Lucas and Stokey’s (1983) analysis of government debt and the timing of taxes. Subsequent theoretical and empirical research has studied how balanced-budget rules or other restrictions on fiscal policy influence government spending, budget deficits, and international indebtedness.

More generally, a vibrant literature has developed in the area of political economics, borrowing insights and methods from Kydland and Prescott’s work. This research program extends the analysis of credibility problems to issues of political incentives and political institutions as central determinants of policy outcomes. Significant cross-disciplinary work has incorporated ideas and tools from political science to analyze conflicts of interest between voters and policymakers, and between different groups of voters or political parties (Kydland and Prescott’s examples all view the private sector through the lens of identical—“representative”—individuals). This area of research extends far beyond stabilization and taxation policy, dealing with trade policy, regulation policy, labor-market policy, economic growth, and so on. Several recent books, aimed at graduate students in economics and po-

political science as well as active researchers in the area, summarize this body of research (see, e.g., Drazen, 2000, Persson and Tabellini, 2000, and Grossman and Helpman, 2001).

2.5 Broader impact

Kydland and Prescott’s analysis of the time consistency problems inherent in monetary policy provides an explanation as to why many countries appeared to be trapped in a vicious spiral of high inflation during the 1970s, despite continuous declarations from governments and central banks that inflation would be brought down. Kydland and Prescott’s recommendation that policies should be rules-based rather than discretionary sparked a debate, where simple rules for money growth, fixed exchange rates, etc., were suggested as solutions to the inflation problem. Following the academic literature reviewed above, the practical policy debate also changed focus. Discussions of isolated policy actions gave away to explicit consideration of the broad institutional framework shaping the incentives of policymakers and thus determining which policies would be credible and politically feasible.

Since the early 1990s, a number of countries have pursued radical institutional reforms of their monetary policy frameworks, especially by increasing the independence of their central banks regarding the operational conduct of policy to achieve the objectives set forth by the political system. At the same time, these objectives have been stated more clearly, usually with price stability as the primary goal. Central bank reforms in countries such as New Zealand, Sweden, and the United Kingdom drew extensively on the conclusions from the academic literature initiated by Kydland and Prescott, as did discussions about the design of the new European Central Bank, ECB, in the European Union. There is also a close connection between the academic research on this topic and the increasing reliance on explicit inflation targets among central banks in developed as well as developing countries.

2.6 Related literature

Time consistency problems in monetary policy, due to the government’s desire to raise revenue from surprise inflation, were pointed out already in Auernheimer (1974). Calvo (1978) examined such time consistency problems but did not derive the time-consistent solution under discretionary decision making or analyze possible solutions. Another type of time consistency problem appears in the literature on savings decisions initiated by Strotz (1956) and later developed by Phelps and Pollak (1968). Here, time inconsistency enters directly through preferences that change over time, whereas in Kydland and Prescott’s analysis, inconsistency is embedded in the constraints.

A time consistency problem more closely related to that studied by Kydland and Prescott may be found in Buchanan’s (1975) discussion of the “Samaritan’s dilemma”, but again without a systematic formal analysis of the time consistency problem associated with government policy. Similar problems are mentioned in Elster (1977). A related problem also appears in the literature on price setting by a durable-goods monopolist. The idea here is that a monopoly producer of a new good would like all consumers to believe that the good will be continued to be sold at a high price. Consumers with a high willingness to pay would then purchase the good at the high initial price, after which it could be sold at a lower
price to the remaining consumers. The so-called Coase conjecture (1972) holds that pricing
will occur at marginal cost because consumers are forward-looking, whereby the monopolist
actually competes in price with his future selves. Formal game-theoretic analyses of this
problem have later been provided by Stokey (1981), Bulow (1982), and Gul, Sonnenschein,
and Wilson (1986).

3 The driving forces behind business cycles

The last two decades have witnessed radical changes in the theory and practice of business
cycle research, and—more generally—in the predominant views on business cycle phenom-
ena. Keynesian analysis from the early postwar period relied on a set of relationships among
aggregate variables (“reduced forms”) that were intended to “sum up” the interactions be-
tween various structural relationships. Although each such relationship was motivated by
microeconomic theory of consumer and firm behavior, it was usually not explicitly derived
from such theory. More importantly, different macroeconomic relationships were not based
on a common microeconomic structure when used together in applied macroeconomic anal-
ysis.

Estimated versions of such business cycle models were widely used in practical forecasting
and policy-oriented evaluations of monetary and fiscal policy interventions. By the mid-
1970s, the Lucas critique (Lucas, 1976) had pointed to serious problems with this approach.
Estimated reduced-form relationships could not be expected to be robust to changes in policy
regimes or in the macroeconomic environment. Macroeconomic developments emphasized
this critique when seemingly stable macroeconomic relationships, based on historical data,
appeared to break down in the 1970s. In particular, the new stagflation—high unemployment
combined with high inflation—played havoc with the Phillips curve, which had earlier seemed
to trace out a stable negative relationship between the rates of inflation and unemployment.
The experiences of the 1970s also called into question the predominant idea that business
cycles are driven mainly by changes in demand. Instead, the contemporary macroeconomic
fluctuations seemed to be caused largely by supply shocks, such as the drastic oil price hikes
in 1973-74 and 1979 and the worldwide slowdown in productivity growth as of the mid-1970s.

Lucas proposed formulating a new macroeconomic theory on firmer ground, i.e., on an ex-
licit microeconomic structure instead of postulated aggregate relationships. This structure
would include assumptions about consumers and their preferences, firms and their technolo-
gies, the information of these agents, in what specific markets they interact, and so on. On
the basis of these deep parameters, general-equilibrium implications for aggregate variables
would be derived and confronted with data. Consumers’ preferences and firms’ technologies
were not likely to be affected by changes in fiscal or monetary policy regimes or in the
macroeconomic environment, even though the behavior of consumers and firms would be
affected. Hence, quantitative analysis based on microeconomic underpinnings is likely to
be more robust to such regime changes and thus more useful in policy analysis than models
based on historical aggregate relationships.

Unfortunately, Lucas’s guidelines were not accompanied by an operational prescription
for implementing them. The development of an alternative macroeconomic modelling ap-
proach, which would satisfy even the minimal requirement of deriving joint predictions for the
main macroeconomic variables from sound microeconomic foundations, seemed a daunting
task. Such a theory would have to be dynamic to properly model investment, consumption,
and other intertemporal decisions on the basis of optimal, forward-looking behavior of firms
and households. Simple dynamic models with rational expectations certainly existed and a
research program on how to econometrically estimate such models was underway, following
the pathbreaking work by Sargent (1977, 1978, 1979). These models involved drastic simpli-
fications, however, and essentially required representing the economy—or parts of it—by a
few linear relationships. Around 1980, traditional (likelihood-based) econometric estimation
of dynamic, non-linear models on a rich enough form to be operationally used in quantitative
macroeconomic analysis seemed out of reach.

Kydland and Prescott’s 1982 paper transformed macroeconomic analysis in several di-
mensions. Indeed, it provided a blueprint for rendering Lucas’s proposal operational. In their
modeling, Kydland and Prescott relied on a stochastic version of the neoclassical growth
model, which has since become the core of much macroeconomic modeling. They showed
that technology shocks, i.e., short-run variations around the positive growth trend for tech-
nology that makes economies grow in the long run, could be an important cause of output
fluctuations. In today’s macroeconomic models, supply shocks typically play an important
role alongside demand shocks. In their model solution, Kydland and Prescott relied on
numerical solution and computer simulation to an extent not previously implemented in eco-
nomics. Nowadays, numerical analysis of economic models is an indispensable element in the
tool kit of graduate students in economics. In their empirical implementation, Kydland and
Prescott relied on so-called calibration, a simple but informative form of estimation when
confronting new models with data. Since then, new macroeconomic theory is frequently
compared with data using these methods. In all these ways, Kydland and Prescott’s work
changed not only the basic methodology of business cycle analysis, but also our perspective
on the importance of various types of shocks and their propagation mechanisms.

3.1 The general idea

We begin by outlining the general features of Kydland and Prescott’s business cycle the-
yory. Next, we review their methodology, and outline a specific simple example of model
formulation, along with a brief look at empirical implementation.

Kydland and Prescott set out to integrate business cycle theory and growth theory. Since
they viewed technology shocks as potentially important sources of short-run output fluctua-
tions, it seemed natural to turn to the neoclassical growth model—the workhorse of growth
theory ever since the research of Robert Solow (1956). Another reason for using the neo-
classical growth model was related to the problem of distinguishing the “short run” (cycles)
from the “long run” (growth); as the long run, by necessity, is a sequence of short runs.
Moreover, most variables of interest in growth theory and business cycle theory coincide.

Kydland and Prescott’s starting point was the fact that the U.S. economy, and many
other Western economies as well, had grown at an average annual rate of around 2 percent
for approximately 100 years, increasing output by a factor of seven. Their hypothesis was
that technology growth might be an important determinant, not only of long-term living
standards, but also of short-term fluctuations, to the extent that technology growth displays
variations over time. One way of measuring technology growth relies on growth accounting, another tool developed by Solow (1957). Based on certain assumptions about the working of the economy (constant returns to scale, perfect competition, and market clearing), consistent with the model economy studied by Kydland and Prescott, this procedure accounts for the part of output growth due to the growth of inputs (labor and capital, notably). The residual component—the “Solow residual”—is then interpreted as technology growth. Kydland and Prescott (1982) assumed a standard deviation for technology shocks of the same magnitude as for the Solow residuals. Measurement based on Solow residuals implies relatively large variations in technology growth over time, a substantial part of which appear at business cycle frequencies. More refined methods have subsequently been used (see Section 3.4 below).

Conceptually, Kydland and Prescott studied a closed-economy dynamic, stochastic general equilibrium model with perfect competition and no market frictions. How do technology shocks translate into output movements in this model? A positive technology shock in period $t$ represents a higher-than-average growth rate of total factor productivity, i.e., a large increase in the economy’s ability to produce output from given supplies of capital and labor. Higher productivity raises wages, so labor supply in period $t$ increases as workers find work more profitable than leisure. Thus, two effects serve to raise period $t$ output: the direct effect of higher productivity and the indirect effect of higher labor input. The return to capital increases as well, but the capital stock in period $t$ is predetermined. Thus, if the technology shock in period $t$ had been foreseen, the implied increase in the period-$t$ return to capital could also have led to higher investment in previous periods, thus raising output in period $t$ through a third, indirect channel.

The boost in period-$t$ output has dynamic consequences. Part of the increase in output is consumed, while the remainder is saved and invested. The split depends on consumers’ preferences and the expected longevity of the productivity shock. Theory and microeconomic evidence indicate a desire to smooth consumption over time, and the portion of a temporary increase in output that is saved depends on the preference for smoothing. The less quickly the productivity shock is expected to die out, the more profitable will it be to save and invest. Kydland and Prescott based their technology growth series on the data, which feature significant positive autocorrelation, thus leading to an investment response to a current shock which is higher than if technology growth were uncorrelated over time. This raises the capital stock in period $t+1$, while technology is still above trend due to autocorrelation. Incentives for higher than normal labor supply thus remain, and—if the increase in the capital stock is large and technology shocks are sufficiently autocorrelated—labor supply in period $t+1$ will be more above trend than in period $t$, as will investment.

These dynamic effects constitute the model’s “propagation mechanism”, whereby an “impulse” of a temporary technology shock shapes the path of future macroeconomic variables. The mechanism is stable, i.e., the effects of an impulse eventually die out, because the technology growth process is mean-reverting and because decreasing returns to capital bring investment back to trend.

The theory delivers time series for macroeconomic variables broadly consistent with data. Due to the propagation mechanism, all macroeconomic aggregates display high autocorrelation and high co-movement, and the volatility of investment is higher than that of output, which is higher than that of consumption. The economy goes through booms and busts,
with recessions caused by lower-than-average technology growth leading workers to work fewer hours and consumers to invest less. Calibrated with parameters from microeconomic studies and simulated with impulses from an estimated technology growth process, Kydland and Prescott’s baseline model generates output fluctuations that amount to around 70 percent of those observed in postwar U.S. data.

3.2 Methods

Kydland and Prescott studied a dynamic, stochastic general equilibrium model. An equilibrium in the model is a stochastic process for quantities and prices such that (i) given the price processes, consumers and firms choose quantities so as to maximize expected utility and maximize profits and (ii) markets clear. Property (i) embeds rational expectations; in a full-fledged dynamic model, unbiased predictions of the future evolution of prices is an element of optimizing behavior. Basic theorems ensuring the existence of an equilibrium—which, mathematically, required solving a fixed-point problem in high-dimensional space—were already provided in the work of Arrow and Debreu (see Debreu, 1959). However, a precise characterization of an equilibrium was very difficult, due to the complexity of dynamic stochastic analysis. Thus, Kydland and Prescott’s 1982 paper made several simplifications of the general structure described by Arrow and Debreu.

Kydland and Prescott considered only one consumption good and one type of “infinitely lived” consumer (to be interpreted as a dynastic family: a sequence of parents and children with altruistic preferences vis-à-vis offspring). Moreover, as in the standard neoclassical growth model, Kydland and Prescott assumed only one type of production technology: an aggregate production function, based on the inputs of capital and labor. They also assumed that markets are devoid of frictions, so that any equilibrium is Pareto optimal. This facilitated matters in the sense that standard welfare theorems allowed them to find and characterize the equilibrium using optimization theory. Since an equilibrium delivered the best possible outcome for the representative consumer, they could sidestep the price mechanism and find the equilibrium quantities directly by solving a “social planning problem”. Based on these quantities, the equilibrium prices were then easily retrieved from the first-order conditions for utility and profit maximization (for details, see Section 3.3.2 below). All of these simplifications have been examined and relaxed in the subsequent literature (see Section 3.4).

In spite of these drastic simplifications, Kydland and Prescott found it necessary to use numerical analysis to characterize the equilibrium. In so doing, they adapted available insights in numerical analysis to the problem at hand and used computer-aided model solution. Today’s state-of-the-art business cycle models are substantially more complex than that analyzed by Kydland and Prescott, and the numerical analysis of economic models has evolved into a subfield of its own in economics.\footnote{For references and an overview, see Amman, Kendrick, and Rust (1996).}

Comparison of the model to data was another challenging task. A standard econometric approach, i.e., to choose the model’s parameters so as to obtain the best possible fit to the business cycle data, could not really be used due to the complexity of the model. To generate model output for even one set of parameter values was quite difficult and time-consuming,
given that it involved numerical solution of a dynamic, stochastic optimization problem. A search among sets of parameter values in order to obtain the best fit was deemed infeasible. Instead, Kydland and Prescott adopted the method of “calibration”. They selected parameter values to match a subset of moments in the data in a way that did not require solution of the entire stochastic model. In particular, they chose parameter values to match certain long-run macroeconomic statistics (such as average postwar interest rates and average capital-output ratios) and microeconomic data (which allowed the parameterization of preferences).

The idea of choosing parameters in accordance with some “basic facts”, rather than the business cycle properties the model was designed to explain, motivated the term calibration. Clearly, calibration is a simple form of estimation, since the model parameters are chosen in a well-specified algorithm to fit a subset of the overall data; in this case, the estimation is based on microeconomic and (long-run) macroeconomic data. However, the method was very practical. It allowed parameterization without solving the full model and it gave clear and useful directions as to whether specific changes in the model could better explain the data. Nowadays, given the advances in computer technology and econometric methods, structural estimation of this class of business cycle models can be carried out and is actively pursued by business cycle analysts (see Section 3.4 below).

3.3 Examples and applications

We now describe in detail a simple special case of Kydland and Prescott’s 1982 setup, while briefly pointing out how their more general model differs from this special case.

3.3.1 The setup

Time is discrete and infinite: 0, 1, 2, … . There is one type of output good at each date, yt, which can be used for consumption or investment: ct + it = yt. Capital accumulation obeys

\[ k_{t+1} = (1 - \delta)k_t + i_t \]

one unit of investment at t adds to the capital stock at t + 1 and then depreciates at a constant rate δ. Output is produced from capital and labor according to the production function \( f: y_t = f(z_t, k_t, l_t) \). Here, f is increasing and concave and is homogeneous of degree one in k and l. The stochastic technology parameter, \( z_t \), follows an AR(1) process, \( z_{t+1} = \rho z_t + \varepsilon_{t+1} \), where \( \rho \in (0, 1) \) indicates positive autocorrelation and \( \varepsilon_t \) is identically and independently distributed over time with zero mean and positive variance denoted \( \sigma^2 \).

---

12 Early applications of similar methodologies, although typically in static models, can be found in the empirical literature on international trade (see Shoven and Whalley, 1984). Dynamic models were used in public finance (see Auerbach and Kotlikoff, 1987).

13 Long-run averages in Kydland and Prescott’s stochastic model are approximately equal to the long-run averages in the non-stochastic version of the model, which are straightforward to solve for analytically.

14 A version of this special case was examined in a multi-sectoral context by Long and Plosser (1983).

15 Kydland and Prescott assumed a so-called “time-to-build” technology, according to which it takes longer than one period for investments to mature into productive capital.

16 The formulation here describes a process where the technology level and population size do not exhibit growth. This is convenient for purposes of illustration, but it is straightforward to allow for positive trends in technology and population.
There is a large number of (identical) consumers, so each consumer chooses quantities and takes prices as given. Each consumer is infinitely lived and derives utility from consumption and leisure. Her preferences from the perspective of time 0 are described by

\[ E[\sum_{t=0}^{\infty} \beta^t u(c_t, 1 - l_t)] , \]

where \( \beta \), the discount factor, is positive and less than unity reflecting a preference for current consumption, and where \( u \) is a strictly increasing and strictly concave function. The total time endowment is 1 and \( l_t \) is the time spent working. The variables \( y_t, c_t, \) and so on, are stochastic—they are driven by the stochastic process for technology \( z_t \)—and \( E \) denotes the expectations operator.

The consumer’s budget constraint reads

\[ c_t + k_{t+1} = (1 + r_t - \delta)k_t + w_t l_t , \]

where \( r_t \) is the market return to capital before depreciation and \( w_t \) the wage rate. Prices in this model are thus given by the stochastic processes for \( r_t \) and \( w_t \). The consumer maximizes the utility function subject to the budget constraint holding at all dates.

Firms maximize profits under perfect competition. Given that the consumers determine capital accumulation, this implies solving the static optimization problem of choosing \( k_t \) and \( l_t \) to maximize

\[ f(z_t, k_t, l_t) - r_t k_t - w_t l_t . \]

Because \( f \) is homogeneous of degree one, equilibrium profits are zero in all time periods.

3.3.2 Analysis

The first-order necessary conditions for consumer and firm optimization govern how the model works. Consumers have an intertemporal first-order condition which determines the consumption-savings choice:

\[ u_1(c_t, 1 - l_t) = \beta E[u_1(c_{t+1}, 1 - l_{t+1})(1 + r_{t+1} - \delta)] . \]

The marginal utility loss of lower consumption in period \( t \) must equal the expected discounted value of the return to higher investment, in terms of next period’s utility. Consumers also have an intratemporal condition which delivers the labor-leisure choice:

\[ u_1(c_t, 1 - l_t)w_t = u_2(c_t, 1 - l_t) . \]

The marginal loss of working one more unit of time in terms of lost leisure (the right-hand side) must thus equal the wage times the marginal utility gain of higher earnings.

Firms’ profit maximization conditions simply state that marginal products equal real factor prices: \( r_t = f_2(z_t, k_t, l_t) \) and \( w_t = f_3(z_t, k_t, l_t) \). Thus, these prices are easy to find as a function of the input quantities and the current value of the technology parameter.

The equilibrium of this model takes the form of stochastic processes for quantities and prices that satisfy all the equilibrium conditions. As in their work on time consistency, Kydland and Prescott used recursive analysis whereby equilibrium processes are expressed
as functions of the economy’s “state variable”, which in this case is \((z_t, k_t)\): the current shock and capital stock. The equilibrium solution for labor will be given by a function \(h_l: \ l_t = h_l(z_t, k_t)\). Thus, this function tells us how much labor will be supplied in equilibrium for any value of the current shock and any value of the capital stock. We let \(h_c\) denote the solution for consumption: \(c_t = h_c(z_t, k_t)\). The two functions \(h_l\) and \(h_c\) are sufficient for describing the equilibrium. As an example, if the current state is \((z_t, k_t)\), the capital stock in period \(t+1\) must satisfy \(k_{t+1} = (1 - \delta)k_t + f(z_t, k_t, h_l(z_t, k_t)) - h_c(z_t, k_t)\), which is itself a function only of the current state. As another example, the wage rate in period \(t\) must equal \(f_3(z_t, k_t, h_l(z_t, k_t))\), which is also a function only of the current state.

In general, it is not possible to find explicit forms for \(h_l\) and \(h_c\). The next step in the analysis is therefore to use numerical analysis in order to approximate these functions under some suitable specific parametric-form assumptions on \(f\) and \(u\) and on the remaining parameters of the model.

In a very special case of the model, it is possible to obtain a full analytical (as opposed to numerical) characterization of the equilibrium. If

\[
\begin{align*}
    u(c, 1 - l) &= (1 - \phi) \log c + \phi \log(1 - l), \\
    f(z, k, l) &= z^{\alpha} k^{1 - \alpha},
\end{align*}
\]

and \(\delta = 1\), then it can be verified that all the equilibrium conditions are satisfied when labor supply is constant over time, i.e., \(h_l(z_t, k_t) = A\), and consumption is a constant fraction of output—so that \(h_c(z_t, k_t) = B z_t^{\alpha} k_t^{1 - \alpha} A^{1 - \alpha}\)—for some constants \(A\) and \(B\).\(^{17}\)

Model output from the specific case which allows a closed-form solution is not directly comparable to business cycle data, because the assumption of full depreciation implies that a period in the model is very long. In the data, physical capital depreciates at a rate on the order of 10 percent per year, and full depreciation occurs after perhaps 20 years. Business cycle models require the period length to be much shorter, and the typical assumption has been to let a period be a quarter, or a year. Moreover, although the special case of the model has a propagation mechanism similar to that discussed in Section 3.1, labor supply does not respond to shocks. But if the rate of depreciation is assigned a realistic value (which necessitates using numerical model solution), then a labor supply mechanism with the properties discussed above appears.

3.3.3 Calibration

We have already touched on the importance of specific parameters for the properties of the model. As outlined above, Kydland and Prescott did not us an econometric approach, but instead chose to calibrate their model. That is, they chose all the parameters based on microeconomic data and long-run macroeconomic data, rather the business cycle frequencies in the data. How, then, does the procedure of calibration work in the case of this model?\(^{18}\)

As already indicated, the length of a time period has to be set and, typically, a time period in

\(^{17}\)To show this, simply insert the asserted functional forms for consumption and labor into the first-order conditions and verify that these conditions hold at all points in time.

\(^{18}\)For details on the calibration undertaken, see, e.g., Prescott (1986).
the model is interpreted to represent one quarter. Following Kydland and Prescott, long-run
data (for example, postwar averages from the U.S. if the model is intended to describe this
economy) are then used for the following variables: the average capital-output ratio (about 8,
when output is measured on a quarterly basis, which helps pin down the rate of depreciation
$\delta$), the average quarterly real interest rate (about 0.015, which pins down the discount rate
$\beta$), the average capital share of income (about 0.35, which pins down the coefficient $\alpha$ in the
production function), and the average amount of hours worked (about 0.2 as a fraction of
total hours, which pins down the weight on leisure in utility $\phi$ in the case of the logarithmic
utility function).

In other words, a version of the equilibrium conditions is used where all variables grow at
a constant rate over time—to depict a long-run, or “steady-state”, situation. The parameter
values are then solved for, given the above long-run statistics. Typically, a somewhat more
general utility function than the logarithmic function is considered. This permits parame-
terization of the curvatures with respect to consumption and leisure such that elasticities
of intertemporal substitution and labor supply conform to those obtained in microeconomic
studies. Finally, the key parameters of the technology process, $z$, are chosen to conform to
the properties of an estimated process for the Solow residual, matching $\rho$ to its first-order
autocorrelation and (the variance of) $\varepsilon$ to its variance.

### 3.3.4 Quantitative evaluation

Once, all of the parameter values have been assigned, the model is fully specified and can be
solved numerically. Given the solution for the equilibrium functions $h_l$ and $h_c$, output and
other variables of interest can be simulated by selecting an initial capital stock and drawing
stochastic shocks from their given statistical distribution, while using the functions $h_l$ and $h_c$
to generate series for all variables of interest. These series can then be compared to business
cycle data.

As an illustration, consider the setup exemplified above, which is also analyzed in Cooley
and Prescott (1995). The calibration is carried out as outlined, where the functional form
for utility is $u(c, l) = \frac{(c^{1-\phi} - 1)^{\gamma}}{\gamma}$. This form implies that the consumer’s attitudes toward
risk—as defined by the “coefficient of relative risk aversion”—is constant over time and equal
to $\gamma$. As in the general discussion above, $\phi$ is determined on the basis of time-use studies.
The parameter $\gamma$ is chosen on the basis of microeconomic studies of risk-taking. The studies
discussed in Prescott (1986) suggest a value for $\gamma$ around 1, which in fact implies that the
utility function reduces to the logarithmic function above. Next, in line with the results of
growth accounting, the process for technology shocks is taken to have a serial correlation
coefficient ($\rho$ above) of 0.95 and a standard deviation ($\sigma$ above) of 0.7 percent.

The data are quarterly data from the postwar U.S. economy. Prior to comparing the
simulated output from the model to data, the business cycle component of the data has
to be extracted. Business cycles are normally thought of as fluctuations around a growth

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19 The framework here and in Cooley and Prescott (1995) are identical except that the latter study allows
long-run growth in population and technology. Because these factors only have a marginal influence on the
results, they are omitted here.

20 The sample begins in the first quarter of 1954 and ends with the second quarter of 1991.
Some statistics from the model (data)

<table>
<thead>
<tr>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
<th>Labor productivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent standard deviations</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.35 (1.72)</td>
<td>0.33 (0.86)</td>
<td>5.95 (8.24)</td>
<td>0.77 (1.59)</td>
<td>0.61 (0.55)</td>
</tr>
<tr>
<td>Correlation with output</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (1)</td>
<td>0.84 (0.83)</td>
<td>0.99 (0.91)</td>
<td>0.86 (0.99)</td>
<td>0.98 (0.03)</td>
</tr>
</tbody>
</table>

Table 1

path that occur with a frequency of three-five years.\textsuperscript{21} Thus, the data are “de-trended”, or “filtered”, so as to suppress their low-frequency (trend) components, while retaining those in the business cycle range. This can be accomplished in several ways. Cooley and Prescott (1995) used a form of difference filter.\textsuperscript{22} Some key statistics of the de-trended data from their study and the associated model output are summarized in Table 1.\textsuperscript{23}

The table shows that the variation in output in the model is somewhat lower than in the U.S. data, but the discrepancy is not large. The model predicts that consumption varies much less and investment much more than output. This is a result of consumption smoothing. The variation in hours in the model is less than half of that in the data. This discrepancy is largely due to the fluctuation in the number of employed workers in the data; the simple model here focuses on variations in the number of hours worked per worker (the intensive margin) and abstracts from variations in labor-force participation and unemployment (the extensive margin). Productivity in the model varies somewhat more than the measure of productivity found in the data, which is based on average compensation per hour.

The table also shows strikingly high correlations between output and other macroeconomic variables. The reason is that there is only one shock—one source of uncertainty—in the model. The correlations between consumption and output and between investment and output are quite similar in the model and in the data. As for the correlation between hours and output, the model does not generate a co-movement as high as that observed in the U.S. economy. This is another consequence of the rudimentary modeling of labor supply in this

\textsuperscript{21}See, e.g., Burns and Mitchell (1946).
\textsuperscript{22}The specific filter used is the one introduced in Hodrick and Prescott (1980).
\textsuperscript{23}The model was simulated 100 times, each with a sample length of 150 time periods. The numbers in the table are averages across simulations. The series for output, consumption, and investment are based on real (1982$) quantities; output is GNP, consumption is consumption on nondurables and services, and investment is gross private domestic investment. Hours are total hours of work based on the U.S. Household Survey. As in the model, labor productivity equals average total compensation per hour, measured from the U.S. National Income and Product Accounts.
simple model. Hourly compensation is around zero in the data whereas the model generates a correlation close to one. This again reflects that the model has only one shock and that labor supply operates only on the intensive margin.

The implications of the model for the dynamic correlations of different variables over the cycle can also be compared to data. For example, the correlation between output in adjacent quarters is 0.70 in the model; in the data, it is 0.85.  

3.4 Subsequent research

Kydland and Prescott’s methodology has been broadly adopted, and adapted to a variety of frameworks for analyzing business cycles. The vast literature that has followed may be organized into five different categories: (1) extensions aimed at evaluating the consequences when the stark simplifying assumptions, such as that of only one type of consumer or abstracting from monetary issues, are relaxed; (2) analyses relying on other impulses, such as monetary shocks or international shocks to the terms of trade; (3) studies considering different propagation mechanisms, such as those implied by imperfections in credit, labor and goods markets; (4) investigations focusing on improved measurement of technology shocks and other variables; and (5) studies aimed at improving the empirical analysis in the direction of structural estimation.

Category 1. A number of simplifying assumptions made by Kydland and Prescott have been relaxed and the main results have been found to be quite robust. These include homogeneity of firms and households (captured by the assumptions of a representative firm and a representative household, as opposed to heterogeneity of firms and households in different dimensions), perfect altruism across generations, the absence of idiosyncratic and uninsurable consumer risk, the aggregation of goods into a single composite good, perfect substitutability between consumption and investment goods, perfect competition in goods markets, no role for money (as opposed to, say, a demand for money based on so-called cash-in-advance constraints), the closed economy, and the exogeneity of technology improvements.

The propagation and serial correlation properties of the model are affected when allowing for costs of changing/reallocating the capital stock and the labor force over time and across sectors. These properties are also affected when nonconvex technologies, such as indivisibilities in labor supply, are introduced to generate adjustment on the extensive margin of employment, which is missing in the simple model of Section 3.3. Extensions of the model to include costs of rapidly adjusting capital and labor in response to shocks help replicate certain asymmetries that characterize the business-cycle data. For instance, recessions are steeper and shorter than booms, estimates of responses to technology shocks suggest that hours worked respond strongly but with a lag, and the components underlying the employment process—job creation and job destruction—have very different time-series properties, with job destruction far more variable than job creation (see, e.g., Gilchrist and Williams, 2000).

24 For a more complete evaluation of the model’s predictions for the structure of leads and lags, see Cooley and Prescott (1995).

25 For an early survey of the literature, see Cooley (1995); for a more recent account, see King and Rebelo (2000).
2000 and Campbell and Fisher, 2000). Moreover, consideration of variable utilization of factors of production has been shown to significantly increase the amplitude of the economy’s response to shocks (for a general discussion of this issue, see King and Rebelo, 2000).

**Category 2.** Many studies have suggested alternatives to aggregate technology shocks as the source of cycles. Real impulses include shocks to relative world prices for raw materials, shocks to the mechanism for wage setting implying changes in firms’ costs (“cost-push” shocks), shocks to fiscal policy and government expenditures, or technology shocks that primarily affect a sector of the economy, rather than aggregate output. Demand shocks include shocks to consumer preferences, e.g., “bursts of impatience”, even though the difficulty of empirically measuring the size of such shocks makes them hard to assess. Preference shocks are perhaps the closest counterpart to those which Keynes perceived as driving the economy, although they may not represent “animal spirits of investors” or “consumer confidence”. One branch of the literature deals with shocks to information, i.e., news about upcoming events, such as technology improvement.

Yet another branch of the literature examines business cycles from an international perspective. Some researchers have studied the reaction of small open economies to international shocks, such as terms-of-trade or interest-rate shocks; see Mendoza (1991) and, for an early application to the case of Sweden, Lundvik (1992). Other researchers have instead considered the transmission of business cycles across similar economies; for the first contribution to this line of research, see Backus, Kehoe, and Kydland (1992).

**Category 3.** Many extensions have been motivated by the belief that a macroeconomic theory where the basic ingredients of the business cycle can be captured in a perfect-market framework misses important real-world features. Naturally, labor-market frictions that give rise to equilibrium unemployment expand the domain of the model, but they do not necessarily imply very different time-series properties for output and investment. The same applies when monopolistic elements are added to competition in goods markets. Despite the added frictions, the basic methodology in this research is typically in line with Kydland and Prescott’s approach, however. For example, the models of credit-market imperfections in Bernanke and Gertler (1989) or Kiyotaki and Moore (1995) are explicitly founded on microeconomics and derive market frictions from “first principles”. Similarly, search and efficiency-wage models of unemployment make assumptions about agents’ behavior derived from microeconomic underpinnings. Some of these models feature qualitatively different propagation mechanisms than those found in Kydland and Prescott’s work. Thus, Kiyotaki and Moore (1995) find pronounced cyclical responses of output to single shocks, whereas Kydland and Prescott’s theory tends to give monotonic responses.

**Category 4.** Renewed measurement of technology shocks has been undertaken using several methods. One such method builds on Solow’s growth accounting (used for example in Prescott (1986) and Kydland and Prescott (1988)), but substantially relaxes the underlying assumptions. First, accounting has been conducted on a more disaggregated level, i.e., relaxing the assumption of an aggregate production function. Second, the assumption that inputs are well measured has been relaxed by allowing (unobserved) factor utilization to

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26 For search frictions, see Merz (1995) or Andolfatto (1996) and for an efficiency-wage formulation, see Danthine and Donaldson (1995). For an early analysis of a real business cycle model with monopolistic competition, see Hornstein (1993).
vary and by econometrically estimating the movement of factor utilization over the cycle using instrumental variables techniques. Third, the assumption of constant returns to scale and perfect competition has been relaxed: based on estimating the size of markups over marginal costs, the same type of accounting method can still be used. These improvements on Kydland and Prescott’s initial measures are discussed in Basu and Fernald (1997, 2000).

A summary of the findings is that estimates of short-run fluctuations in technology remain significant; whether they have the specific effect on the economy predicted by Kydland and Prescott’s original model is a more open question.

Another, and quite different, approach to measuring technology shocks is to estimate a stochastic, vector-autoregressive (VAR) system of aggregate time series containing output, measured labor productivity, and a set of other variables. In the VAR system, the shocks driving the observed variables are then classified as supply (technology) shocks if they have a permanent impact on labor productivity, which technology shocks should have; any remaining shocks are classified as demand shocks (see Blanchard and Quah, 1989). For this approach, see, e.g., Gali (1999) and Fisher (2002). The results of this approach appear sensitive to specification details, but call into question the generality of Kydland and Prescott’s original findings.

Category 5. Econometric estimation has gradually been adopted. As mentioned earlier, traditional econometric estimation of a fully micro-founded business cycle model was not really an option when Kydland and Prescott made their original contribution, and may not have been very productive in the early stages of the research program. But over time, as the theory has become richer, computers have become more powerful, and econometric methods have become more advanced, the situation has changed (category (5) above). Econometric estimation of the stochastic growth model began with Altug (1989), but was undertaken in a linearized version of the business cycle model. Later developments estimated key parts of the model—such as first-order conditions for savings and labor supply—using so-called generalized method of moments estimators (see, e.g., Hansen, 1982). Full structural estimation of stochastic nonlinear dynamic general equilibrium models has now been undertaken as well. Smets and Wouters (2003) provide a recent example of successful full-fledged (Bayesian) estimation of a model for the Euro area based on Kydland and Prescott’s work.

An important new literature, which deserves special mention, encompasses several of the extensions discussed above. Commonly referred to as new-Keynesian business cycle research, this literature examines monetary models of business cycles based on frictions in the process of price and/or wage adjustment (see e.g., Rotemberg and Woodford, 1997, Clarida, Gali, and Gertler, 2000, and Dotsey, King, and Wolman, 1999). These new-Keynesian models are built around a core very similar to Kydland and Prescott’s model, but they also include microeconomic assumptions on the costs of changing prices for firms, which typically are assumed to interact under monopolistic competition. Price-setting and/or wage-setting decisions are explicitly modeled as forward-looking and based on rational expectations. This renders the analytical structure similar to that in Kydland and Prescott’s original work and the new-Keynesian researchers have also borrowed their analytical tools. The resulting models can give rise to a Phillips-curve. Monetary shocks produce potentially large effects on output, and monetary policy can produce, or stabilize, short-run fluctuations. Models of this type have also proved very useful for more complex analyses of the time consistency
problems of monetary policy uncovered by Kydland and Prescott.

New-Keynesian frameworks have been applied to both positive and normative analysis of different monetary rules and institution design. Since the underlying theory rests on explicit microeconomic assumptions, the evaluation of policy experiments is straightforward: simulating the model for different policy scenarios, the resulting welfare levels for different agents can easily be compared. Thus, the modeling allows not only qualitative, but also quantitative welfare statements. Such policy evaluations are also attractive in view of the Lucas critique. Since the models are formulated on the basis of deep parameters, they should be more robust to the conduct of policy than aggregate reduced-form relations. In sum, the new-Keynesian approach has synthesized earlier Keynesian analysis with the real business cycle analysis originating in Kydland and Prescott’s work.

3.5 Broader impact

Kydland and Prescott’s 1982 paper transformed the academic research on business cycles. Extensions and refinements in the subsequent literature improved the ability of the original model to match the macroeconomic data and allowed meaningful analyses of macroeconomic policy. Models used in actual policy contexts have increasingly adopted Kydland and Prescott’s methodology. Gradually, many models used in policy organizations and central banks have come to incorporate microeconomic foundations in the form of rational savings and labor supply behavior combined with rational expectations. One typical procedure has been to formulate deterministic versions of Kydland and Prescott’s model—to be used for medium-term counterfactual analyses—and to add an ad-hoc stochastic structure that allows for rich short-run dynamics. Today, some organizations have complete operational versions of Kydland-Prescott-style business cycle models incorporating full short-term dynamics. An alternative—although closely related—approach has been to look for ways of summarizing full-fledged Kydland-Prescott models using (approximate) reduced-form systems that are easy to analyze and therefore convenient tools in policy analysis (see, e.g., Woodford, 2003 and Schmitt-Grohé and Uribe, 2003). Computer technology has defined the frontier of Kydland and Prescott’s research program, and the rapid recent advances in this technology have greatly expanded the ability to solve and estimate highly complex versions of new business cycle theories.

3.6 Related literature

The core of the real business cycle model is a neoclassical growth model with optimal savings decisions. Cass (1965) and Koopmans (1965) added optimal savings decisions to Solow’s neoclassical setup, although they did not model labor supply. Stochastic shocks were introduced into the optimal growth model by Brock and Mirman (1972), but their model was not given an equilibrium interpretation and was not used to analyze technology-driven short-run cycles. In their early contribution to the literature initiated by Kydland and Prescott, Long and Plosser (1983) examined co-movement across sectors due to aggregate technology shocks and coined the term “real business cycles”. Bruno and Sachs (1979) analyzed supply shocks,

27 For an application to short-term forecasting, see del Negro and Schorfheide (2004).
but not in the context of a fully dynamic and stochastic macroeconomic model based on microeconomic foundations. More generally, Kydland and Prescott’s approach to business cycle analysis is linked to early articles by Frisch (1933) and Slutsky (1937), which showed how an economy’s adjustment to a sequence of random shocks can give rise to cyclical fluctuations reminiscent of business cycles.

4 Recommended reading

Although rather technically demanding, Kydland and Prescott’s original articles from 1977 and 1982 are highly recommended. Several of the early articles on time consistency and macroeconomic policy are reprinted in Persson and Tabellini (1994). For a survey of this research, see Drazen (2000). For readings on Kydland and Prescott’s business cycle theory, see the volume by Cooley (1995), which contains a series of surveys on different aspects of real business cycles. King and Rebelo (2000) provide an up-to-date comprehensive review, while Prescott (1986) gives a non-technical introduction to the topic.

5 References


28


