Econ 510a (second half)
Yale University
Fall 2005
Prof. Tony Smith

HOMEWORK #1

This homework assignment is due at the beginning of class on Wednesday, October 26.

1. **(a)** Consider a version of the static model that we discussed in class on October 19 in which a typical consumer's preferences are given by: $u(c, \ell) = \log(c) + A\log(\ell)$, where $A > 0$, and the firm's production function is $y = zk^{\alpha}n^{1-\alpha}$, where $0 < \alpha < 1$. (Let both the number of consumers and the number of firms be equal to 1.) Find an algebraic expression for the competitive equilibrium amount of labor supply in terms of primitives (i.e., in terms of the parameters describing technology and preferences). How do changes in z and A affect equilibrium output, consumption, and labor supply? Explain (i.e., try to give some economic intuition for your answers).

(b) Now introduce a government that taxes labor income (i.e., income from wages) at a proportional rate τ and returns the proceeds in a lump-sum fashion to consumers. A typical consumer's budget constraint reads:

$$c = rk_0 + (1-\tau)w(1-\ell) + T,$$

where $\tau > 0$ and $T > 0$ is a lump-sum transfer to consumers. Each consumer takes the prices r and w and the transfer T as given when solving his decision problem. The government must satisfy its budget constraint: $T = \tau wn$, i.e., total transfers to consumers must be equal to total revenues from the tax on labor income. Find an algebraic expression for the competitive equilibrium amount of labor supply. How do changes in z, A, and τ affect equilibrium output, consumption, and labor supply? Explain.

(c) Is the competitive equilibrium allocation in part (b) identical to the Pareto optimal allocation? Explain why or why not.

(d) Suppose instead that the transfer is financed by a proportional tax on capital income. Do the competitive equilibrium and Pareto optimal allocations coincide in this case? Explain why or why not.

2. Consider a version of the static model in the first problem (without taxes) in which a typical consumer's preferences are given by:

$$u(c, \ell) = \frac{c^{1-\sigma} - 1}{1-\sigma} + A\frac{\ell^{1-\gamma} - 1}{1-\gamma},$$

where both σ and γ are positive. (Note that, as σ and γ both approach 1, this utility function converges to the one in the first problem.) Find an equation that determines the competitive equilibrium amount of labor supply. (You will not able to solve explicitly for labor supply as a function of primitives.) How do increases in z and A affect equilibrium output, consumption, and labor supply? Explain. (Hint: To determine the effect of changes in z on labor supply, totally differentiate the expression determining labor supply with respect to n and z, compute the derivative dn/dz, and then try to "sign" the derivative.)

3. Consider an economy with two time periods (labelled 0 and 1) in which a typical consumer has preferences given by: $\log(c_0) + \beta \log(c_1)$, where c_t is consumption in period t. Each consumer is endowed with k_0 units of capital at the beginning of period 0 and with one unit of time in each period.

In each period, there is a price-taking, profit-maximizing firm that produces goods using capital and labor. These goods can be either consumed or saved in the form of capital that can be used in production in the next period. Let the firm's production function be: $y = zk^{\alpha}n^{1-\alpha}$, where k is the amount of capital rented by the firm and n is the amount of labor rented by the firm in a given period.

Because leisure is not valued (leisure does not appear in the utility function), each consumer supplies labor inelastically, i.e., he supplies one unit of labor in each time period. The only interesting decision that a consumer makes, then, is how much to save in period 0. Let k_1 be the amount of capital that a typical consumer saves in period 0. Then each consumer faces a pair of budget constraints:

$$c_0 = r_0 k_0 + w_0 - k_1$$

and

$$c_1 = r_1 k_1 + w_1,$$

where r_t is the rental price of capital in period t (expressed in terms of period-t consumption goods) and w_t is the wage rate in period t (again expressed in terms of period-t consumption goods). Each consumer takes these prices as given when deciding how much to save. Because period 1 is the last period of his life, each consumer consumes all of his resources in period 1. (Note that, as in the first two problems, there is an implicit assumption that capital is used up entirely in the process of production, i.e., the depreciation rate of capital is equal to one.)

In equilibrium, the markets for goods, labor, and capital must clear in both time periods.

(a) Find an explicit expression (in terms of primitives) for the competitive equilibrium capital stock in period 1. Use your answer to determine the equilibrium rate of return on savings between periods 0 and 1. In addition, determine the equilibrium allocation of consumption across the two time periods. How do changes in z affect the equilibrium allocation? Explain.

(b) Formulate a social planning problem for this economy and show that the allocation chosen by the social planner is identical to the competitive equilibrium allocation that you determined in part (a). Illustrate this result using an appropriate diagram.