Econ 510a (second half)
Yale University
Fall 2007
Prof. Tony Smith

HOMEWORK #1

This homework assignment is due at noon on Friday, November 2. Please put your assignment in Evrim Aydin's mailbox.

1. (a) In the Solow growth model that we developed in lecture, suppose that the rate of depreciation is zero. Show that, in this case, there is no steady state but, nonetheless, the growth rate of the capital stock converges to zero.

   (b) Does the neoclassical growth model have a steady state if the rate of depreciation is zero? Explain.

2. In the neoclassical growth model that we developed in lecture, assume that u has constant elasticity of intertemporal substitution σ^{-1} (i.e., let $u(c) = (1-\sigma)^{-1}c^{1-\sigma}$) and that $F(k) = k^\alpha + (1-\delta)k$. Discuss how changes in σ and α affect the speed of convergence to the steady state. (Recall that, near the steady state, the speed of convergence is inversely related to the slope of the optimal decision rule at the steady state.) Try to give economic intution for your findings.

3. Consider the planning problem for a simple finite-horizon neoclassical growth model:

$$\max_{\{c_t,\, k_{t+1}\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t \log(c_t),$$

   given $k_0 = 10$ and subject to the constraint that $c_t + k_{t+1} = Ak_t^\alpha + (1-\delta)k_t$. Set $\beta = 0.95$, $\delta = 0.1$, and $\alpha = 0.4$. Choose A so that the steady-state value of capital in the corresponding infinite-horizon model is 100.

   (a) Set $T = 20$ and solve the model numerically (say, in Matlab) using the "shooting" method described in lecture: start by guessing a value for k_1, solve for k_2 from the Euler equation at time 0, then solve for k_3 from the Euler equation at time 1, and so on, until k_{T+1} is found. Then vary k_1 and repeat until the appropriate value of k_{T+1} (what is it?) is found. Calculate the savings rate at each point along the optimal path.

   (b) Find the lowest value for T such that the highest value of capital between periods 0 and T exceeds 90.

4. By differentiating the functional Euler equation in the neoclassical growth model, we showed in lecture that the derivative of the optimal decision rule, evaluated at the steady state, satisfies a quadratic equation with two roots, one between 0 and 1 and one larger than β^{-1}. (Aside: Be sure that you know how to show this!) One way to rule out the explosive root (the one larger than β^{-1}) is to argue that the optimal path for capital converges to a (unique) steady state for any (positive) initial capital stock. This problem explores another way to rule out the explosive root, under the assumption that both the value function and the decision rule are twice differentiable (this does not hold in general, but does hold for many specific choices for u and F, and we will assume it here).

   Given the optimal decision rule $k' = g(k)$, the value function solves the following functional equation:

$$v(k) = u(F(k) - g(k)) + \beta v(g(k)).$$

   Differentiate this equation twice, evaluate it at the steady state k^*, and then solve for $v''(k^*)$. (Hint: Use the first-order condition for capital to eliminate some terms.) Then use the fact that v is strictly concave to argue that $(g'(k^*))^2 < \beta^{-1}$, implying that $g'(k^*)$ cannot be larger than β^{-1}.