Econ 511b (Part I)
Yale University
Spring 2004
Prof. Tony Smith

HOMEWORK #1

This homework assignment is due at the beginning of lecture on Friday, January 16.

1. Consider the dynamic programming problem

$$\max_{\{x_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t F(x_t, x_{t+1}), \quad \text{given } x_0,$$

subject to $x_{t+1} \in \Gamma(x_t)$. Let F be a quadratic function of its two arguments.

(a) Show that the Euler equation for this problem is a linear second-order difference equation. (You may assume that this problem has an interior solution.)

(b) Show that if one of the two eigenvalues associated with the second-order difference in part (a) is less than one in absolute value, then the other eigenvalue is larger than β^{-1} in absolute value.

2. **(a)** For the neoclassical growth model that we developed in lecture, show that the derivative of the policy function at the steady state can be found by differentiating the recursive (or functional) version of the Euler equation. In particular, show that this derivative (evaluated at the steady state) solves a quadratic equation whose two roots correspond to the two eigenvalues of the linearized second-order difference equation that we discussed in class.

(b) In the neoclassical growth model developed in lecture, assume that the felicity function u has constant elasticity of intertemporal substitution σ^{-1} and that $f(k) = k^{\alpha} + (1 - \delta)k$. Discuss how β, α, δ, and σ affect the speed of convergence to the steady state.