

Econ 510a (second half)
Yale University
Fall 2004
Prof. Tony Smith

HOMEWORK #2

This homework assignment is due at the beginning of the help session on Thursday, November 4.

1. Consider again the two-sector economy that you studied in the first problem on Homework #1, but replace the ad hoc behavioral assumptions with neoclassical assumptions. Specifically, imagine that a social planner seeks to maximize $\sum_{t=0}^{\infty} \beta^t u(c_t)$, given an initial condition $k_0 > 0$, subject to the technological constraints specified in the problem on Homework #1. Assume that u satisfies the usual regularity conditions. Note that although leisure is not valued (i.e., the total amount of labor supply L does not appear in the planner's objective), the planner must nonetheless decide in each period how to allocate L across the two sectors.
 - (a) Formulate the planner's optimization problem as a dynamic programming problem. Be sure to distinguish clearly between state variables and control (or choice) variables.
 - (b) Find a set of Euler equations and first-order conditions that an optimal solution to the planning problem must satisfy.
 - (c) Suppose that $F(K_{ct}, L_{ct}) = K_{ct}^{\alpha} L_{ct}^{1-\alpha}$, where $0 < \alpha < 1$, and that $F = G$. Use your answer from part (b) to find the steady state for this economy as a function of the structural parameters α , β , and δ .
 - (d) The solution to the planner's problem is a pair of decision rules describing how to allocate capital and labor across the two sectors given the current state variable(s). Use your answer from part (b) to find a pair of functional equations that determine the planner's two decision rules. (Hint: Once you have eliminated the derivative(s) of the value function, you will be left with one *intertemporal* equation—an Euler equation governing the consumption/savings tradeoff—and one *intra-temporal* equation governing the intersectoral allocation of labor at a point in time.)
 - (e) Show how you can use the two functional equations that you derived in part (d) to determine the derivatives of the two decision rules at the steady state. If the algebra gets too nasty, you do not need to solve explicitly for these derivatives as a function of primitives (that is, simply display the equations that determine the two derivatives but do not solve them for the derivatives).

2. Consider a neoclassical growth model similar to the one that we have discussed in lecture, but in which the level of technology oscillates deterministically between two values A_H and A_L , where $A_H > A_L$. In particular, period- t output y_t equals $A_H F(k_t)$ if t is even and equals $A_L F(k_t)$ if t is odd. The planner seeks to maximize $\sum_{t=0}^{\infty} \beta^t u(c_t)$, given $k_0 > 0$, subject to $c_t + k_{t+1} = y_t + (1 - \delta)k_t$ and $k_{t+1} \geq 0$ for all t .
- (a) Formulate the planner's problem recursively. (Hint: Consider two value functions, one for periods in which the level of technology is high and one for periods in which the level of technology is low. Find a pair of Bellman equations that these functions must satisfy.)
- (b) Let the felicity function u be logarithmic, let $F(k_t) = k_t^\alpha$, and assume that capital depreciates fully in one period (i.e., set $\delta = 1$). Use a guess-and-verify method to find the two value functions in part (a). Describe fully the dynamic behavior of the capital stock starting from any initial condition for the capital stock.
3. Consider an exchange economy with two (types of) consumers named A and B . The two consumers have identical preferences: they each value consumption streams according to $\sum_{t=0}^{\infty} \beta^t u(c_t)$, where u has a constant elasticity of intertemporal substitution σ^{-1} . Consumer i 's endowment of consumption goods is $\{\omega_{it}\}_{t=0}^{\infty}$, $i = A, B$. Consumption goods are perishable (i.e., they cannot be stored and used for consumption in future periods).
- (a) Carefully define a competitive equilibrium with date-0 trading for this economy.
- (b) Suppose that $\omega_{At} = 2$ for all t and $\omega_{Bt} = 1$ for all t . Find the competitive equilibrium allocations and prices.
- (c) Suppose now that the endowments fluctuate deterministically: consumer A 's endowment stream is $\{2, 1, 2, 1, 2, 1, \dots\}$ and consumer B 's endowment stream is $\{1, 2, 1, 2, 1, 2, \dots\}$. Find the competitive equilibrium allocations and prices. (Hint: Guess that each consumer's consumption is constant across time and verify that this guess is correct.)
- (d) In parts (b) and (c) there is no variation in the *aggregate* endowment across time. Suppose that, as in part (b), consumer A 's endowment is 2 in every period but that consumer B 's endowment fluctuates: his endowment stream is $\{1, 3, 1, 3, 1, 3, \dots\}$. Find the competitive equilibrium allocations and prices. To simplify the algebra, set $\sigma = 1$ (i.e., let the felicity function u be logarithmic).
- (e) Carefully define a competitive equilibrium with sequential trading for this economy. Use your results from parts (b), (c), and (d) to determine the equilibrium interest rates for each pair of endowment streams. In addition, for each case

determine how each consumer's asset holdings vary over time (assume that each consumer starts with zero assets in period 0).