Econ 510a (second half)
Yale University
Fall 2006
Prof. Tony Smith

HOMEWORK #2

This homework assignment is due at 5PM on Friday, November 10 in Marnix Amand's mailbox.

1. Consider a consumer with the following sequence of budget constraints:

$$c_t + a_{t+1} = R_t a_t + w_t, \quad t = 0, 1, 2, \ldots,$$

where $w_t \geq 0$ is the consumer's labor income in period t. The consumer's consumption cannot be negative. The period-t gross interest rate R_t is greater than 1 for all t.

(a) Find the no-Ponzi-game restriction and use it to derive a consolidated, or intertemporal, budget constraint for the consumer. Interpret your answer.

(b) Suppose that $w_t = w > 0$ and $R_t = R > 1$ for all t. Show that the nPg restriction is equivalent to imposing a constraint that the consumer's asset holdings never fall below a fixed amount B, where B is allowed to be negative. In other words, show that there is a borrowing limit B such that the set of feasible consumption levels defined by the sequential budget constraints and the nPg restriction is identical to the set of feasible consumption levels defined by the sequential budget constraints and the borrowing constraint $a_{t+1} \geq B$ for all t. Express B in terms of R and w. (Hint: Imagine a consumer who has zero consumption and whose asset holdings do not change over time.)

(c) Suppose again that w_t and R_t are time-varying. What would the borrowing constraint have to look like in order to obtain a result like the one in part (b)?

2. Consider a consumer with the following optimization problem:

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t), \quad \text{given } a_0 > 0,$$

subject to the sequence of budget constraints in the first problem and a no-Ponzi-game restriction. The felicity function u is strictly increasing, strictly concave, twice continuously differentiable, and satisfies the Inada condition $\lim_{c \to 0} u'(c) = \infty$.

(a) Find the transversality condition for this problem. Show that the nPg restriction is met if the transversality condition and the Euler equation are both satisfied.

(b) Modify the proof of Proposition 3.4 on p. 23 of the lecture notes by Per Krusell to prove that a sequence $\{a_t^*\}_{t=0}^{\infty}$ that satisfies the transversality condition and the Euler equation maximizes the consumer's objective, subject to the sequence of budget constraints and the nPg restriction. (Note that the proposition in the lecture notes imposes the requirement that the consumer's asset holdings be nonnegative in each period; in this problem, we are imposing instead the nPg restriction.) Before considering the general case in which labor income and the interest vary over time, you might want to study the special case in which they are constant.

3. Consider an exchange economy with two consumers named A and B. The two consumers have identical preferences: they each value consumption streams according to $\sum_{t=0}^{\infty} \beta^t \log(c_t)$. Consumer i's endowment of consumption goods is $\{\omega_{it}\}_{t=0}^{\infty}$, $i = A, B$. Consumption goods are perishable (i.e., they cannot be stored and used for consumption in future periods).

(a) Carefully define a competitive equilibrium with date-0 trading for this economy.

(b) Suppose that $\omega_{At} = 4$ for all t and $\omega_{Bt} = 1$ for all t. Find the competitive equilibrium allocations and prices.

(c) Suppose now that the endowments fluctuate deterministically: consumer A's endowment stream is $\{4, 1, 4, 1, 4, 1, \ldots\}$ and consumer B's endowment stream is $\{1, 4, 1, 4, 1, 4, \ldots\}$. Find the competitive equilibrium allocations and prices.

(d) In parts (b) and (c) there is no variation in the *aggregate* endowment across time. Suppose that, as in part (b), consumer A's endowment is 4 in every period but that consumer B's endowment fluctuates: his endowment stream is $\{1/2, 2, 1/2, 2, 1/2, 2, \ldots\}$. Find the competitive equilibrium allocations and prices.

(e) The social planning problem for this economy is:

$$\max_{\{c_{At}\}_{t=0}^{\infty}, \{c_{Bt}\}_{t=0}^{\infty}} \left\{ \alpha^A \sum_{t=0}^{\infty} \beta^t \log(c_{At}) + \alpha^B \sum_{t=0}^{\infty} \beta^t \log(c_{Bt}) \right\},$$

subject to the resource constraint $c_{At} + c_{Bt} = \omega_{At} + \omega_{Bt}$ for all t. The numbers α^A and α^B are called *Negishi weights*. For each of the pairs of endowment streams in parts (b), (c), and (d), show that the consumption allocation chosen by the planner coincides with the allocation that arises in competitive equilibrium, provided that the weight α^i is set equal to the inverse of consumer i's marginal utility of income in competitive equilibrium.

(f) Carefully define a competitive equilibrium with sequential trading for this economy. Use your results from parts (b), (c), and (d) to determine the equilibrium interest rates for each pair of endowment streams. In addition, for each case determine how each consumer's asset holdings vary over time (assume that each consumer starts with zero assets in period 0).