Econ 511b (Part I)
Yale University
Spring 2004
Prof. Tony Smith

HOMEWORK #2

This homework assignment should be handed in by 5PM on Friday, January 23 to Jinhui Bai's mailbox in the basement of 28 Hillhouse.

1. Consider the planning problem for a simple finite-horizon neoclassical growth model:

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t \log(c_t),$$

given $k_0 = 10$ and subject to the constraint that $c_t + k_{t+1} = Ak_t^\alpha + (1-\delta)k_t$. Set $\beta = 0.95$, $\delta = 0.1$, and $\alpha = 0.4$. Choose A so that the steady-state value of capital in the corresponding infinite-horizon model is 100.

Solve the model numerically (say, in Matlab) using the "shooting" method described in lecture on January 14: start by guessing a value for k_1, solve for k_2 from the Euler equation at time 0, then solve for k_3 from the Euler equation at time 1, and so on, until k_{T+1} is found. Then vary k_1 and repeat until the appropriate value of k_{T+1} (i.e., 0) is found. Find the lowest value for T such that the highest value of capital between periods 0 and T exceeds 90.

2. Consider a neoclassical growth model with two sectors, one producing consumption goods and one producing investment goods. Consumption is given by $C_t = F(K_{Ct}, L_{Ct})$ and investment is given by $I_t = G(K_{It}, L_{It})$, where K_{jt} is the amount of capital in sector j at the beginning of period t and L_{jt} is the amount of labor used in sector j in period t. The total amount of labor in each period is equal to L (leisure is not valued). Labor can be freely allocated in each period between the two sectors: $L = L_{Ct} + L_{It}$. Capital, on the other hand, is sector-specific: once it is installed in a given sector, it cannot be moved to the other sector. Investment goods, however, can be used to augment the capital stock in either sector. In particular, the capital stocks in the two sectors evolve according to:

$$K_{j,t+1} = (1-\delta)K_{jt} + I_{jt}, \quad j = C, I,$$

where $I_t = I_{Ct} + I_{It}$.

The social planner seeks to maximize $\sum_{t=0}^{\infty} \beta^t u(C_t)$, given K_{C0} and K_{I0}, subject to the constraints on technology. Note that although leisure is not valued (i.e., the total amount of labor supply L does not appear in the planner's objective), the planner must nonetheless decide in each period how to allocate L across the two sectors.

(a) Formulate the planner's optimization problem as a dynamic programming problem. Be sure to distinguish clearly between state variables and control (or choice) variables.

(b) Find a set of Euler equations and first-order conditions that an optimal solution to the planning problem must satisfy.

(c) Suppose that $F(K_{Ct}, L_{Ct}) = K_{Ct}^{\alpha} L_{Ct}^{1-\alpha}$ and $G(K_{It}, L_{It}) = K_{It}^{\gamma} L_{It}^{1-\gamma}$. Use your answer from part (b) to find the steady state for this economy as a function of the structural parameters.

3. Consider an exchange economy with two consumers named A and B. The two consumers have identical preferences: they each value consumption streams according to $\sum_{t=0}^{\infty} \beta^t u(c_t)$, where u has a constant elasticity of intertemporal substitution σ^{-1}. Consumer i's endowment of consumption goods is $\{\omega_{it}\}_{t=0}^{\infty}$, $i = A, B$. Consumption goods are perishable (i.e., they cannot be stored and used for consumption in future periods).

(a) Carefully define a competitive equilibrium with date-0 trading for this economy.

(b) Suppose that $\omega_{At} = 3$ for all t and $\omega_{Bt} = 1$ for all t. Find the competitive equilibrium allocations and prices.

(c) Suppose now that the endowments fluctuate deterministically: consumer A's endowment stream is $\{3, 1, 3, 1, 3, 1, \ldots\}$ and consumer B's endowment stream is $\{1, 3, 1, 3, 1, 3, \ldots\}$. Find the competitive equilibrium allocations and prices. (Hint: Guess that each consumer's consumption is constant across time and verify that this guess is correct.)

(d) In parts (b) and (c) there is no variation in the *aggregate* endowment across time. Suppose that, as in part (b), consumer A's endowment is 3 in every period but that consumer B's endowment fluctuates: his endowment stream is $\{1/2, 3/2, 1/2, 3/2, 1/2, 3/2, \ldots\}$. Find the competitive equilibrium allocations and prices. To simplify the algebra, set $\sigma = 1$ (i.e., let the felicity function u be logarithmic).

(e) Carefully define a competitive equilibrium with sequential trading for this economy. Use your results from parts (b), (c), and (d) to determine the equilibrium interest rates for each pair of endowment streams. In addition, for each case determine how each consumer's asset holdings vary over time (assume that each consumer starts with zero assets in period 0).