Econ 510a (second half)
Yale University
Fall 2004
Prof. Tony Smith

HOMEWORK #3

This homework assignment is due at the beginning of the help session on Thursday, November 11.

1. Consider a neoclassical growth model with valued leisure. The (representative) consumer values streams of consumption and leisure according to $\sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t)$, where ℓ_t is hours of leisure in period t. The felicity function u is strictly concave and strictly increasing in both of its arguments. Output is produced according to $y_t = F(k_t, n_t)$, where $n_t = L - \ell_t$ is hours of labor supply in period t (L is the total number of hours available for either leisure or work in period t) and F exhibits constant returns to scale. Output in period t can used for either consumption c_t or investment i_t, and capital accumulates according to $k_{t+1} = (1 - \delta)k_t + i_t$.

(a) Display Bellman's equation for the problem faced by the social planner in this economy. Identify clearly the state and control (or choice) variable(s).

(b) Derive the first-order and envelope conditions for the planning problem.

(c) Use the conditions from part (b) to determine the economy's steady state. Show how the steady state depends on primitives and compare your results to those for a growth model without valued leisure.

(d) Let $F(k, n) = k^\alpha n^{1-\alpha}$ and

$$u(c, \ell) = \frac{(c^\theta \ell^{1-\theta})^{1-\sigma} - 1}{1 - \sigma},$$

where $\sigma > 0$ and $0 < \theta < 1$. Solve explicitly for the steady state in terms of parameters.

2. Consider an exchange economy populated by identical consumers whose preferences exhibit "habit persistence":

$$\sum_{t=0}^{\infty} \beta^t \frac{(c_t - \lambda c_{t-1})^{1-\sigma} - 1}{1 - \sigma},$$

where $\sigma > 0$, $\beta \in (0, 1)$, and λ is positive and bounded. Each consumer has the same endowment ω_t in period t. Assume that ω_t grows deterministically according to: $\omega_{t+1} = g\,\omega_t$, with $g > 1$. There is a single asset, a one-period riskless bond whose price is q_t in period t. A one-period riskless bond, purchased at price q_t in period t, pays one unit of the consumption good in period $t+1$. That is, the consumer's budget constraint in period t takes the form: $c_t + q_t b_{t+1} = b_t + \omega_t$, where b_t is the quantity of bonds purchased in period $t-1$. (Note: The price q_t can be viewed as the inverse of the gross interest rate.) There is no restriction on borrowing except for a no-Ponzi-game condition. Assume that each consumer has zero bonds in period 0.

- **(a)** Carefully define a sequential competitive equilibrium for this economy.
- **(b)** Formulate the consumer's optimization problem as a dynamic programming problem under the conjecture that the price $q_t = q$ for all t. (You will verify below that this conjecture is correct in equilibrium.)
- **(c)** Derive the Euler equation for the consumer's problem.
- **(d)** Use your answer from part (c) to find the equilibrium bond price as a function of the structural parameters.

3. Consider an exchange economy with two (types of) consumers. Type-A consumers comprise fraction λ of the economy's population and type-B consumers comprise fraction $1-\lambda$ of the economy's population. Each consumer has (constant) endowment ω in each period. A consumer of type i has preferences over consumption streams of the form $\sum_{t=0}^{\infty} \beta_i^t u(c_t)$. Assume that $1 > \beta_A > \beta_B > 0$: type-$A$ consumers are more patient than type-B consumers. Consumers trade a one-period riskless bond in each period (see the second problem for the definition of a one-period riskless bond). There is no restriction on borrowing except for a no-Ponzi-game condition. Assume that each consumer has zero bonds in period 0.

- **(a)** Carefully define a sequential competitive equilibrium for this economy. (Hint: The market-clearing condition can be written $\lambda c_{At} + (1-\lambda) c_{Bt} = \omega$, where c_{it} is the consumption of a typical type-i consumer in period t.)
- **(b)** Show that this economy has no steady state: in particular, show that the type-B consumers become poorer and poorer over time and consume zero in the limit. (Hint: Use the strict concavity of the felicity function u.)