Econ 510a (second half)
Yale University
Fall 2007
Prof. Tony Smith

HOMEWORK #3

This homework assignment is due at 4PM on Friday, November 16. Please put your assignment in Evrim Aydin's mailbox.

1. In lecture on Monday, November 5, we studied a two-period exchange economy with two types of consumers, A and B, who differ in their endowment streams. This problem considers several variations on this economy and asks you to ascertain in each case whether the first welfare theorem holds (i.e., whether the competitive equilibrium allocation is Pareto optimal).

   (a) Suppose that, in each period, a government taxes the consumption of a typical type-i consumer at rate τ^i. The government uses its tax proceeds in each period to give each consumer an equal lump-sum transfer of the consumption good. A typical consumer's lifetime budget constraint, therefore, reads:

   $$p_0(1+\tau^i)c_0^i + p_1(1+\tau^i)c_1^i = p_0\omega_0^i + p_1\omega_1^i + p_0\bar{S}_0 + p_1\bar{S}_1,$$

   where $\bar{S}_t$ is the lump-sum transfer in period t. The left-hand side of the budget constraint is the consumer's total expenditures (including taxes that he pays, which in turn depend on how much he consumes); the right-hand side is the consumer's total resources (including lump-sum transfers from the government). The government's budget constraint in period t is:

   $$\bar{S}_t = \theta\,\tau^A\,\bar{c}_t^A + (1-\theta)\,\tau^B\,\bar{c}_t^B,$$

   where $\bar{c}_t^i$ is the equilibrium consumption of a typical type-i consumer in period t. Although, in equilibrium, all consumers of a given type will choose the *same* level of consumption in period t, each consumer is free to choose *any* consumption level when solving his individual maximization problem, taking as given both prices and the behavior of other consumers in the economy (i.e., $\bar{c}_t^A$ and $\bar{c}_t^B$).

   For this economy, you have two tasks. First, carefully define a competitive equilibrium and display a set of conditions that determine the competitive equilibrium allocations and prices. Second, determine whether the competitive equilibrium allocation is Pareto optimal and justify your answer appropriately.

(b) Suppose instead that the government imposes both date- and type-specific taxes on consumption: the tax rate on the consumption of a type-i consumer in period t is τ_t^i. As in part (a), the government's tax proceeds are used to fund equal lump-sum transfers to consumers. For this economy, complete the same two tasks as in part (a).

(c) Suppose now that the government eliminates consumption taxes but instead imposes both date- and type-specific taxes on consumers' *endowments*: in period t, the government confiscates fraction τ_t^i of the endowment of a typical type-i consumer. As in parts (a) and (b), the government's tax proceeds are used to fund equal lump-sum transfers to consumers. For this economy, complete the same two tasks as in part (a).

(d) Eliminate government altogether and instead assume that consumers' preferences exhibit *consumption externalities*: each consumer's utility in period t is *increasing* in his own consumption, c_t^i, but *decreasing* in average (or per capita) consumption, $\theta\bar{c}_t^A+(1-\theta)\bar{c}_t^B$. Recall that each consumer takes $\bar{c}_t^A$ and $\bar{c}_t^B$ as *given* when choosing his own consumption c_t^i. For such an economy, complete the same two tasks as in part (a).

2. For a neoclassical growth model in which consumers value leisure, carefully define:

   (a) a competitive equilibrium with date-0 trading;

   (b) a competitive equilibrium with sequential trading; and

   (c) a recursive competitive equilibrium (hint: you need *two* functions to describe the behavior of the aggregate economy).

   In addition, use the the recursive competitive equilibrium formulation to show that the competitive equilibrium allocation is Pareto optimal.

3. Consider a neoclassical growth model in which consumers live for only two time periods, 0 and 1. There are two types of consumers: fraction θ of consumers have initial capital holdings equal to $k_0^L > 0$ and fraction $1-\theta$ of consumers have initial capital holdings equal to $k_0^H > k_0^L$ (where 'L' and 'H' denote, 'low' and 'high', respectively).

   (a) Carefully define a competitive equilibrium for this economy (assume that consumers have time-separable preferences that depend on consumption but not on leisure).

   (b) Show that if the felicity function u exhibits constant relative risk aversion, then redistributions of the initial endowments of capital (i.e., changes in k_0^L and k_0^H that leave the total amount of capital in period 0 unchanged) have no effect either on equilibrium aggregate savings in period 0 or on equilibrium prices in either period.

This is a version of an *aggregation theorem* for this economy: holding the total amount of capital in period 0 constant, the behavior of the aggregates in this economy does not depend on the distribution of capital in period 0.

(c) Suppose now that consumers do value leisure. Can you find a felicity function (that now depends on both consumption and leisure) for which the aggregation theorem in part (b) holds? How large is the class of such felicity functions?