Econ 511b (Part I)
Yale University
Spring 2004
Prof. Tony Smith

HOMEWORK #3

This homework assignment should be handed in by 5PM on Friday, January 30 to Jinhui Bai's mailbox in the basement of 28 Hillhouse.

1. Consider an exchange economy with two (types of) consumers. Type-A consumers comprise fraction λ of the economy's population and type-B consumers comprise fraction $1 - \lambda$ of the economy's population. Each consumer has (constant) endowment ω in each period. A consumer of type i has preferences over consumption streams of the form $\sum_{t=0}^{\infty} \beta_i^t \log(c_t)$. Assume that $1 > \beta_A > \beta_B > 0$: type-$A$ consumers are more patient than type-B consumers. Consumers trade a risk-free bond in each period. There is no restriction on borrowing except for a no-Ponzi-game condition. Each consumer has zero assets in period 0.

   (a) Carefully define a sequential competitive equilibrium for this economy.

   (b) Carefully define a recursive competitive equilibrium for this economy.

   (c) Show that this economy has no steady state: in particular, show that the type-B agents become poorer and poorer over time and consume zero in the limit.

2. Consider an infinite-horizon one-sector growth model with an externality in production. Leisure is not valued and the (representative) consumer has time-separable preferences with discount factor $\beta \in (0, 1)$. Consumers own the factors of production. Capital depreciates at rate δ. There is a large number of identical firms each of which has the following production technology:

   $$F(k, \ell, \bar{k}) = Ak^{\alpha}\ell^{1-\alpha}\bar{k}^{\gamma},$$

   where k is the capital rented by the firm, $\bar{k}$ is the aggregate capital stock, and the parameters α and γ satisfy $0 < \gamma < 1 - \alpha$ and $\alpha \in (0, 1)$. Thus there is a productive externality from the rest of the economy: a higher aggregate capital stock increases the firm's productivity. A typical (small) firm takes the aggregate capital stock as given when choosing its inputs.

   (a) Define a recursive competitive equilibrium for this economy. Be clear about which variables consumers and firms take as given when they solve their optimization

problems. Find the competitive equilibrium steady-state aggregate capital stock as a function of primitives.

(b) Write the planning problem for this economy in recursive form. The planner internalizes the externality in production: his production technology is

$$F(\bar{k}, \ell, \bar{k}) = A\bar{k}^{\alpha+\gamma}\ell^{1-\alpha}.$$

Find the steady-state aggregate capital stock implied by the planning problem. Show that it is higher than the competitive equilibrium steady-state aggregate capital stock.

(c) Now introduce a government into the competitive equilibrium that you defined in part (a). The government subsidizes investment expenditures at a proportional rate τ and finances these subsidies by means of a lump-sum tax on consumers. The investment subsidy is constant across time but the lump-sum tax varies over time so as to balance the government's budget in every period. Define a recursive competitive equilibrium for this economy.

(d) For what subsidy rate τ is the competitive equilibrium steady-state aggregate capital stock the same as the steady-state aggregate capital stock in the planning problem?

3. Consider a neoclassical growth model with logarithmic utility, Cobb-Douglas production, full depreciation of the capital stock in one period, and inelastic labor supply (leisure is not valued). In this problem, you will solve explicitly for the recursive competitive equilibrium of this economy (assuming that the economy is decentralized in the same manner as in the second problem).

(a) Suppose that aggregate capital evolves according to $\bar{k}' = G(\bar{k}) = sf(\bar{k}, 1)$, where f is the economy's production function. (Why is this a reasonable conjecture?) Find explicit formulas for the value function $v(k, \bar{k})$ and the decision rule $k' = g(k, \bar{k})$ of a "small" (or typical) consumer who takes the law of motion of aggregate capital as given. The functions v and g depend on s as well as on primitives of technology and preferences.

(b) Find the competitive equilibrium value of s by imposing the consistency condition $G(\bar{k}) = g(\bar{k}, \bar{k})$. Verify that the resulting law of motion for aggregate capital solves the planning problem for this economy. Display v and g for the equilibrium value of s.

(c) How does an increase in aggregate capital affect the savings behavior and the (indirect) utility of a typical consumer (holding fixed the consumer's holdings of capital)?

(d) How does the equilibrium utility of a typical consumer vary with aggregate capital (taking into account that the consumer's holdings of capital equal aggregate capital in equilibrium)?