Econ 510a (second half)
Yale University
Fall 2006
Prof. Tony Smith

HOMEWORK #4

This homework assignment is due at 5PM on Friday, December 1 in Marnix Amand's mailbox.

1. Consider a two-period exchange economy with two types of consumers of equal measure. Each consumer maximizes $u(c_0^i) + \beta E[u(c_1^i)]$, where c_t^i, $i = 1, 2$, is the consumption of a type-i consumer in period t. In period 0, type-i consumers are endowed with ω_0^i units of the (nonstorable) consumption good. Endowments in period 1 are random: with probability π_j, $j = 1, 2$, a type-i consumer receives ω_{1j}^i units of the consumption good in period 1. In period 0, consumers trade Arrow securities whose payoffs depend on the state of the world in period 1.

(a) Carefully define a competitive equilibrium for this economy. Are markets complete? Explain why or why not.

(b) Suppose that $\bar{\omega}_0 = \bar{\omega}_{11} = \bar{\omega}_{12}$, where $\bar{\omega}_0 \equiv \sum_{i=1}^2 \omega_0^i$ and $\bar{\omega}_{1j} \equiv \sum_{i=1}^2 \omega_{1j}^i$. Prove an aggregation theorem for this economy: that is, show that redistributions of the period-0 endowments do not affect the equilibrium prices of the Arrow securities. (Note that a redistribution leaves the aggregate endowment unchanged.) In addition, characterize the equilibrium consumption allocation as much as possible.

(c) Now suppose that $\bar{\omega}_0 \neq \bar{\omega}_{11} \neq \bar{\omega}_{12}$. Prove an aggregation theorem for this economy under the assumption that u has a constant elasticity of intertemporal substitution equal to σ^{-1}. In addition, characterize the equilibrium consumption allocation as much as possible.

(d) Now suppose that in period 0 consumers cannot trade Arrow securities but instead can trade only a riskfree bond (i.e., a sure claim to one unit of the consumption good in period 1). Carefully define a competitive equilibrium for this economy.

(e) For the economy in part (d), set $\omega_0^1 = \omega_0^2$, $\omega_{11}^1 = \omega_{12}^2$, $\omega_{12}^1 = \omega_{11}^2$, and $\pi_1 = \pi_2 = 1/2$, but do not assume a functional form for u. Find the equilibrium consumption allocation and the equilibrium bond price. How does the bond price compare to the one in part (c)? Explain.

2. Consider an exchange economy with two (types of) consumers. Type-A consumers comprise fraction λ of the economy's population and type-B consumers comprise fraction $1 - \lambda$ of the economy's population. Each consumer has (constant) endowment ω

in each period. A consumer of type i has preferences over consumption streams of the form $\sum_{t=0}^{\infty} \beta_i^t u(c_t)$. Assume that $1 > \beta_A > \beta_B > 0$: type-$A$ consumers are more patient than type-B consumers. Consumers trade a one-period riskfree bond in each period There is no restriction on borrowing except for a no-Ponzi-game condition. Assume that each consumer has zero bonds in period 0.

(a) Carefully define a sequential competitive equilibrium for this economy.

(b) Show that this economy has no steady state: in particular, show that the type-B consumers become poorer and poorer over time and consume zero in the limit.

3. Consider an exchange economy with two infinitely-lived consumers with identical preferences given by:

$$E\left(\sum_{t=0}^{\infty} \beta^t \log(c_t)\right).$$

Both of the consumers have random endowments that depend on an (exogenous) sequence of state variables $\{s_t\}_{t=0}^{\infty}$. The s_t's are statistically independent random variables with identical probability distributions. Specifically, for each t, $s_t = H$ with probability π and $s_t = L$ with probability $1 - \pi$, where π does not depend on time or on the previous realization of states. If $s_t = H$, then the first consumer's endowment is 2 and the second consumer's endowment is 1; if $s_t = L$, then the first consumer's endowment is 1 and the second consumer's endowment is 0. Markets are complete.

(a) Carefully define a competitive equilibrium with date-0 trading for this economy. (Assume that consumers make decisions before observing the realization of the state in period 0.)

(b) Determine the competitive equilibrium allocation in terms of primitives.

(c) Determine the prices of the Arrow securities in terms of primitives.

(d) Use your answer from part (c) to determine the average rate of return on a (one-period) riskfree bond in this economy.

4. Read the following writings by Robert E. Lucas, Jr. on the methodology of modern macroeconomics:

(a) "What Economists Do" (available on the course web site).

(b) "Methods and Problems in Business Cycle Theory" (available on the course web site).

(c) Sections 1–4 and 6–7 of "Econometric Policy Evaluation: A Critique" (handed out in lecture).