Econ 510a (second half)
Yale University
Fall 2007
Prof. Tony Smith

HOMEWORK #4

This homework assignment is due at 4PM on Friday, November 30. Please put your assignment in Evrim Aydin's mailbox.

1. In lecture on Wednesday, November 14, we studied an overlapping generations model in which consumers live for two periods and are endowed with one unit of the consumption good when young and zero units when old. A consumer born in period t values consumption goods according to: $c_{1t} + c_{2,t+1}$, where c_{1t} is his consumption when young and $c_{2,t+1}$ is his consumption when old.

   Suppose that "fiat" money is introduced into this economy. In particular, the initial old in period 0 are endowed with $M > 0$ units of money. In period t, consumers can exchange q_t units of the date-t consumption good for one unit of money (or, equivalently, $1/q_t$ units of money for one unit of the date-t consumption good); in other words, q_t is the price in period t of a unit of money in terms of date-t consumption goods. A consumer born in period t solves the following problem:

$$\max_{c_{1t}, c_{2,t+1}, m_{t+1}} \quad c_{1t} + c_{2,t+1}$$

   subject to:

$$\begin{aligned} c_{1t} + q_t m_{t+1} &= 1 \\ c_{2,t+1} &= q_{t+1} m_{t+1} \\ m_{t+1} &\geq 0, \end{aligned}$$

   where m_t is the amount of money that a consumer owns at the beginning of period t. Because the initial old in period 0 are endowed with M units of money, the consumption of the initial old is given by $c_{20} = q_0 M$.

   A competitive equilibrium in this economy is a set of allocations c_{20} and $\{(c_{1t}, c_{2,t+1})\}_{t=0}^{\infty}$ and a sequence of prices $\{q_t\}_{t=0}^{\infty}$ such that: one, consumers behave optimally taking prices as given; two, the goods market clears in every period: $c_{1t} + c_{2t} = 1$ for $t \geq 0$; and, three, the money market clears in every period: $m_t = M$ for $t \geq 0$.

(a) Show that this economy has a multiplicity of competitive equilibria. Specifically, show that, for any $q \in [0, M^{-1}]$, $q_t = q$ for all $t \geq 0$ is a competitive equilibrium sequence of prices.

(b) For which (if any) of the equilibria in part (a) is the competitive equilibrium allocation of consumption Pareto optimal?

2. Consider a Diamond overlapping-generations economy in which consumers supply (inelastically) one unit of labor when young and $\lambda \in [0, 1]$ units of labor when old. The preferences of a consumer born in period t are given by:

$$\log(c_{1t}) + \beta \log(c_{2,t+1}),$$

where c_{1t} is consumption when young, $c_{2,t+1}$ is consumption when old, and $\beta \in (0, 1)$. Assume that the aggregate production function is Cobb-Douglas with exponent on capital equal to $\alpha \in (0, 1)$.

(a) Find the equilibrium law of motion of the aggregate capital stock.

(b) Show that the steady-state of this economy is dynamically efficient provided that λ is greater than some "cutoff" value $\lambda^* \in (0, 1)$.

3. Consider a Diamond overlapping-generations economy like the one described in the second problem, but set $\lambda = 0$. Introduce a "pay-as-you-go" social security system into this economy: in every period, the government collects d units of the consumption good from the young and gives the proceeds of this lump-sum tax to the old. Suppose that d is zero initially and that the government increases d by a small amount. Show that this increase reduces the steady-state aggregate capital stock in competitive equilibrium. Under what conditions can the introduction of a pay-as-you-go social security system be Pareto-improving (i.e., under what conditions can it improve the welfare of all generations, including the initial old)?