Econ 511b (Part I)
Yale University
Spring 2004
Prof. Tony Smith

**HOMEWORK #4**

*This homework assignment should be handed in by 5PM on Friday, February 6 to Jinhui Bai's mailbox in the basement of 28 Hillhouse.*

**1.** Consider a neoclassical growth model with valued leisure. The (representative) consumer values streams of consumption and leisure according to $\sum_{t=0}^{\infty} \beta^t U(c_t, \ell_t)$, where $\ell_t$ is hours of leisure in period $t$. The felicity function $U$ is strictly concave and strictly increasing in both of its arguments. Output is produced according to $y_t = F(k_t, n_t)$, where $n_t = L - \ell_t$ is hours of labor supply in period $t$ ($L$ is the total number of hours available for either leisure or work in period $t$) and $F$ exhibits constant returns to scale. Output in period $t$ can used for either consumption $c_t$ or investment $i_t$ and capital accumulates according to $k_{t+1} = (1 - \delta)k_t + i_t$.

**(a)** Display Bellman's equation for the problem faced by the social planner in this economy. Identify clearly the state and control (or choice) variable(s).

**(b)** Derive the first-order and envelope conditions for the planning problem.

**(c)** Use the conditions from part (b) to determine the economy's steady state. Show how the steady state depends on primitives and compare your results to those for a growth model without valued leisure.

**(d)** Let $F(k, n) = k^\alpha n^{1-\alpha}$ and $U(c, \ell) = \lambda \log(c) + (1 - \lambda) \log(\ell)$, where $0 < \alpha < 1$ and $0 < \lambda < 1$. Solve explicitly for the steady state in terms of parameters.

**2.** **(a)** Carefully define a recursive competitive equilibrium for the neoclassical growth model with valued leisure. (Hint: You need *two* functions to describe the behavior of the aggregate economy.)

**(b)** Find the (functional) first-order conditions of a typical consumer who takes as given the economy's aggregate laws of motion.

**(c)** Impose equilibrium conditions on the first-order conditions from part (b) and verify that the resulting equations are identical to the first-order conditions associated with the planning problem for this economy.

**3.** Consider a competitive equilibrium one-sector growth model without valued leisure in which consumers own capital and labor and rent their services to firms. There is no uncertainty and the felicity function of a typical consumer has constant elasticity of intertemporal substitution $\sigma^{-1}$.

**(a)** Show that the decision rule of a typical consumer takes the form:

$$k' = \mu(\bar{k}) + \lambda(\bar{k})k,$$

where the functions $\mu$ and $\lambda$ satisfy the following pair of functional equations (i.e., these two equations must hold for all values of $\bar{k}$):

$$\mu(\bar{k}) + \frac{w(\bar{k}') - \mu(\bar{k}')}{r(\bar{k}') - \lambda(\bar{k}')} = \frac{w(\bar{k}) - \mu(\bar{k})}{r(\bar{k}) - \lambda(\bar{k})}\lambda(\bar{k})$$

and

$$\frac{1}{\beta r(\bar{k}')} = \left(\frac{(r(\bar{k}') - \lambda(\bar{k}'))\lambda(\bar{k})}{r(\bar{k}) - \lambda(\bar{k})}\right)^{-\sigma},$$

where $\bar{k}' \equiv \mu(\bar{k}) + \lambda(\bar{k})\bar{k}$. In these equations, $k$ is the individual's holdings of capital, $\bar{k}$ is aggregate capital, $r(\bar{k})$ is the rental rate of capital plus one minus the depreciation rate, and $w(\bar{k})$ is the wage rate. (Hint: Obtain the Euler equation for a typical consumer, guess that the consumer's decision rule takes the conjectured form, and then find restrictions that the "coefficients" $\mu(\bar{k})$ and $\lambda(\bar{k})$ must satisfy in order for the Euler equation to hold for all values of $k$ and $\bar{k}$.)

**(b)** Suppose now that there are two (types of) consumers in the economy who differ only in their initial capital holdings. Each consumer represents half of the economy's population. Use the result from part (a) to argue that a redistribution of capital (holding aggregate capital constant) across the two consumers at time 0 has no effect on equilibrium interest rates and wages.