Econ 510a (second half)
Yale University
Fall 2004
Prof. Tony Smith

HOMEWORK #5

This homework assignment is due at the beginning of the help session on Monday, November 29.

1. **(a)** Carefully define a recursive competitive equilibrium for the neoclassical growth model with valued leisure. (Hint: You need *two* functions to describe the behavior of the aggregate economy.)

(b) Find the (functional) first-order conditions of a typical consumer who takes as given the economy's aggregate laws of motion.

(c) Impose equilibrium conditions on the first-order conditions from part (b) and verify that the resulting equations are identical to the first-order conditions associated with the planning problem for this economy (see the first problem on Homework #3).

2. This problem introduces a "disaster" state (such as a Great Depression) into a Lucas "tree" economy like the one we have developed in lecture. Let the tree's dividend take on one of three values: $\bar{d}_1 = 1 + \delta$, $\bar{d}_2 = 1 - \delta$, and $\bar{d}_3 = 0.5$, where $\delta = 0.02$. Let the transition probability matrix for the dividend be specified as follows:

$$\begin{bmatrix} 0.98 & 0.02 - p & p \\ 0.02 - p & 0.98 & p \\ 0.5 & 0.5 & 0 \end{bmatrix},$$

where p is close 0. Thus the dividend usually fluctuates between $1 + \delta$ and $1 - \delta$ but occasionally drops by roughly 50% to 0.5. After such a crash, the dividend immediately returns to its "normal" range. The purpose of this problem is to investigate whether the introduction of a rare disaster state can improve the ability of the Lucas tree model to match the equity premium. (This idea was first proposed, in a slightly different form, by T.A. Rietz (1988), "The Equity Risk Premium: A Solution," *Journal of Monetary Economics* 22, 117–131. See also the critical response by R. Mehra and E.C. Prescott (1988), "The Equity Risk Premium: A Solution?", *Journal of Monetary Economics* 22, 133–136.)

(a) Read the notes on the Mehra-Prescott numerical example posted on the course web site! (Be sure to download the revised set of notes that I posted on Nov. 18.) You won't be able to do this problem until you have worked through these notes.

(b) Set $p = 0.001$. Compute the invariant distribution for the dividend. Use the invariant distribution to verify that the coefficient of variation of the dividend is close to 0.02, matching the standard deviation of the detrended log of aggregate consumption in U.S. data.

(c) Let the consumer's discount factor equal 0.998. Assume that the consumer has constant-relative-risk-aversion (CRRA) preferences, and let the coefficient of relative risk aversion equal 2. Compute the prices of all of the Arrow securities.

(d) Find the prices of a one-period risk-free bond and of a perpetual claim to the tree's dividend in each of the three states of the world.

(e) Compute (the unconditional expected value of) the equity premium. How close is it to 1.5%, its value in quarterly U.S. data? Find the value of the coefficient of relative risk aversion for which the theoretical equity premium matches its observed value.

(f) Using the preference parameters from part (c), compute the prices of a two-period risk-free bond in each of the three states of the world. (A two-period risk-free bond is a promise to pay 1 unit of the consumption good in all states of the world two periods from now.) Use your answer to compute the unconditional expected value of the rate of return on a two-period risk-free bond (call it r_2). How does $(1 + r_2)^{1/2} - 1$ compare to r_1, the unconditional expected value of the rate of return on a one-period risk-free bond? (Optional question: The difference between $(1+r_2)^{1/2}$ and r_1 is a measure of the slope of the term structure of interest rates. How does this slope compare to the slope in the two-state model that we developed in lecture?)

3. (a) Read Section 2.3 in the *Lecture Notes on Economic Growth* by Per Krusell. This section shows how to introduce exogenous labor-augmenting technological progress into the neoclassical growth model. Specifically, it shows how to transform the model into a stationary one that behaves "almost" like a neoclassical growth model without technological progress. In the transformed model, capital per efficiency unit of labor converges to a steady state. Consequently, in the original (untransformed) non-stationary model, all variables (capital, consumption, investment, and output) converge to a balanced growth path.

(b) Suppose that the government taxes capital income (net of depreciation) at rate τ and returns the proceeds to consumers in a lump-sum fashion (so as to balance its budget in every period). How does capital income taxation affect the economy's long-run rate of growth (i.e., the rate of growth along the balanced growth path)? (To answer this question, you must consider a decentralized version of the neoclassical growth model with labor-augmenting technological progress. For

simplicity, assume that leisure is not valued.)

(c) Suppose that the economy is on its balanced growth path and that the capital income tax rate τ is positive. Suppose now that the government suddenly (and unexpectedly) eliminates capital income taxation. Describe as fully as you can what happens to the rate of growth of output, both in the short run and in the long run.

4. Answer the questions in parts (b) and (c) of the third problem for an Ak model like the one we discussed in lecture on Wednesday, November 18.