Econ 510a (second half)
Yale University
Fall 2005
Prof. Tony Smith

HOMEWORK #5

This homework assignment is due at the beginning of class on Wednesday, November 30.

1. Consider an exchange economy populated by identical consumers whose preferences exhibit "habit persistence": a typical consumer's lifetime utility is given by

$$\sum_{t=0}^{\infty} \beta^t \, \frac{(c_t - \lambda c_{t-1})^{1-\sigma} - 1}{1-\sigma},$$

where $\sigma > 0$, $\beta \in (0,1)$, and λ is positive and bounded. Each consumer has the same endowment ω_t in period t. Assume, for simplicity, that ω_t grows deterministically according to: $\omega_{t+1} = g\,\omega_t$, where $g > 1$. There is a single asset, a one-period riskless bond (i.e., a sure claim to one unit of the consumption good in the next period) whose price is q_t in period t. There is no restriction on borrowing except for a no-Ponzi-game condition. Assume that each consumer has zero bonds in period 0.

(a) Carefully define a sequential competitive equilibrium for this economy.

(b) Formulate the consumer's optimization problem as a dynamic programming problem under the conjecture that the price $q_t = q$ for all t. (You will verify below that this conjecture is correct in equilibrium.)

(c) Derive the Euler equation for the consumer's problem.

(d) Use your answer from part (c) to find the equilibrium bond price in this model as an explicit function of the structural parameters (i.e., β, γ, g, and λ). Explain how changes in these parameters affect the bond price.

2. Consider a two-period exchange economy with identical consumers. Each consumer maximizes $u(c_0) + \beta E[u(c_1)]$, where c_t is consumption in period t. Each consumer is endowed in period 0 with one tree. Each tree yields one unit of the (nonstorable) consumption good in period 0 and a random amount of the consumption good in period 1. In particular, with probability π, each tree yields d_H units of the consumption good in period 1; with probability $1-\pi$, each tree yields d_L units of the consumption good in period 1, where $d_H > d_L$. (The trees are identical, so they all yield the same number of units of the consumption good—either d_H or d_L—in period 1.) In period 0, consumers trade two assets in competitive markets: trees and riskfree bonds (i.e., sure claims to one unit of the consumption good in period 1).

(a) Are markets complete in this economy? Explain why or why not.

(b) Carefully define a competitive equilibrium for this economy. Find the equilibrium consumption allocation and the equilibrium prices of a tree and of the riskfree bond.

(c) Now suppose that the market for trees is shut down so that consumers can trade only the riskfree bond. (Are markets complete in this case?) Carefully define a competitive equilibrium for this economy and show that the equilibrium consumption allocation and the equilibrium price of a riskfree bond are the same as in part (b). Explain intuitively why this result holds.

3. Consider a two-period exchange economy with two types of consumers of equal measure. Each consumer maximizes $u(c_0^i) + \beta E[u(c_1^i)]$, where c_t^i, $i = 1, 2$, is the consumption of a type-i consumer in period t. In period 0, type-i consumers are endowed with ω_0^i units of the (nonstorable) consumption good. Endowments in period 1 are random: with probability π_j, $j = 1, 2$, a type-i consumer receives ω_{1j}^i units of the consumption good in period 1. In period 0, consumers trade Arrow securities whose payoffs depend on the state of the world in period 1.

(a) Carefully define a competitive equilibrium for this economy. Are markets complete? Explain why or why not.

(b) Suppose that $\bar{\omega}_0 = \bar{\omega}_{11} = \bar{\omega}_{12}$, where $\bar{\omega}_0 \equiv \sum_{i=1}^{2} \omega_0^i$ and $\bar{\omega}_{1j} \equiv \sum_{i=1}^{2} \omega_{1j}^i$. Prove an aggregation theorem for this economy: that is, show that redistributions of the period-0 endowments do not affect the equilibrium prices of the Arrow securities. (Note that a redistribution leaves the aggregate endowment unchanged.) In addition, characterize the equilibrium consumption allocation as much as possible.

(c) Now suppose that $\bar{\omega}_0 \neq \bar{\omega}_{11} \neq \bar{\omega}_{12}$. Prove an aggregation theorem for this economy under the assumption that u has a constant elasticity of intertemporal substitution equal to σ^{-1}. In addition, characterize the equilibrium consumption allocation as much as possible.

(d) Now suppose that in period 0 consumers cannot trade Arrow securities but instead can trade only a riskfree bond (i.e., a sure claim to one unit of the consumption good in period 1). Carefully define a competitive equilibrium for this economy.

(e) For the economy in part (d), set $\omega_0^1 = \omega_0^2$, $\omega_{11}^1 = \omega_{12}^2$, $\omega_{12}^1 = \omega_{11}^2$, and $\pi_1 = \pi_2 = 1/2$, but do not assume a functional form for u. Find the equilibrium consumption allocation and the equilibrium bond price. How does the bond price compare to the one in part (c)? Explain.