HOMEWORK #5

This homework assignment is due at 5PM on Friday, December 8 in Marnix Amand’s mailbox.

1. (a) In the Mehra-Prescott model that we discussed in lecture on November 30, for what value of \( \gamma \) (the degree of risk aversion) does the model generate an average (annual) equity premium of 6%? (Set \( \beta = 0.99 \) and fix the other parameters at the values that we used in lecture.) For this degree of risk aversion, what is the average (annual) risk-free rate? Is it close to the historical average of 1%?

(b) In the Mehra-Prescott model, set \( \gamma = 1 \) (log utility) and \( \beta = 0.99 \), but suppose that the high-growth state lasts longer (on average) than the low-growth state: set \( \phi_{11} = 0.6 \) (but keep \( \phi_{22} = 0.43 \)). Recalibrate the values of \( \mu \) and \( \delta \) so that the average (annual) growth rate of per capita consumption is 0.018 and the standard deviation of the (annual) growth rate of per capita consumption is 0.036. Calculate the average equity premium and the average risk-free rate for this new parametrization.

(c) In the Mehra-Prescott model with \( \gamma = 1 \), \( \beta = 0.99 \), and the parameters of the process for consumption growth set equal to their values from class (i.e., \( \phi_{11} = \phi_{22} = 0.43 \), \( \mu = 0.018 \), and \( \delta = 0.036 \)), find the average rate of return on a two-period risk-free bond, i.e., a sure claim to one unit of the consumption good two periods from now.

Let the average (net) rate of return on the two-period bond be given by \( r_2 \). Similarly, let \( r_1 \) be the average (net) rate of return on a one-period risk-free bond. The gross two-period return \( 1 + r_2 \) can be decomposed into the product of two successive (gross) one-period returns \( 1 + \tilde{r}_2 \), where \( \tilde{r}_2 = (1 + r_2)^{1/2} - 1 \). The term structure of interest rates is upward-sloping if \( \tilde{r}_2 > r_1 \); otherwise, it is downward-sloping. Which way does the term structure slope if the Mehra-Prescott model is calibrated as in part (a)?

2. Consider the planning problem for a neoclassical growth model with logarithmic utility, full depreciation of the capital stock in one period, and a production function of the form \( y = zk^\alpha \), where \( z \) is a random shock to productivity. The shock \( z \) is observed before making the current-period savings decision. Assume that the capital stock can take on only two values: i.e., \( k \) is restricted to the set \( \{\bar{k}_1, \bar{k}_2\} \). In addition, assume that
z takes on values in the set \{\bar{z}_1, \bar{z}_2\} and that z follows a Markov chain with transition probabilities \(p_{ij} = P(z' = \bar{z}_j | z = \bar{z}_i)\).

(a) Let \(\bar{z}_1 = 0.9, \bar{z}_2 = 1.1, p_{11} = 0.95, \text{ and } p_{22} = 0.9\). Find the invariant distribution associated with the Markov chain for z. Use the invariant distribution to compute the long-run (or unconditional) expected value of z; that is, compute \(E(z) = \pi_1 \bar{z}_1 + \pi_2 \bar{z}_2\), where \(\pi_1\) and \(\pi_2\) determine the invariant distribution.

(b) Let \(\beta = 0.9, \alpha = 0.36, \bar{k}_1 = 0.95 k_{ss}, \text{ and } \bar{k}_2 = 1.05 k_{ss}\), where \(k_{ss}\) is the steady-state capital stock in a version of this model without shocks and with no restrictions on capital (i.e., \(k_{ss} = (\alpha/\beta)^{-1/2}\)). Let \(g(k, z)\) denote the planner’s optimal decision rule. Prove that \(g(k_i, z_j) = k_j\) for all \(i\) and \(j\).

(c) The decision rule from part (b) and the law of motion for z jointly determine an invariant distribution over \((k, z)\)-pairs. Find this distribution. (That is, find probabilities \(\pi_{ij} = P(k = k_i, z = z_j)\) that “reproduce” themselves: if \(\pi_{ij}\) is the unconditional probability that the economy is in state \((k_i, z_j)\) today, then it is also the unconditional probability that the economy is in this state tomorrow. For a more complete discussion of this concept, see pp. 78 and 79 in the lecture notes by Per Krusell.) Use your answer to compute the long-run (or unconditional) expected values of the capital stock and of output.

(d) In Matlab, use the optimal decision rule, the law of motion for z, and a random number generator to create a simulated time series \(\{k_t, y_t\}_{t=0}^T\), given an initial condition \((k_0, z_0)\). Compute \(T^{-1} \sum_{t=1}^T k_t\) and \(T^{-1} \sum_{t=1}^T y_t\) for a suitably large value of \(T\) and confirm that these sample means are close to the corresponding population means that you computed in part (c). (You may find useful the Matlab code by Ljunqvist and Sargent for simulating a Markov chain that I have posted on the course web site.)

3. (a) Consider a neoclassical growth model with shocks to total factor productivity and valued leisure. Carefully define a sequential competitive equilibrium for this economy in which households own the factors of production and rent them to firms in competitive markets.

(b) Carefully define a recursive competitive equilibrium for the economy in part (a). (Hint: You need two functions to describe the aggregate economy, one for aggregate capital and one for aggregate labor supply.)

(c) Carefully describe a procedure (analogous to the one that we will, or did, discuss in lecture on December 5 for a similar economy without valued leisure) for computing a linear approximation to the competitive equilibrium behavior of the economy in parts (a) and (b). You do not need to implement the procedure, but you need
to explain it in enough detail that someone who knows nothing about economics, but who can take derivatives and manipulate algebra, could implement it. (Hint: There are two unknown decision rules, one for savings and one for labor supply, that must satisfy simultaneously two first-order conditions.)