Econ 510a (second half)
Yale University
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Prof. Tony Smith

HOMEWORK #6

This homework assignment does not have to be turned but you are responsible for the material it covers.

1. **(a)** Recall that the growth rate g of the dividend of the "tree" in the Mehra-Prescott model follows a two-state Markov chain: $P(g' = \lambda_j | g = \lambda_i) = \pi_{ij}$, where $\pi_{11} = \pi_{22} = \pi = 0.43$, $\lambda_1 = 1 + \gamma + \delta$, $\lambda_2 = 1 + \gamma - \delta$, $\gamma = 0.018$ and $\delta = 0.036$. Assume logarithmic utility and discount factor $\beta = 0.99$. Compute the average (or long-run) rate of return on a riskfree bond, the average rate of return on equity (i.e., a claim to the dividend stream generated by the "tree"), and the average equity premium.

(b) What happens to the rates of return that you calculated in part (a) if the standard deviation of the consumption growth rate doubles (i.e., if δ equals 0.072 rather than 0.036)?

(c) (This part is optional.) For the parameter values in part (a), find the average rate of return on a *two-period* riskfree bond, i.e., a sure claim to one unit of the consumption good two periods from now. (Recall that the price today of such a claim is the conditional expected value of the intertemporal marginal rate of substitution between consumption today and consumption two periods from now. The gross rate of return on the two-period riskfree bond is the inverse of this price.)

Let the (net) rate of return on the two-period bond be given by r_2. Similarly, let r_1 be the (net) rate of return on a one-period riskfree bond. The gross two-period return $1 + r_2$ can be decomposed into the product of two successive (gross) one-period returns $1 + \tilde{r}_2$, where $\tilde{r}_2 = (1 + r_2)^{1/2} - 1$. The *term structure of interest rates* is upward-sloping if $\tilde{r}_2 > r_1$; otherwise, it is downward-sloping. Which way does the term structure slope if the Mehra-Prescott model is calibrated as in part (a)?

2. The first part of this problem studies a simple endogenous growth model known as the "Ak" model. The rest of the problem then develops a microfoundation for the "Ak" model.

(a) Consider a growth model in which the production function does not exhibit decreasing returns to capital: $f(\bar{k}) = A\bar{k} + (1 - \delta)\bar{k}$, where $\bar{k}$ is the aggregate capital stock and the constant A is positive. There is no leisure in this model: it enters neither the production function nor the utility function. Let the felicity function $u(c)$ have a constant elasticity of intertemporal substitution equal to σ^{-1}.

For this environment, show that the equilibrium law of motion for aggregate capital is: $\bar{k}' = B\bar{k}$, where B is a constant that depends on the structural parameters A, β, and σ. (Hint: Find the Euler equation for the social planning problem, conjecture that $\bar{k}' = B\bar{k}$, and then find the value of B that sets the Euler equation to zero for all values of $\bar{k}$.) Show that B could be either greater or smaller than one, depending on the values of the structural parameters.

Note that in this model there are no transitional dynamics: the aggregate capital stock is always on the balanced growth path (with growth rate determined by B).

(b) Now consider again the growth model with an externality in production that you studied in the second problem on Homework #4. In particular, let $\gamma = 1 - \alpha$ in this model. Show that, in competitive equilibrium, this economy behaves like an "Ak" model. Express the competitive equilibrium growth rate in terms of the primitives of technology and preferences.

(c) Show that, for the economy in part (b), the growth rate that would be chosen by a social planner is larger than the competitive equilibrium growth rate.

(d) Find a tax policy that would induce the competitive equilibrium growth rate to coincide with the growth rate chosen by a social planner.

3. This problem shows that, under some assumptions, the competitive equilibrium of the standard neoclassical growth model is invariant to how the government finances its expenditures. To this end, suppose that the government seeks to finance a fixed (deterministic) stream of expenditures $\{g_t\}_{t=0}^{\infty}$. The economy's aggregate resource constraint then reads:

$$\bar{c}_t + \bar{k}_{t+1} - (1-\delta)\bar{k}_t + g_t = \bar{y}_t,$$

where $\bar{k}$ denotes aggregate capital, $\bar{c}$ denotes aggregate consumption, and $\bar{y} = F(\bar{k}, \bar{n})$ denotes aggregate output. The production function F has constant returns to scale. Leisure is not valued, so without loss of generality, set $\bar{n} = 1$. Government expenditures are not valued by consumers either, i.e., they do not enter the consumer's utility function.

The government uses a combination of debt and (lump-sum) taxation to finance its expenditure stream. In particular, the government issues a sequence of one-period debt $\{b_t\}_{t=0}^{\infty}$, with $b_0 = 0$. This debt is a promise by the government to pay one unit of the consumption good in the next period; let the price of such a promise in period t be q_t. The government also engages in lump-sum taxation of consumers; let τ_t be the lump-sum tax in period t. In every period, the government satisfies the following budget constraint:

$$\tau_t + q_t b_{t+1} = b_t + g_t.$$

The left-hand side of this constraint is the government's inflows in period t (measured in terms of period-t consumption goods), while the right-hand side is the government's outflows in period t (again, measured in terms of period-t consumption goods).

The representative consumer takes prices as given and seeks to maximize the lifetime utility of consumption subject to a lifetime budget constraint given by:

$$\sum_{t=0}^{\infty} p_t(c_t + k_{t+1} + q_t a_{t+1}) = \sum_{t=0}^{\infty} p_t((r_t + 1 - \delta)k_t + w_t + a_t - \tau_t),$$

where p_t is the price of period-t consumption goods in terms of period-0 consumption goods (whose price p_0 is normalized to 1). In this budget constraint, a_{t+1} is the amount of government debt (i.e., promises by the government to deliver consumption goods in period $t+1$) that the consumer purchases in period t (assume that $a_0 = 0$). In equilibrium, the demand for government debt is equal to the supply of government debt in every period: $a_t = b_t$ for all t.

(a) Carefully define a competitive equilibrium with date-0 trading for this economy.

(b) Use the no-arbitrage condition $p_t q_t = p_{t+1}$ to derive the government's consolidated budget constraint:

$$\sum_{t=0}^{\infty} p_t g_t = \sum_{t=0}^{\infty} p_t \tau_t.$$

(c) Use the result from part (a) to show that the consumer's lifetime budget constraint can be written as follows:

$$\sum_{t=0}^{\infty} p_t(c_t + k_{t+1} + g_t) = \sum_{t=0}^{\infty} p_t((r_t + 1 - \delta)k_t + w_t).$$

Because the sequences $\{\tau_t\}_{t=0}^{\infty}$ and $\{b_t\}_{t=0}^{\infty}$ do not appear in this budget constraint, it is evident that the way in which the government finances its expenditure stream $\{g_t\}_{t=0}^{\infty}$ is irrelevant to the consumer's optimization problem. Instead, all that matters to the consumer is the net present value of government expenditures, i.e., $\sum_{t=0}^{\infty} p_t g_t$. This implies in turn that the government's financing decisions do not affect the determination of equilibrium prices and quantities. This result is known as the *Ricardian equivalence theorem*.

For further reading on Ricardian equivalence, you may find it helpful to read Section 3 of Chapter 1 in Williamson's notes and pp. 165–168 of Chapter 10 in Krusell's notes.