STUDY QUESTION

The purpose of this study question is to give you practice applying the tools and methods covered in the course.

The neoclassical growth model, like the Solow growth model before it, assumes that there are diminishing returns to capital in production: holding other factors constant, the marginal product of capital declines as capital increases. As a result, the optimal path for capital converges to a steady state (or, if there is exogenous growth in the productivity of labor and preferences satisfy certain restrictions, to a balanced growth path along which all variables grow at the same rate as labor productivity).

Suppose instead that capital does not have diminishing returns: output at any point in time is given by \( y = A k \), where \( A \) is a positive constant.

(a) Suppose that the representative consumer has preferences given by \( \sum_{t=0}^{\infty} \beta^t \log(c_t) \) and that capital accumulates according to \( k_{t+1} = (1 - \delta) k_t + x_t \), where \( x_t \) is investment in period \( t \). Show that the solution to the usual planning problem takes the form: \( k' = \exp(g) k \). Express the growth rate \( g \) in terms of \( A \), \( \beta \), and \( \delta \). This simple model of endogenous growth is known as the ‘\( Ak \)’ model; for further discussion, see Chapter 8 of the Lecture Notes by Per Krusell.

(b) Show that the growth model with an externality in production that you studied in the third problem of Problem Set #3 behaves like an \( Ak \) model if \( \gamma = 1 - \alpha \).