Econ 510a (second half)
Yale University
Fall 2007
Prof. Tony Smith

STUDY QUESTIONS

The purpose of these study questions is to give you practice applying the tools and methods covered in the course.

1. The neoclassical growth model, like the Solow growth model before it, assumes that there are diminishing returns to capital in production: holding other factors constant, the marginal product of capital declines as capital increases. As a result, the optimal path for capital converges to a steady state (or, if there is exogenous growth in the productivity of labor and preferences satisfy certain restrictions, to a balanced growth path along which all variables grow at the same rate as labor productivity).

   Suppose instead that capital does not have diminishing returns: output at any point in time is given by $y = Ak$, where A is a positive constant.

   (a) Suppose that the representative consumer has preferences given by $\sum_{t=0}^{\infty} \beta^t \log(c_t)$ and that capital accumulates according to $k_{t+1} = (1 - \delta)k_t + x_t$, where x_t is investment in period t. Show that the solution to the usual planning problem takes the form: $k' = \exp(g)k$. Express the growth rate g in terms of A, β, and δ. This simple model of *endogenous* growth is known as the 'Ak' model; for further discussion, see Chapter 8 of the Lecture Notes by Per Krusell.

   (b) Show that the growth model with an externality in production described in the fourth (optional) problem on Homework #5 behaves like an Ak model if $\gamma = 1-\alpha$.

2. In the second problem on Homework #3, you defined a recursive competitive equilibrium for a neoclassical growth model with value leisured and no uncertainty. Modify this definition to allow for an aggregate production function of the form $y = e^z F(k, n)$, where z is a shock to aggregate (or total factor) productivity that evolves according to: $z' = \rho z + \epsilon'$, where $\epsilon' \sim iidN(0, \sigma_\epsilon^2)$ and $|\rho| < 1$.