Econ 511b (Part I)
Yale University
Spring 2004
Prof. Tony Smith

HOMEWORK #6

This homework assignment should be handed in by 5PM on Friday, February 20 to Jinhui Bai's mailbox in the basement of 28 Hillhouse.

1. **(a)** Find the planner's optimal decision rule in the stochastic one-sector growth model without valued leisure by linearizing the Euler equation. Let the production function take the form $f(k_t, n_t, z_t) = e^{z_t} k_t^{\alpha} n_t^{1-\alpha}$, let the consumer's felicity function have a constant elasticity of intertemporal substitution γ^{-1}, and let z_t follow an AR(1) process: $z_{t+1} = \rho z_t + \epsilon_{t+1}$, $\epsilon_{t+1} \sim iidN(0, \sigma^2)$. Set $\alpha = 0.36$, $\rho = 0.95$, $\sigma = 0.007$, the discount factor $\beta = 0.99$, and the depreciation rate $\delta = 0.025$. Solve the model for two different values of γ: 1 (the log case) and 2.

(b) Using Matlab, compute *impulse response functions* for output, consumption, and investment in response to an innovation in the level of technology of size σ. That is, suppose that the economy is at the deterministic steady state and the innovation ϵ suddenly jumps in the current period to σ and then returns to 0 in all subsequent periods. Determine (numerically) the values of output, consumption, and investment in the current period and in the next, say, 10 periods. Express the impulse responses as percentage deviations from the (deterministic) steady-state values. Compare the impulse response functions for the two values of γ.

2. Consider a stochastic neoclassical growth model with the following structure:

- Each consumer has preferences of the form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t + B\frac{(1-n_t)^{1-\nu}-1}{1-\nu}\right)^{1-\sigma} - 1}{1-\sigma},$$

  where labor supply $n_t < 1$.

- The economy's technology is described by:

$$c_t + k_{t+1} - (1-\delta)k_t = z_t k_t^{\alpha} n_t^{1-\alpha},$$

  where $\log(z_t)$ is stochastic and evolves according to a stationary AR(1) process:

$$z_{t+1} = \rho z_t^{\rho} \epsilon_{t+1},$$

with $\log(\epsilon_t) \sim iidN(0, \sigma_\epsilon^2)$.

(a) Derive the Euler equation for the savings decision.

(b) Derive the first-order condition for the labor-leisure decision.

(c) Show that the first-order condition for the labor-leisure decision can be written as a simple function relating $\log(1-n)$ to $\log(w)$, where w is the wage rate.

(d) Use the following facts from the hypothetical economy Pekrland to calibrate all of the model's parameters except ρ and σ_ϵ^2:

- The average value of the capital-output ratio (in annual terms) is 2.
- Pekrlanders work (on average) one-half of their total available time.
- Experimental evidence on Pekrlanders' attitudes towards risk shows that they all have a coefficient of relative risk aversion equal to 2.
- Capital's share of income is one-third.
- The average value of the investment-to-output ratio is 0.2.
- Labor economists in Pekrland have found that, in a log-log regression of a typical Perklander's hours of leisure on the wage, the coefficient on the log of the wage is -0.5: if wages go up by 1%, a typical Pekrlander works 0.5% more hours.

You do not need to compute all of the parameters numerically, but you do need to describe how you would compute them.

3. Consider again the growth model with an externality in production that you studied in problem 2 of Homework #3. In particular, let $\gamma = 1 - \alpha$ and let the consumer's felicity function have constant elasticity of intertemporal substitution σ^{-1}.

(a) Show that, in competitive equilibrium, this economy behaves like an "AK" model. Express the competitive equilibrium growth rate in terms of primitives of technology and preferences.

(b) Show that the growth rate that would be chosen by a social planner is larger than the competitive equilibrium growth rate.

(c) Find a tax policy that would induce the competitive equilibrium growth rate to coincide with the growth rate chosen by a social planner.