

Asset Pricing

The objective of this section of the course is to introduce the asset pricing formula developed by Lucas [1978]. We will study the pricing of assets that is consistent with the neoclassical growth model. More generally, this is the pricing methodology that is implied by the "microfoundations" approach to macroeconomics - which we endorse!

Lucas works out his formula using an endowment economy inhabited by one agent. The reason for doing so is that in such an environment the allocation problem is trivial; therefore only the prices that support a no-trade general equilibrium need to be sorted out.

In the second part of this section, we study the application of the Lucas pricing formula performed by Mehra and Prescott [1985]. The authors utilized the tools developed by Lucas [1978] to determine the asset prices that would prevail in an economy whose endowment process mimicked the consumption pattern of the United States economy during the last century. They then compared the theoretical results with real data. Their findings were striking and have produced an extensive literature.

1 Lucas Asset Pricing Formula

The Model

The representative agent in Lucas' economy solves:

$$\begin{aligned} & \max_{\{c_t(z^t)\}_{t=0}^{\infty}, \forall z^t} \left\{ \sum_t \sum_{z^t} \pi(z^t) \cdot u[c_t(z^t)] \right\} \\ \text{s.t.} \quad & \sum_t \sum_{z^t} p_t(z^t) \cdot c_t(z^t) = \sum_t \sum_{z^t} p_t(z^t) \cdot \varpi_t(z^t) \\ & c_t(z^t) = \varpi_t(z^t) \quad \forall t, \forall z^t \quad (\text{market clearing}) \end{aligned}$$

The last condition is the feasibility condition. Notice that it implies that the allocation problem is trivial, and only the prices $p_t(z^t)$ supporting this allocation as a (competitive) equilibrium must be sought. (*Note:* Lucas' paper uses continuous probability.)

Therefore, we have two tasks:

1st Task: Find an expression for $p_t(z^t)$ in terms of the primitives.

2nd Task: Apply the resulting formula $p_t(z^t)$ to price arbitrary assets.

1st Task

First order conditions from the consumer's problem:

$$c_t(z^t) : \beta^t \cdot \pi(z^t) \cdot u'[c_t(z^t)] = \lambda \cdot p_t(z^t)$$

where $c_t(z^t) = \varpi_t(z^t)$ will need to hold, and λ will be endogenous. We can get rid of this shadow value of income by normalizing $p_0 = 1$:

$$c_0 : u'(\varpi_0) = \lambda \cdot p_0 \equiv \lambda$$

Then

$$p_t(z^t) = \beta^t \cdot \pi(z^t) \cdot \frac{u'[\varpi_t(z^t)]}{u'(\varpi_0)} \quad (\text{LPF})$$

The Lucas Pricing Formula (LPF) shows that $p_t(z^t)$ is the price of a claim on consumption goods at t that yields 1 unit if the realized state is z^t , and 0 units otherwise.

We can distinguish three separate components in the price of this claim:

1. *Time*: $p_t(z^t)$ is decreasing in t (since $\beta < 1$).
2. *Likelihood*: $p_t(z^t)$ is increasing in the probability of occurrence of z^t .
3. *Marginal rate of substitution*: $p_t(z^t)$ is increasing in the marginal rate of substitution between goods at (t, z^t) and $t = 0$ (don't forget that $p_t(z^t)$ is in fact a *relative* price).

For the case of a concave felicity index $u(\cdot)$ (which represents risk averse behavior), the third effect will be high if the endowment of goods is *scarce* at (t, z^t) relative to $t = 0$.

2nd Task

Any asset is in essence nothing but a sum of contingent claims. Therefore pricing an asset consists of summing up the prices of these rights to collect goods. You may already (correctly) suspect that the key is to properly identify the claims to which the asset entitles its owner. This involves specifying the time and state of nature in which these rights get activated, and the quantities.

We must find the price at (t, z^t) of an asset that pays 1 unit at $t + 1$ for every possible realization z^{t+1} such that $z^{t+1} = (z_{t+1}, z^t)$ for $z_{t+1} \in Z$.

The date-0 price of such an asset is given by

$$q_0^{rf}(z^t) = \sum_{z' \in Z} \underbrace{p_{t+1}(z', z^t)}_{\text{price of claim}} \cdot \underbrace{1}_{\text{quantity}}$$

The date- t price is computed by

$$\begin{aligned} q_t^{rf}(z^t) &= \frac{q_0^{rf}(z^t)}{p_t(z^t)} \\ q_t^{rf}(z^t) &= \frac{\sum_{z' \in Z} p_{t+1}(z', z^t) \cdot 1}{p_t(z^t)} \end{aligned}$$

Using (LPF) to replace for $p_t(z^t)$, $p_{t+1}(z_{t+1}, z^t)$:

$$\begin{aligned} q_t^{rf}(z^t) &= \frac{\beta^{t+1} \cdot \sum_{z' \in Z} \pi(z', z^t) \cdot \frac{u'[\varpi_{t+1}(z', z^t)]}{u'[\varpi_0]}}{\beta^t \cdot \pi(z^t) \cdot \frac{u'[\varpi_t(z^t)]}{u'[\varpi_0]}} \\ q_t^{rf}(z^t) &= \beta \cdot \sum_{z' \in Z} \frac{\pi(z', z^t)}{\pi(z^t)} \cdot \frac{u'[\varpi_{t+1}(z', z^t)]}{u'[\varpi_t(z^t)]} \end{aligned}$$

Notice that three components identify before now have the following characteristics:

1. *Time*: Only one period discounting must be considered between t and $t+1$.
2. *Likelihood*: $\frac{\pi(z', z^t)}{\pi(z^t)}$ is the *conditional probability* of the state z' occurring at $t+1$, given that z^t is the history of realizations up to t .
3. *Marginal rate of substitution*: The relevant rate is now between goods at (t, z^t) and $(t+1, z^{t+1})$ for each possible z^{t+1} of the form (z_{t+1}, z^t) with $z_{t+1} \in Z$.

For more intuition, you could also think that $q_t^{rf}(z^t)$ is the price that would obtain if the economy, instead of starting at $t=0$, was "rescheduled" to begin at date t (with the stochastic process $\{z_t\}_{t=0}^\infty$ assumed to start at z^t).

Next we price a stock that pays out dividends according to the process $d_t(z^t)$ (a *tree* yielding $d_t(z^t)$ units of fruit at date-state (t, z^t)). The date- t price of

this portfolio of contingent claims is given by

$$\begin{aligned}
q_t^{tree}(z^t) &= \frac{\sum_{s=t+1}^{\infty} \sum_{z^s} p_s(z^s) \cdot d_s(z^s)}{p_t(z^t)} \\
q_t^{tree}(z^t) &= \sum_{s=t+1}^{\infty} \sum_{z^s} \beta^{s-t} \cdot \frac{\pi(z^s)}{\pi(z^t)} \cdot \frac{u'[\varpi_s(z^s)]}{u'[\varpi_t(z^t)]} \cdot d_s(z^s) \\
q_t^{tree}(z^t) &= E_t \left[\sum_{s=t+1}^{\infty} \beta^{s-t} \cdot \frac{u'[\varpi_s]}{u'[\varpi_t(z^t)]} \cdot d_s \right]
\end{aligned}$$

Notice that the price includes the three components enumerated above, multiplied by the quantity of goods to which the asset entitles in each date-state. This quantity is the dividend process $d_t(z^t)$.

We can also write the price of the tree in a recursive way. In the deterministic case, this would mean that

$$p_t = \frac{p_{t+1} + d_{t+1}}{R_{t+1}}$$

where R_{t+1} is the (gross) interest rate between periods t and $t + 1$. This is recursive because the formula for p_t involves p_{t+1} .

The uncertainty analogue to this expression is

$$\begin{aligned}
q_t^{tree}(z^t) &= \sum_{z^s} \beta \cdot \frac{\pi(z', z^t)}{\pi(z^t)} \cdot \frac{u'[\varpi_{t+1}(z', z^t)]}{u'[\varpi_t(z^t)]} \cdot [d_{t+1}(z', z^t) + q_{t+1}^{tree}(z', z^t)] \\
q_t^{tree} &= \beta \cdot E_t \left[\frac{u'[\varpi_{t+1}]}{u'[\varpi_t]} \cdot [d_{t+1} + q_{t+1}^{tree}] \right]
\end{aligned}$$

You can check that this corresponds to the previous formula by iteratively substituting for $q_{t+1}^{tree}(z', z^t)$. More importantly, notice that the price includes the usual three components: what about quantities? This expression reads like the price of a one period tree that entitles to the dividend $d_{t+1}(z', z^t)$, plus the amount of "money" needed to purchase the one-period tree again next period.

If you think about how this price fits into the endowment economy, then the amount $q_{t+1}^{tree}(z', z^t)$ will have to be such that at date-state $[t + 1, (z', z^t)]$, the consumer is marginally indifferent between purchasing the tree again, or using the proceeds to buy consumption goods. Given an equilibrium price $q_{t+1}^{tree}(z', z^t)$ for each date-state, the equilibrium price $q_t^{tree}(z^t)$ will be determined recursively.

Finally, the jargon has it that

$$\beta \cdot \sum_{z^s} \frac{\pi(z', z^t)}{\pi(z^t)} \cdot \frac{u'[\varpi_{t+1}(z', z^t)]}{u'[\varpi_t(z^t)]} \equiv \beta \cdot E_t \left[\frac{u'[\varpi_{t+1}(z', z^t)]}{u'[\varpi_t(z^t)]} \right]$$

be called the "stochastic discount factor" or "pricing kernel".

2 The Equity Premium Puzzle

The equity premium is the name of an empirical regularity observed in the United States asset markets during the last century. It consists of the difference between the returns on stocks and on government bonds. Investors who had always maintained a portfolio of shares with the same composition as Standard and Poor's SP500 index would have obtained, if patient enough, a return around 6% higher than if investing all their money in government bonds. Since shares are riskier than bonds, this fact should be explainable by the "representative agent's" dislike for risk. In the usual CES utility function, the degree of risk aversion (but notice that also the inter-temporal elasticity of substitution!) is captured by the σ parameter.

Mehra and Prescott's exercise was intended to verify the theory against the observations. To that effect, they computed statistics of the realization of (de-trended) aggregate consumption in the United States, and used those statistics to generate an endowment process in their model economy. That is, their endowment economy mimics the United States economy for a single agent.

Using parameters consistent with microeconomic behavior (drawn from micro, labor, other literature, and "introspection"), they *calibrated* their model to simulate the response of a representative agent to the assumed endowment process. Their results were striking in that the model predicts an equity premium that is significantly lower than the actual one observed in the United States. This incompatibility could be interpreted as evidence against the neoclassical growth model (and related traditions) in general, or as a signal that some of the assumptions used by Mehra and Prescott (profusely utilized in the literature) need to be revised. It is a "puzzle" that actual behavior varies so widely with predicted behavior because we believe that the microfoundations tradition is essentially correct and should provide accurate predictions.

2.1 The Model

The economy is modelled as in the Lucas [1978] paper. It is an endowment economy, inhabited by a representative agent, and there are complete markets. The endowment process is characterized by two parameters, that were picked so that the implied statistics matched aggregate US consumption data between 1889 and 1978.

Preferences

Preferences are modelled by the utility function

$$U = E_0 \left[\sum_{t=0}^{\infty} \beta^t \cdot u(c_t) \right]$$

And the felicity index u is of the CES type:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

Therefore preferences involve two parameters: β and σ , the values of which will need to be calibrated (and play an essential role in the "puzzle"). β measures the time impatience of agents; what does σ measure? In a deterministic environment, σ^{-1} is the coefficient of inter-temporal substitution. But in the uncertainty case, σ is also the coefficient of relative risk aversion (CRRA):

$$CRRA \equiv -\frac{u''(c) \cdot c}{u'(c)} = -\frac{-\sigma \cdot c^{-1-\sigma} \cdot c}{c^{-\sigma}} = \sigma$$

Therefore the same parameter measures two (distinct?) effects: the willingness to substitute consumption over time, and also across states of nature. The higher the σ , the less variability the agent wants his consumption pattern to show, whatever the source of this variability: deterministic growth, or stochastic deviation.

Endowment Process

Let y_t denote income (in equilibrium, consumption) at time t . Then

$$y_{t+1} = x_{t+1} \cdot y_t$$

The growth rate is stochastic: x_{t-1} is a finite random variable that can take n values. The stochastic process is modelled by a 1st order Markov chain, where:

$$\phi_{ij} \equiv \Pr [x_{t+1} = \lambda_j | x_t = \lambda_i]$$

Asset Prices

Applying the Lucas pricing formula to the *tree* that yields $d_t = y_t$ at time t , we have that

$$p_t^e = E_t \left[\sum_{s=t+1}^{\infty} \beta^{s-t} \cdot \frac{u'(y_s)}{u'(y_t)} \cdot d_s \right]$$

p_t^e is the price of equity, that is, of the "market portfolio". It is the price of contingent claims on the whole produce of the economy at t , therefore it should be interpreted as the value of a portfolio of claims on all possible productive investments. The closest measure that we have of such a portfolio is the stock

market, where shares in companies involved in almost all productive activities are traded.

We will solve for these prices using a recursive formulation. First notice that, due to the 1st order Markov assumption on x_t , the likelihood of changing states is invariant over time, therefore we can drop the time subscript and write p^e as a function of the state:

$$p_t^e = p_e(x_t, y_t)$$

All the information of the economy is summarized by the level of the endowment process, y_t , and the last realization of the shock, x_t , so we guess that prices will end up being a function of those two variables only. The reason why y_t is informative is that, since in equilibrium consumption is equal to the endowment, then y_t will provide the level of marginal utility against which future consumption streams will be compared when setting prices. And x_t conveys information on the part of the Markov process where the economy is standing. Only x_t is relevant, and not lagged values of x_t , because of the first order assumptions. Then the recursive formulation of the price of equity is:

$$p_e(x_t, y_t) = E \left[\sum_{s=t+1}^{\infty} \beta^{s-t} \cdot \left(\frac{y_t}{y_s} \right)^{\sigma} \cdot y_s \middle| x_t, y_t \right]$$

(Notice that the time subscript has been dropped from the E expectation operator since the probability of changing states do not depend on time under the Markov 1st order assumption.)

For each state x_i , $i = 1, \dots, n$, this price (at any date t) is given by:

$$p_i^e(y) = \beta \cdot \sum_{j=1}^n \phi_{ij} \cdot \left(\frac{y}{y \cdot \lambda_j} \right)^{\sigma} \cdot [y \cdot \lambda_j + p_j^e(y \cdot \lambda_j)] \quad \forall y, \forall i$$

Where $p_j^e(y \cdot \lambda_j)$ will be the price of equity next period if the realized state is j , since this will imply that consumption (the endowment) growth will be $x_{t+1} = \lambda_j$.

We guess a linear solution to this functional equation:

$$p_i^e(y) = p_i^e \cdot y$$

And this yields a system of equations, with the unknowns being the coefficients p_i^e :

$$\begin{aligned} p_i^e &= \beta \cdot \sum_{j=1}^n \phi_{ij} \cdot (\lambda_j)^{-\sigma} \cdot [\lambda_j + p_j^e \cdot \lambda_j] \\ p_i^e &= \beta \cdot \sum_{j=1}^n \phi_{ij} \cdot (\lambda_j)^{1-\sigma} \cdot [1 + p_j^e] \end{aligned}$$

This equation relating p_i^e to the (weighted) summation of the p_j^e needs to hold for all i , therefore we have a linear system of n equations and n unknowns.

Similarly, the price of a risk-free asset paying off 1 unit in every state is given by

$$p_i^{rf}(y) = \beta \cdot \sum_{j=1}^n \phi_{ij} \cdot x_j^{-\sigma} \cdot 1$$

$x_j^{-\sigma}$ is the ratio of consumption next period to consumption today - the growth rate of consumption. Notice that the level of the endowment, y , does not enter this formula, whereas it did enter the formula for equity prices.

Returns on Assets

Given the prices, we can compute the returns that an investor would perceive by purchasing them. This will be a random variable induced by the randomness in prices and (in the case of equity) by the variability of the endowment process also. The (net) return realized at state j by an investor who purchased equity in state i is given by:

$$r_{ij}^e = \frac{(1 + p_j^e) \cdot \lambda_j}{p_i^e} - 1$$

To understand where this formula comes from, just multiply through by y :

$$\frac{(1 + p_j^e) \cdot \lambda_j \cdot y}{p_i^e \cdot y} - 1 = \frac{\lambda_j \cdot y + p_j^e \cdot \lambda_j \cdot y - p_i^e \cdot y}{p_i^e \cdot y} \equiv \frac{d_{t+1,j} + p_{t+1,j} - p_{t,i}}{p_{t,i}}$$

The amount $d_{t+1} + p_{t+1}$ is the pay off from the tree next period (if the state is j). By subtracting the investment size $p_{t,i}$, the numerator yields the net result perceived by the investor. Dividing by the invested amount gives the (net) rate of return.

The conditional expected return, therefore, is

$$r_i^e = E_i[r_{ij}^e] = \sum_{j=1}^n \phi_{ij} \cdot r_{ij}^e$$

And the unconditional expected return,

$$r^e = E[r_i^e] = \sum_{i=1}^n \pi_i \cdot r_i^e = \sum_{i=1}^n \pi_i \cdot \sum_{j=1}^n \phi_{ij} \cdot r_{ij}^e$$

r^e is not a random variable. It is an expectation, taken with respect to the invariant (long run) distribution π_i of the Markov process. Recall that this is the probability vector that satisfies:

$$\Pi = \begin{pmatrix} \pi_1 \\ \vdots \\ \pi_n \end{pmatrix} = \Phi' \cdot \Pi$$

r^e will be compared to actual, long run US data as the return on equity. The equity premium will be given by this r^e minus the long run return on government bonds (proxy for risk-free assets). In the model, this return on the risk-free assets is given by:

$$r_i^{rf} = \frac{1}{p_i^{rf}} - 1$$

This is a random variable. The long run return is:

$$r^{rf} = \sum_{i=1}^n \pi_i \cdot r_i^{rf}$$

The US data shows the following value of the equity premium:

$$r^e - r^{rf} \sim 6\%$$

Where r^e is the return on the S&P500 from 1889 to 1978, and r^{rf} is the yield on government bonds throughout that period.

Calibration

Mehra & Prescott calibrate the Markov process assuming that there are two states: $n = 2$. The value of each possible realization of the endowment growth rate is:

$$\begin{aligned} \lambda_1 &= 1 + \mu + \delta \\ \lambda_2 &= 1 + \mu - \delta \end{aligned}$$

μ is the average growth rate $\frac{c_{t+1}-c_t}{c_t}$. Its value, to match that of aggregate consumption in the US in the period under study, was $\mu = .018$. δ is the variation in the growth rate.

The transition matrix was assumed symmetric, so that the probability of changing state are the same at each state:

$$\Phi = \begin{pmatrix} \phi & 1 - \phi \\ 1 - \phi & \phi \end{pmatrix}$$

Then δ and ϕ are picked so as to match: $\left\{ \begin{array}{l} \text{the standard deviation of } \frac{c_{t+1}-c_t}{c_t}, \text{ equal to } .036 \\ \text{the 1}^{\text{st}} \text{ order serial correlation of } \frac{c_{t+1}-c_t}{c_t}, \text{ equal to } .43 \end{array} \right.$

The resulting parameter values are: $\delta = .036$, and $\phi = .43$

The remaining parameters are β and σ , that represent preferences. A priori, by introspection economists believe that β must lie in the interval $(0, 1)$. With respect to σ , Mehra and Prescott cite several different studies and opinions on

its likely values. Most micro literature suggests that σ must be approximately equal to 1 (this is the logarithmic utility case). However, some economists also believe that it could take values as high as 2 or even 4. Certainly, there seems to be consensus that σ has to be lower than 10.

Then instead of picking values for β and σ , Mehra and Prescott plotted the level of equity premium that the model would predict for different, *reasonable* combinations of values. The following chart is approximately what they obtained:

The model can only produce the equity premium observed in actual data at the expense of a very high risk-free interest rate, or highly *unreasonable* parameter values (such as $\beta > 1$; how do you feel about your own β ?). When compared to actual data, the risk premium is too low in the model, and the risk-free rate too high. In fact, these are two puzzles.

Suggested Solutions to the Puzzle

There is one "solution" that consists of solving for parameter values that will yield the same equity premium - risk free rate as the data. You may realize that by fixing one of the preference parameters, the other can be solved for these values. An example is $\sigma \sim 15$, and $\beta \sim 1.08$. Are these values *reasonable*? What can you say from introspection? Is the total sum of instantaneous utility values bounded for these parameters?

We will enumerate other solutions that have been proposed in the literature.

Solution 1 - Epstein - Zin preferences

One of the issues that seem to be crucial in the puzzle is that the CES utility function mixes the time structure of preferences and the aversion for risk. Both are measured by (functions of) the same parameter σ . In some sense, this is consistent with microeconomists' way of modelling risk: Remember that uncertainty is just the expansion of the decision making scenario to a multiplicity of "states of nature"; total utility is hence just the expected value of optimal decision making in each of these states. You may notice that nothing in essence differs between "time" and "states of nature". "Time" is just another subindex to identify states of the world.

However, people seem to regard time and uncertainty as essentially different phenomena. It is natural then to seek a representation of preferences that can treat these two components of reality separately. This has been addressed by Epstein and Zin, who axiomatically worked on non-expected utility and came up with the following (non-expected) utility function representation for a preference relation that considers time and states of nature as more than just two indices of the state of the world:

$$U_t = \left[c_t^{1-\rho} + \beta \cdot (E_t [U_{t+1}^{1-\sigma}])^{\frac{1-\rho}{1-\sigma}} \right]^{\frac{1}{1-\rho}}$$

where ρ measures inter-temporal elasticity of substitution, and σ is the coefficient of relative risk aversion. Notice that if $\rho = \sigma$, then this formula reduces to

$$U_t^{1-\rho} = c_t^{1-\rho} + \beta \cdot E_t [U_{t+1}^{1-\rho}]$$

If there is no uncertainty, then the expectation operator is redundant, and we are back to the CES function.

This proposed solution is able to account for the risk-free rate puzzle. However, to match the equity premium it still requires an *unreasonably* high σ .

Solution 2 - Habit Persistence

Suppose that each instant's felicity value depends not only on current, but also on past consumption amounts (people might be reluctant to see their consumption fall from one period to the other):

$$U = E_t \left[\sum_{t=0}^{\infty} \beta^t \cdot u(c_t, c_{t-1}) \right]$$

For instance,

$$U = E_t \left[\sum_{t=0}^{\infty} \beta^t \cdot \frac{(c_t - \lambda \cdot c_{t-1})^{1-\sigma}}{1-\sigma} \right]$$

This preference representation can solve the risk free puzzle with *reasonable* parameter values. A related version of this type of utility function is that where felicity depends on external effects (people might be happy if others around them enjoy high levels of consumption ... or quite the opposite!). A possible felicity index showing those characteristics could be:

$$u(c_t, \bar{c}_t, \bar{c}_{t-1}) = \frac{c_t^{1-\sigma}}{1-\sigma} \cdot \bar{c}_t^\gamma \cdot \bar{c}_{t-1}^\lambda$$

In this example, a high value of γ can produce an equity premium value close to that in the data, with a *reasonable*, low σ . The \bar{c}_{t-1}^λ component in preferences can be used to solve the risk-free puzzle. However, in spite of its ability to solve both puzzles with reasonable parameter values, this preference representation has the shortfall that it generates too variable non-stationary returns: r_i^{rf} is too variable compared to actual data, even though r^{rf} may be accurately explained.

Solution 3 - Peso Problem

Suppose everybody believed that with some small probability there could be a huge disaster (a nuclear war, say). This would be accounted for in prices (and hence, returns). Such a factor might explain the equity premium.

Solution 4 - Incomplete Markets

A key assumption in the Mehra and Prescott model is that there is a representative agent whose consumption equals aggregate consumption. This can be generalized to a numerous population if we assume that all individuals are perfectly insured - the maximum variability their consumption can show is aggregate variability. However, it is not true that every person's consumption has exactly the same variability as aggregate consumption. Individuals' productivity could also be subject shocks by itself (for instance, becoming handicapped after an accident).

Such a mismatch would imply that trying to explain the puzzles by a model based on a representative agent could not be successful. If markets are incomplete, equity holding decisions are taken by individuals who suffer "idiosyncratic" stochastic shocks that may differ from one another, and due to the incompleteness, consumers are not able to insure themselves against this idiosyncratic risk. Return differentials between risky and risk-free assets then must lure into equity individuals whose consumption variability is larger than the aggregate.

Solution 5 - Transaction Costs

Some authors have tried to explain the high risk premium as the consequence of high transaction costs to buy shares. However, this needs unrealistic cost levels to match the data.

Solution 6 - Production

Mehra and Prescott's model is an endowment economy. Could the introduction of production into the model affect its results? The answer is no: it is consumption that we are interested in; it does not really matter how consumption is provided for. This approach is not really relevant.

Solution 7 - Leverage

In Mehra and Prescott's model, equity is the price of the "tree" that yields the whole economy's production. However, actual equity does not exactly give its owner rights to the whole product of a company. Other parties have rights over a company's economic surplus, that come before shareholders. Creditors' claims have priority in case of bankruptcy.

Therefore, actual share dividends are more risky than consumption. There are "legal" risks involved in investing in shares, that are not reflected in Mehra and Prescott's formulation. Finance people tend to believe in this explanation more than economists. In finance, the total compensation for risk ("EP") satisfies:

$$EP = \text{amount of risk} \times \underbrace{\frac{\text{compensation}}{\text{unit of risk}}}_{\text{Sharpe ratio}}$$

The Sharpe ratio is a measure of the market price of risk. This approach to the equity premium puzzle emphasizes the explanation of the "amount of risk" component of EP.

3 References

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