

Econ 525a (first half)  
Fall 2007  
Yale University  
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## HOMEWORK #2

*This homework assignment is due on Friday, October 26.*

Consider a two-period growth model with idiosyncratic shocks to labor productivity. There are two types of consumers, each of whom has measure one-half. Both types of consumers have the same initial wealth  $\omega$  in period 1. Each consumer's labor productivity in period 2 is random. There are two states of the world in period 2. With probability one-half, type-1 consumers receive shock  $e^1$  and type-2 consumers receive shock  $e^2$ ; with probability one-half, type-1 consumers receive shock  $e^2$  and type-2 consumers receive shock  $e^1$ . Assuming that consumers do not value leisure, total labor supply in period 2 is  $L_2 \equiv (e^1 + e^2)/2$ .

Output in period 2 is produced by a profit-maximizing competitive firm with (aggregate) production function  $F(K_2, L_2)$ , where  $K_2$  is the total amount of capital in period 2. The production function exhibits constant-returns-to-scale and is strictly increasing and strictly concave in both of its arguments.

Consumers can save in the form of capital and they can purchase state-contingent claims to consumption. They also (inelastically) supply labor in a competitive labor market in period 2. Assuming a complete set of state-contingent claims, a typical consumer's budget constraints read:

$$c_1 = \omega - k_2 - q^1 a^1 - q^2 a^2$$

and

$$c_2^i = r(K_2, L_2)k_2 + w(K_2, L_2)e^i + a^i,$$

where  $c_1$  is consumption in period 1,  $c_2^i$  is consumption in state  $i$  in period 2,  $k_2$  is savings in capital, and  $a_i$  is the number of Arrow securities paying one unit of the consumption good in state  $i$  (and zero units in state  $j \neq i$ ). The rental rate of capital  $r$  and the wage rate  $w$  are the appropriate marginal products of the production function;  $q^i$  is the price of an Arrow security that pays off in state  $i$ . Consumers seek to maximize  $u(c_1) + \beta E[u(c_2)]$  and they face a no-borrowing constraint on capital:  $k_2 \geq 0$ .

The equilibrium conditions are that  $K_2 = k_2$  (i.e., total savings in period 2 is the sum of the savings of both types of consumers—which are the same for both consumers since they face symmetric decision problems) and that the markets for the two Arrow securities clear.

- (a) Prove that the competitive equilibrium prices and allocations for this economy are the same as those that obtain in an economy without uncertainty (i.e., one in which

$e_1 = e_2$ ).

- (b) Now consider an economy with incomplete markets. In particular, assume that consumers cannot trade Arrow securities, but that they can save in the form of capital. Find an equation that determines aggregate savings  $K_2$  in period 2. Is aggregate savings higher or lower than in part (a)? (In the absence of a proof, try a numerical example with logarithmic utility.)
- (c) In the economy with incomplete markets, suppose that the government introduces a proportional tax on investment and returns the proceeds to consumers in a lump-sum fashion. The consumer's budget constraints now read:

$$c_1 = \omega - (1 + \tau)k_2 + \tau K_2$$

and

$$c_2^i = r(K_2, L_2)k_2 + w(K_2, L_2)e^i.$$

A typical consumer takes the lump-sum subsidy  $\tau K_2$  as given when making his choices. Prove that increasing  $\tau$  from 0 reduces aggregate savings. (Hint: Find an equation that determines  $K_2$  implicitly as a function of  $\tau$ , totally differentiate it with respect to  $\tau$ , and then evaluate the resulting expression at  $\tau = 0$ .)

- (d) Prove that increasing  $\tau$  from 0 increases a typical consumer's ex ante welfare. Try to provide intuition for this result. (Hint: You may need to use the fact that if  $F$  is homogeneous of degree 1, then  $KF_{KK} + LF_{KL} = 0$ .)