Econ 525a
Fall 2014
Yale University
Prof. Tony Smith

HOMEWORK #1

This homework assignment is due on September 19.

1. Consider a two-period neoclassical growth model with endogenous leisure but no uncertainty. There are two types of consumers, A and B. Type-A consumers comprise fraction θ of the population and type-B consumers comprise fraction $1-\theta$ of the population. All consumers have time-separable preferences with a felicity function given by:

$$\log(c) + D\log(\ell),$$

where c is consumption and $\ell \in [0,1]$ is hours of leisure in any given time period. A typical type-A consumer has initial holdings of capital equal to k_A, while a typical type-B consumer has initial holdings of capital equal to $k_B < k_A$.

Find the equilibrium prices (i.e., the rental rate of capital and the wage rate in both periods) and quantities (i.e., capital holdings and hours of leisure for both types in both periods) in this economy. Show that changes in k_A and k_B that leave the initial aggregate capital stock unchanged have no effect either on prices or on aggregate quantities (i.e., total capital accumulation and total labor supply in either period).

2. Imagine a consumer who either has a low wage ($w = w_1$) or a high wage ($w = w_2$) and whose asset holdings a are restricted to lie in the set $\{a_1, a_2, a_3\}$, where $a_1 < a_2 < a_3$. The wage w follows a discrete-state Markov chain with transition probabilities $\pi_{ij} = P(w' = w_j | w = w_i)$. Let the consumer's savings decision rule $a' = g(a, w)$ take the following form:

(i) $a_1 = g(a_1, w_1)$, $a_1 = g(a_2, w_1)$, $a_2 = g(a_3, w_1)$

(ii) $a_2 = g(a_1, w_2)$, $a_3 = g(a_2, w_2)$, $a_3 = g(a_3, w_2)$

(a) Suppose that $\pi_{22} = 0.95$ and $\pi_{11} = 0.5$. Find the invariant distribution over the two wage levels w_1 and w_2. That is, find two numbers p_1 and p_2 such that if fraction p_i of consumers have wage equal to w_i today, then these fractions replicate themselves in the next period.

(b) Now find the invariant distribution p_{ij} over the discrete state space $\{a_1, a_2, a_3\} \times \{w_1, w_2\}$.

(c) Suppose that consumers are distributed initially according to the invariant distribution that you computed in part (b). What is the probability that a (randomly chosen) consumer with the lowest level of asset holdings today has the highest level of asset holdings three periods from now?

(d) Suppose that initially consumers are spread uniformly over the state space. Compute the dynamics of the distribution of consumers as time evolves and verify numerically that this distribution converges to the one that you computed in part (b). (You may want to write a program, say, in Matlab, to automate the numerical calculations.)