

Econ 525a
Fall 2014
Yale University
Prof. Tony Smith

PROBLEM SET #6

This problem set is due on November 10; please put it in my mailbox at 28 Hillhouse.

1. Write a program that uses Chebyshev interpolation to approximate the function $f(x) = \log(x)$ on the interval $[0.1, 1]$. How does the approximation error change as the degree of the approximation increases from 1 (linear) to 4 (quartic)? How well does the interpolating polynomial perform outside the interval of approximation? (Recall that Chebyshev interpolation means to construct a polynomial of degree n that matches f at $n + 1$ grid points that are the roots of the Chebyshev polynomial of degree $n + 1$. One way to construct this polynomial is to write it as a linear combination of Chebyshev polynomials, with coefficients calculated using the formulas presented in lecture. Chebyshev polynomials are defined only on $[-1, 1]$, so evaluate them at $z = 2(x - a)/(b - a) - 1$, where $x \in [a, b]$ and $[a, b]$ is the interval of approximation.)
2. Write a program that uses Gauss-Hermite quadrature to compute $E[e^X]$, where $X \sim N(\mu, \sigma^2)$. Set $\mu = 1$ and $\sigma = 1, 2, 3$. How does your answer change as you vary the number of quadrature points from 2 to 10? (You can check your numerical answer either by using the analytical formula $E[e^X] = e^{\mu + \sigma^2/2}$ or by estimating the expectation using Monte Carlo integration. To obtain the Gauss-Hermite weights and abscissas, you can use the program `gauher` in Chapter 4.5 of *Numerical Recipes*.)