HOMEWORK #1

1. Write a program (in either Matlab or Fortran) to solve the neoclassical growth model using value iteration on a discrete grid (this is the method that we discussed in lecture on September 11). Let the production function take the form $f(k) = Ak^\alpha + (1 - \delta)k$, where $A > 0$, $0 < \alpha < 1$, and $0 \leq \delta \leq 1$. Let the utility (or felicity) function be $U(c) = \log(c)$. Center your grid at the steady-state capital stock $\bar{k}$, as defined by $f'(\bar{k}) = \beta^{-1}$. Start with a small number (say, 11) of equally-spaced grid points, and then increase this number to, say, 101. Obtain numerical results both for the case of full depreciation ($\delta = 1$) and for the case of less-than-full depreciation ($\delta < 1$). For $\delta = 1$, compare your numerical findings to the analytical (closed-form) solutions for the value function and the decision rule.

2. Use one-sided finite differences to compute an approximation to the first derivative of $g(p) \equiv 0.5p^{-0.5} + 0.5p^{-0.2}$ at $p = 1.5$. Let the increment $\epsilon$ in the finite differences range across all the values in the set $\{10^{-1}, 10^{-2}, \ldots, 10^{-10}\}$. For which value of $\epsilon$ is the approximate first derivative the most accurate?

3. Repeat the third problem using two-sided finite differences to approximate the first derivative.

4. Use the bisection, secant, and Newton’s methods to compute an estimate of $p^*$, where $g(p^*) = 0.75$ (and $g$ is defined in the second problem). For each method, report how many iterations are required to compute an estimate $\hat{p}$ satisfying $|f(\hat{p}) - f(p^*)| < 10^{-6}$.

5. Repeat the fourth problem using Brent’s method as described in Chapter 9.3 of *Numerical Recipes in Fortran*. 