

Econ 561b
Yale University
Spring 2010
Prof. Tony Smith

Syllabus for
COMPUTATIONAL METHODS FOR ECONOMIC DYNAMICS
ECON 561b

Course Objectives: Most of the dynamic models used in modern quantitative research in economics do not have analytical (closed-form) solutions. For this reason, the computer has become an indispensable tool for conducting research in economics. The goal of this set of lectures is to provide an introduction to computational tools for conducting numerical analysis of dynamic economic models. These tools have applications in all areas of economics, including macroeconomics, labor economics, industrial organization, financial economics, public finance, and political economy.

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Office hours: by appointment (or just stop by and see if I am free)

Class Meetings: The course meets on Mondays and Wednesdays from 10:30AM to 11:50AM in Room 106 (28 Hillhouse). This is a half-semester (seven-week) course; the last lecture is on Wednesday, February 24.

Prerequisites: This course is designed for graduate students in economics who have taken first-year graduate courses in microeconomics, macroeconomics, and econometrics. No prior knowledge of either numerical methods or computer programming is assumed, but some familiarity with a programming language would prove helpful.

Texts: The lectures will be largely self-contained, but there are several good texts that provide useful complements to the material in the lectures. An especially valuable book is: *Numerical Recipes in Fortran 77: The Art of Scientific Computing, Second Edition* (Volume 1 of Fortran Numerical Recipes) by William H. Press, Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery (Cambridge University Press, 1992). This book is available online (for free) at: www.nrbook.com/a/bookfpdf.php. Its companion, *Numerical Recipes in Fortran 90: The Art of Parallel Scientific Computing, Second Edition* (Volume 2 of Fortran Numerical Recipes), is also available online at: www.nrbook.com/a/bookf90pdf.php. (Note: The third edition of *Numerical Recipes*, with code available entirely in C++, is available online too—with a paid subscription—at www.nr.com. The third edition covers a few more topics than the second edition, but its text overlaps substantially with the second edition.)

Other useful books include:

- *Applied Computational Economics and Finance* by Mario J. Miranda and Paul L. Fackler (MIT Press, 2002).
- *Numerical Methods in Economics* by Kenneth L. Judd (MIT Press, 1998).
- *Dynamic Economics: Quantitative Methods and Applications* by Jérôme Adda and Russell Cooper (MIT Press, 2003).
- *Computational Methods for the Study of Dynamic Economies*, edited by Ramon Marimon and Andrew Scott (Oxford University Press, 1999).
- *Handbook of Computational Economics (Volume 1)*, edited by Hans M. Amman, David A. Kendrick, and John Rust (North-Holland, 1996).

Exercises: The best (and really the only) way to learn numerical methods is to use them in actual problems. Accordingly, each week of lectures will be accompanied by a set of problems for students to solve. It is highly recommended that students attempt to work these problems!

SCHEDULE OF LECTURES

Week 1

Introduction (built around some simple examples from economics, including the stochastic-growth model and a canonical consumption-savings model).

General considerations in numerical analysis: convergence, roundoff error, truncation error.

Numerical differentiation.

Root-finding in one or more dimensions: bisection, secant method, Newton's method, fixed-point iteration, Gauss-Jacobi, Gauss-Seidel, Brent's method.

Suggested readings:

Chapters 1, 5.7, and 9 in *Numerical Recipes*; Appendix 2A, Chapter 3, and Chapter 5.6 in Miranda and Fackler; Chapters 1, 2, 5, and 7.7 in Judd.

Huggett, M. (1993), "The Risk-Free Rate in Heterogeneous-Agents, Incomplete Markets Economies," *Journal of Economic Dynamics and Control* 17, 953–969.

Taylor, J.B. and H. Uhlig (1990), "Solving Nonlinear Stochastic Growth Models: A Comparison of Alternative Solution Methods," *Journal of Business and Economic Statistics* 8, 1–18.

Week 2

Minimization in one or more dimensions: golden section search, Brent's method with or without derivatives, simplex method, Newton-Raphson, variable metric methods.

Suggested readings: Chapter 10 in *Numerical Recipes*; Chapter 5 in Miranda and Fackler; Chapter 4 in Judd.

Week 3

Interpolation and approximation of functions: linear interpolation in several dimensions, cubic splines, polynomial interpolation, orthogonal polynomials.

Suggested readings: Chapters 3 and 6 in *Numerical Recipes*; Chapter 5 in Miranda and Fackler; Chapter 6 in Judd.

Week 4

Numerical integration: cubic spline integration, Gaussian quadrature, Monte Carlo integration, integration of multivariate normal densities.

Suggested readings: Chapters 4 and 7 in *Numerical Recipes*; Chapter 5 in Miranda and Fackler; Chapters 7 and 8 in Judd.

Week 5

Numerical dynamic programming: value iteration, Euler equation methods, rules of thumb, perturbation methods, parameterized expectations, linear-quadratic (first-order) and second-order methods.

Suggested readings:

Chapters 7, 8, and 9 in Miranda and Fackler; Chapters 12, 13, 16, and 17 in Judd.

Benitez-Silva, H., G. Hall, G. Hitsch, G. Pauletto, and J. Rust (2005), “A Comparison of Discrete and Parametric Approximation Methods for Continuous-State Dynamic Programming Problems,” manuscript (ms.cc.sunysb.edu/~hbenitezsilv/dpa2005.pdf).

Christiano, L.J. and J.D.M. Fisher (2000), “Algorithms for Solving Dynamic Models with Occasionally Binding Constraints,” *Journal of Economic Dynamics and Control* 24, 1179–1232.

Coleman, W.J. II (1990), “Solving the Stochastic Growth Model by Policy Function Iteration,” *Journal of Business and Economic Statistics* 8, 27–29.

Keane, M.P. and K. Wolpin (1994), “The Solution and Estimation of Discrete Choice Dy-

dynamic Programming Models by Simulation and Interpolation: Monte Carlo Evidence,” *The Review of Economics and Statistics* 76, 648–672.

Kim, J., S. Kim, E. Schaumburg, and C.A. Sims (2008), “Calculating and Using Second Order Accurate Solutions of Discrete Time Dynamic Equilibrium Models,” *Journal of Economic Dynamics and Control* 32, 3397–3414.

Schmitt-Grohé, S. and M. Uribe (2004), “Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function,” *Journal of Economic Dynamics and Control* 28, 755–775.

Smith, Jr., A.A. (1991), “Solving Stochastic Dynamic Programming Problems Using Rules of Thumb,” Queen’s University Discussion Paper No. 816.

Uhlig, H. (1999), “A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily,” in: *Computational Methods for the Study of Dynamic Economies*.

Weeks 6 and 7

Computation of dynamic equilibrium models with heterogeneous actors.

Suggested readings:

Aiyagari, S.R. (1994), “Uninsured Idiosyncratic Risk and Aggregate Saving,” *Quarterly Journal of Economics* 109, 659–684.

Bachmann, R. (2009), “Understanding Jobless Recoveries,” manuscript (umich.edu/~rudib/jobmarketpaper_sentout.pdf).

Den Haan, W.J. (2010), “Comparison of Solutions to the Incomplete Markets Model with Aggregate Uncertainty,” *Journal of Economic Dynamics and Control* 34, 4–27.

Devereux, M. and A. Sutherland (2008), “Country Portfolios in Open Economy Macro Models,” forthcoming in *Journal of the European Economic Association* (NBER Working Paper No. 14372).

Güvenen, M.F. (2009), “A Parsimonious Macroeconomic Model for Asset Pricing,” *Econometrica* 77, 1711–1750.

Hopenhayn, H. and R. Rogerson (1993), “Job Turnover and Policy Evaluation: A General

- Equilibrium Analysis,” *Journal of Political Economy* 101, pp. 915–938.
- Huggett, M. (1996), “Wealth Distribution in Life-Cycle Economies,” *Journal of Monetary Economics* 38, 469–494.
- Khan, A. and J.K. Thomas (2002), “Nonconvex Factor Adjustments in Equilibrium Business Cycle Models: Do Nonlinearities Matter?”, *Journal of Monetary Economics* 50, 331–360.
- Khan, A. and J.K. Thomas (2003), “Inventories and the Business Cycle: An Equilibrium Analysis of (S,s) Policies,” *American Economic Review* 97, 1165–1188.
- Krueger, D. and F. Kubler (2003), “Computing Equilibrium in OLG Models with Stochastic Production,” *Journal of Economic Dynamics and Control* 28, 1411–1436.
- Krusell, P. and A.A. Smith, Jr. (1997), “Income and Wealth Heterogeneity, Portfolio Selection, and Equilibrium Asset Returns,” *Macroeconomic Dynamics* 1, 387–422.
- Krusell, P. and A.A. Smith, Jr. (1998), “Income and Wealth Heterogeneity in the Macroeconomy,” *Journal of Political Economy* 106, 867–896.
- Krusell, P., T. Mukoyama, A. Şahin, and A.A. Smith, Jr. (2009), “Revisiting the Welfare Effects of Eliminating Business Cycles” *Review of Economic Dynamics*, 12, 393–404.
- Kubler, F. and K. Schmedders (2005), “Approximate versus Exact Equilibria in Dynamic Economies,” *Econometrica* 73, 1205–1235.
- Midrigan, V. (2009), “Menu Costs, Multi-Product Firms and Aggregate Fluctuations,” manuscript (homepages.nyu.edu/~vm50/scannerprices_july09.pdf).
- Reiter, M. (2009), “Solving Heterogenous Agent Models by Projection and Perturbation,” *Journal of Economic Dynamics and Control* 33, 649–665.
- Reiter, M. (2009), “Approximate and Almost-Exact Aggregation in Dynamic Stochastic Heterogeneous-Agent Models,” manuscript (elaine.ihs.ac.at/~mreiter/appraggr.pdf).
- Ríos-Rull, J.V. (1999), “Computation of Equilibria in Heterogeneous-Agent Models,” in: *Computational Methods for the Study of Dynamic Economies*.
- Telmer, C., K. Storesletten, and A. Yaron (2007), “Asset Pricing with Idiosyncratic Risk and Overlapping Generations,” *Review of Economic Dynamics* 10, 519–548.
- Thomas, J.K. (2002), “Is Lumpy Investment Relevant for the Business Cycle?” *Journal of*

Political Economy 110, 508–534.

Weintraub, G., L. Benkard, and B. Van Roy (2008), “Markov Perfect Industry Dynamics with Many Firms,” *Econometrica* 76, 1375–1411.

Weintraub, G., L. Benkard, and B. Van Roy (2009), “Computational Methods for Oblivious Equilibrium,” forthcoming in *Operations Research*
(www.econ.yale.edu/~lanierb/research/CompOR090818-posted.pdf).