Econ 561b
Yale University
Spring 2004
Prof. Tony Smith

**Syllabus for**
**COMPUTATIONAL METHODS FOR ECONOMIC DYNAMICS**
**ECON 561b**

**Course Objectives**: The goals of this course are to teach students the basic tools of numerical analysis and to illustrate how these tools can be used to address analytically intractable problems in economics and econometrics. The underlying theme of this course is that computational methods belong in every economist's toolkit. These methods can be used not only to explore the theoretical implications of economic models in the absence of analytical solutions, but also to assess the *quantitative* importance of the various forces at play in an economic model. Thus computation can help to advance both theoretical and empirical work in economics. In order to teach students how to wield computational tools in an informed and intelligent way, this course endeavors to explain not only *when and how* to use various numerical algorithms but also *how and why* they work; in other words, the course opens up the "black boxes".

Another theme of this course is that computational methods are vital to all types of research in economics, from the most theoretical to the most applied. To this end, the substantive applications in the course are drawn from a wide range of fields, including macroeconomics, finance, game theory, industrial organization, public finance, contract theory, and econometrics. The course will pay special attention to dynamic economic problems, including methods for solving stochastic dynamic programming problems, for computing equilibria with heterogeneous firms and consumers, and for estimating the parameters of dynamic economic models.

**Contact Information**
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Office hours: Thursdays from 10AM–noon, or by appointment

**Class Meetings**: The course meets on Mondays from 4PM to 5:20PM in Room B8 (28 Hillhouse) and Wednesdays from 4PM to 5:20PM in Room 106 (28 Hillhouse).

**Texts:** The required textbooks for this course are: *Numerical Methods in Economics* by Kenneth L. Judd (MIT Press, 1998) and *Numerical Recipes in Fortran 77 (Second Edition)* by William H. Press, Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery (Cambridge University Press, 1992). Both of these books are available in the Yale bookstore. Students who are familiar with the C or C++ programming languages may want to use versions of *Numerical Recipes* geared towards these languages. *Numerical Recipes in Fortran 90: The Art of* Parallel *Scientific Computing* is a useful companion volume to *Numerical Recipes in Fortran 77*; in addition to using a more modern version (or standard) of the Fortran programming language, this book shows how to write Fortran programs that take advantage of parallel computing. The Numerical Recipes books are also available online at: www.nr.com.

Other (optional) books that students might find useful are: the *Handbook of Computational Economics (Volume 1)*, edited by Hans M. Amman, David A. Kendrick, and John Rust (North-Holland, 1996); *Computational Methods for the Study of Dynamic Economies*, edited by Ramon Marimon and Andrew Scott (Oxford University Press, 1999); *Frontiers of Business Cycle Research*, edited by Thomas F. Cooley (Princeton University Press, 1995); and *Dynamic Economics: Quantitative Methods and Applications* by Jérôme Adda and Russell Cooper (MIT Press, 2003).

**Grading**: Occasional problem sets will constitute 30% of the course grade and a project (described in more detail below) will constitute 70% of the course grade.

Students may use any programming language to complete the problem sets, including Fortran, C, Matlab, and Gauss. If you do not already know a programming language, Matlab is probably the easiest to learn. If you want to do state-of-the-art research using computational methods, however, you will need ultimately to learn a fast high-level programming language such as Fortran or C. This course will not teach programming *per se*, but it will teach and emphasize general principles of programming, such as simplicity, clarity, structure, replicability, and testing. Since one of the goals of this course is to teach students what is going on *inside* the "black boxes" of numerical algorithms, students are asked to avoid the use of such black boxes except for routine tasks.

The project will consist of the application of computational methods to a problem in economics or econometrics. This problem could be original research, perhaps as a first step towards a Ph.D. dissertation. However, the problem need not consist of original research: one acceptable option for the project is, in fact, to attempt to replicate the computational results in an existing paper.

The written report for the project should consist of three parts: a brief description of the problem, a detailed description of the computational methods used to solve the problem (including a copy of the code), and a thorough description of the numerical results.

# COURSE OUTLINE

## 1 Basic numerical methods

- Introduction (built around the Huggett-Aiyagari model; see references in Section 3 below); general considerations in numerical analysis (convergence, roundoff error, truncation error), numerical differentiation.
  [Judd: Chapters 1, 2, and 7.7; *Numerical Recipes*: Chapters 1 and 5.7]
- Root-finding in one or more dimensions (bisection, secant method, Newton's method, fixed-point iteration, Gauss-Jacobi, Gauss-Seidel, Brent's method).
  [Judd: Chapter 5; *Numerical Recipes*: Chapter 9]
- Minimization in one or more dimensions (golden section search, Brent's method with or without derivatives, simplex method, Newton-Raphson, conjugate gradient methods, variable metric methods, simulated annealing).
  [Judd: Chapter 4; *Numerical Recipes*: Chapter 10]
- Interpolation and approximation of functions (linear interpolation in several dimensions, cubic splines, polynomial interpolation, searching a table, orthogonal polynomials).
  [Judd: Chapter 6; *Numerical Recipes*: Chapters 3 and 6]
- Random numbers, simulation, and introduction to asymptotic theory.
  [Judd: Chapter 8; *Numerical Recipes*: Chapter 7]
- Integration (cubic spline integration, Gaussian quadrature, Monte Carlo integration, integration of multivariate normal densities).
  [Judd: Chapter 7; *Numerical Recipes*: Chapter 4]

## 2 Numerical dynamic programming

- Linear-quadratic methods
  - Uhlig, H. (1999), "A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily," in: *Computational Methods for the Study of Dynamic Economies*.
  - Anderson, E., L.P. Hansen, E.R. McGrattan, and T.J. Sargent (1996), "Mechanics of Forming and Estimating Dynamic Linear Economies," in: *Handbook of Computational Economics*.
- Second-order methods

- Kim, J., S. Kim, E. Schaumburg, and C.A. Sims (2003), "Calculating and Using Second Order Accurate Solutions of Discrete Time Dynamic Equilibrium Models," manuscript (www.princeton.edu/∼sims/#gensys2).

- Nonlinear methods for models with continuous choices: value iteration, Euler equation methods, rules of thumb, perturbation methods, Coleman's method, parameterized expectations.
  [Judd: Chapters 12, 13, 16, and 17]

  - Benitez-Silva, H., G. Hall, G. Hitsch, G. Pauletto, and J. Rust (2000), "A Comparison of Discrete and Parameteric Approximation Methods for Continuous-State Dynamic Programming Problems," manuscript (www.econ.yale.edu/∼econ681/cefbcn.html)
  - Christiano, L.J. and J.D.M. Fisher (2000), "Algorithms for Solving Dynamic Models with Occasionally Binding Constraints," *Journal of Economic Dynamics and Control* 24, 1179–1232.
  - Coleman, W.J. II (1990), "Solving the Stochastic Growth Model by Policy Function Iteration," *Journal of Business and Economic Statistics* 8, 27–29.
  - Krusell, P., Kuruşçu, B., and A.A. Smith, Jr. (2002), "Time Orientation and Asset Prices," *Journal of Monetary Economics* 49, 107–135.
  - Rust, J. (1997), "Using Randomization to Break the Curse of Dimensionality," *Econometrica* 65, 487–516.
  - Smith, Jr., A.A. (1991), "Solving Stochastic Dynamic Programming Problems Using Rules of Thumb," Queen's University Discussion Paper No. 816.

- Nonlinear methods for models with discrete choices

  - Keane, M.P. and K.I. Wolpin (1994), "The Solution and Estimation of Discrete Choice Dynamic Programming Models by Simulation: Monte Carlo Evidence," *Review of Economics and Statistics* 76, 648–672.

# 3 Computing equilibria of heterogeneous-actor models

- Updating distributions, computing invariant distributions, finding equilibrium prices.

  - Aiyagari, S.R. (1994), "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics* 109, 659–684.
  - Burstein, A. (2003), "Inflation and Output Dynamics with State Dependent Pricing Decisions," manuscript (www.econ.ucla.edu/people/defaultpapers.cfm?NAME=Burstein).

– Campbell, J.R. (1998) “Entry, Exit, Embodied Technology, and Business Cycles,” *Review of Economic Dynamics* 1, 371–408.

– Campbell, J.R. and J.D.M. Fisher (2000), “Aggregate Employment Fluctuations with Microeconomic Asymmetries,” *American Economic Review* 90, 1323–1345.

– Castañeda, A., J. Díaz-Giménez, and J.-V. Ríos-Rull (2003), “Accounting for the U.S. Earnings and Wealth Inequality,” *Journal of Political Economy* 111, 814–857.

– Chatterjee, S., D. Corbae, M. Nakajima, and J.-V. Ríos-Rull (2002),“A Quantitative Theory of Unsecured Consumer Credit with Risk of Default,” manuscript (www.ssc.upenn.edu/∼vr0j/).

– Dotsey, M. R.G. King, A.L. Wolman (1999), “State-Dependent Pricing and the General Equilibrium Dynamics of Money and Output,” *Quarterly Journal of Economics* 114, 655–690.

– Golosov, M. and R.E. Lucas, Jr. (2003), “Menu Costs and Phillips Curves,” manuscript (www.econ.umn.edu/%7Egolosov/research.htm).

– Guvenen, M.F. (2002), “Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective,” manuscript (www.econ.rochester.edu/guvenen/guvenen.htm).

– Guvenen, M.F. (2003) “A Parsimonious Macroeconomic Model for Asset Pricing: Habit Formation or Cross-Sectional Heterogeneity?” manuscript (www.econ.rochester.edu/guvenen/guvenen.htm).

– Huggett, M. (1993), “The Risk-Free Rate in Heterogeneous-Agents, Incomplete Markets Economies,” *Journal of Economic Dynamics and Control* 17, 953–969.

– Huggett, M. (1996), “Wealth Distribution in Life-Cycle Economies,” *Journal of Monetary Economics* 38, 469–494.

– Khan, A. and J.K. Thomas (2002), “Nonconvex Factor Adjustments in Equilibrium Business Cycle Models: Do Nonlinearities Matter?” *Journal of Monetary Economics* 50 331–360.

– Khan, A. and J.K. Thomas (2003), “Inventories and the Business Cycle: An Equilibrium Analysis of (S,s) Policies,” manuscript (www.econ.umn.edu/∼jkt/).

– Krusell, P. and A.A. Smith, Jr. (1997), “Income and Wealth Heterogeneity, Portfolio Selection, and Equilibrium Asset Returns,” *Macroeconomic Dynamics* 1, 387–422.

– Krusell, P. and A.A. Smith, Jr. (1998), “Income and Wealth Heterogeneity in the Macroeconomy,” *Journal of Political Economy* 106, 867–896.

– Krusell, P. and A.A. Smith, Jr. (1999), “On the Welfare Effects of Eliminating Business Cycles,” *Review of Economic Dynamics* 2, 245–272.

- Krusell, P. and A.A. Smith, Jr. (1999), "Revisiting the Welfare Effects of Eliminating Business Cycles," manuscript (fasttone.gsia.cmu.edu/tony)
- Nakajima, M. and J.-V. Ríos-Rull (2002), "Default and Aggregate Fluctuations in Storage Economies," manuscript (www.ssc.upenn.edu/∼vr0j/).
- Ríos-Rull, J.V. (1999), "Computation of Equilibria in Heterogeneous-Agent Models," in: *Computational Methods for the Study of Dynamic Economies*.
- Smith, Jr., A.A. and C. Wang (2000), "Dynamic Credit Relationships in General Equilibrium," manuscript (fasttone.gsia.cmu.edu/tony).
- Telmer, C., K. Storesletten, and A. Yaron (2001), "Asset Pricing with Idiosyncratic Risk and Overlapping Generations," manuscript (bertha.gsia.cmu.edu/telmerc/research.html).
- Thomas, J.K. (2002), "Is Lumpy Investment Relevant for the Business Cycle?" *Journal of Political Economy* 110, 508–534.
- Veracierto, M.L. (2002), "Plant-Level Irreversible Investment and Equilibrium Business Cycles," *American Economic Review* 92, 181–197.

# 4 Computing Markov-perfect equilibria

- Applications to hyperbolic (quasi-geometric) discounting, industrial organization, and time-consistent fiscal policy.
  - Ericson, R. and A. Pakes (1995), "Markov-Perfect Industry Dynamics: A Framework for Empirical Work," *Review of Economic Studies* 62, 53–82.
  - Klein, P. P. Krusell, and J.-V. Ríos-Rull (2003), "Time-Consistent Public Expenditures," manuscript (www.ssc.upenn.edu/∼vr0j/).
  - Klein, P. and J.-V. Ríos-Rull (2003), "Time-Consistent Optimal Fiscal Policy," *International Economic Review* 44, 1217–1246.
  - Krusell, P., Kuruşçu, B., and A.A. Smith, Jr. (2002), "Equilibrium Welfare and Government Policy with Quasi-Geometric Discounting," *Journal of Economic Theory* 105, 42–72.
  - Pakes, A. and P. McGuire (1994), "Computing Markov-Perfect Nash Equilibria: Numerical Implications of a Dynamic Differentiated Product Model," *RAND Journal of Economics* 25, 555–589.
  - Pakes, A. and P. McGuire (2001), "Stochastic Algorithms, Symmetric Markov Perfect Equilibrium, and the 'Curse' of Dimensionality," *Econometrica* 69, 1261–1281.

# 5 Estimation by simulation

- Simulated method of moments, simulated minimum distance estimation, and indirect inference


  - Duffie, D. and K.J. Singleton (1993), "Simulated Moments Estimation of Markov Models of Asset Prices," *Econometrica* 61, 929–952.
  - Gallant, A.R. and G.E. Tauchen (1996), "Which Moments to Match?" *Econometric Theory* 12, 657–681.
  - Gourieroux, C., A. Monfort, and E. Renault (1993), "Indirect Inference," *Journal of Applied Econometrics* 8, S85–S118.
  - Keane, M.P. and J. Geweke (2001), "Computationally Intensive Methods for Integration in Econometrics," in: *Handbook of Econometrics*, Volume 5, J.J. Heckman and E.E. Leamer (eds.), Elsevier Science B.V., 3463-3568.
  - Keane, M.P. and A.A. Smith, Jr. (2003), "Generalized Indirect Inference for Discrete Choice Models," manuscript (fasttone.gsia.cmu.edu/tony).
  - Lee, B. and B.F. Ingram (1991), "Simulation Estimation of Time Series Models", *Journal of Econometrics* 47, 197–205.
  - McFadden, D.L. (2001), "A Method of Simulated Moments for Estimation of Discrete Response Models Without Numerical Integration," *Econometrica* 57, 995-1026.
  - Smith, Jr., A.A. (1993), "Estimating Nonlinear Time–series Models Using Simulated Vector Autoregressions," *Journal of Applied Econometrics* 8, S63–S84.