TEMPTATION AND TAXATION

BY PER KRUSELL, BURHANETTIN KURUŞÇU, AND ANTHONY A. SMITH, JR.[1]

We study optimal taxation when consumers have temptation and self-control problems. Embedding the class of preferences developed by Gul and Pesendorfer into a standard macroeconomic setting, we first prove, in a two-period model, that the optimal policy is to subsidize savings when consumers are tempted by "excessive" impatience. The savings subsidy improves welfare because it makes succumbing to temptation less attractive. We then study an economy with a long but finite horizon which nests, as a special case, the Phelps–Pollak–Laibson multiple-selves model (thereby providing guidance on how to evaluate welfare in this model). We prove that when period utility is logarithmic, the optimal savings subsidies increase over time for any finite horizon. Moreover, as the horizon grows large, the optimal policy prescribes a constant subsidy, in contrast to the well known Chamley–Judd result.

KEYWORDS: Temptation, self-control, consumption-savings, optimal taxation.

1. INTRODUCTION

EXPERIMENTAL AND INTROSPECTIVE EVIDENCE suggest that consumers exhibit preference reversals as time passes. Such evidence has led to the development of models in which consumers have time-inconsistent preferences (see Laibson (1997), who built on earlier work by Strotz (1956) and Phelps and Pollak (1968)). In models with time-inconsistent preferences, a sequence of the consumer's different "selves," each valuing consumption streams in a unique way, plays a dynamic game. In this game of conflict across selves, one can define Pareto frontiers among selves and discuss non-cooperative equilibria of the dynamic game relative to this frontier. Consequently, policy proposals by an outside authority, such as the government, do not, in general, lead to unambiguous recommendations without deciding how to assign welfare weights to the different selves.

In contrast, Gul and Pesendorfer (2001, 2004, 2005) developed an alternative, axiomatic, approach to modeling preference reversals. This approach does not necessitate splitting up the consumer into multiple selves. To address reversals, Gul and Pesendorfer formalized the ideas of temptation and self-control: they defined preferences over consumption sets rather than over consumption sequences, and then discussed temptation and self-control in terms of preferences over these sets. The axiomatization delivers a representation theorem with utility over consumption sets expressed in terms of two utility functions: one describes *commitment utility* (*u*), which gives the ranking that the consumer uses to compare consumption bundles, as opposed to

consumption sets; the other describes *temptation utility* (v), which plays a key role in determining how actual consumption choices depart from what commitment utility would dictate. In this framework, the consumer's *welfare of a given set B*, where B, for example, could be a normal budget set, is given by $\max_{x \in B}[u(x) + v(x)] - \max_{\tilde{x} \in B} v(\tilde{x})$, the sum of the commitment and temptation utilities *less* the temptation utility evaluated at the most tempting choice (i.e., the maximal level of temptation). The consumer's actual choice maximizes the sum of the commitment and temptation utilities. In addition, the consumer experiences a (utility) cost of self-control, $\max_{\tilde{x} \in B} v(\tilde{x}) - v(x)$, which increases to the extent that the consumer's actual choice deviates from succumbing completely to the temptation. The consumer's actual choice then represents a compromise between commitment utility and the cost of self-control.

Using the Gul–Pesendorfer model, it is straightforward to ask normative questions. The purpose of this paper is to examine optimal tax policy with their model. In particular, we look at Ramsey taxation, that is, we consider whether and how linear tax-transfer schemes can be used to improve consumer welfare. Linearity here is a restriction in our analysis: if any nonlinear taxation scheme is allowed, one could (trivially) circumvent the self-control/temptation problems by implementing a command policy in which the consumer's choice set is reduced to a singleton. There are obvious reasons why such schemes are not attractive to use in practice, but more importantly we show that even with the rather weak instrument we offer the government, it is possible to improve utility in most cases, and that in some cases a linear tax can actually achieve the full optimum. We discuss and motivate linearity in more detail in Section 2.3.

Overall, the question here is how different (linear) distortion rates affect the welfare of consumers who suffer from temptation and self-control problems. To begin our analysis, we look at a two-period model with general preferences, except that we specialize temptation utility to reflect impatience, since this is the object of our study. In the two-period model, the consumption set faced by an agent is the usual triangle, and taxes alter the precise nature of the triangle. We first analyze a partial equilibrium economy (i.e., prices are exogenous) in which we let the government use a tax-transfer scheme that—for the consumer's actual choice—uses up no net resources and thus is self-financing. For example, the government can make consumption in period 1 more expensive relative to consumption in period 2 by subsidizing period-2 consumption, and to the extent the consumer responds by buying more consumption in period 2 than his endowment, the government must use a lump-sum tax in period 1 to balance its budget. We show that, in general, taxation can improve welfare and that a temptation toward impatience calls for subsidizing consumption in period 2. We also examine the size of these subsidies.

Subsidizing period-2 consumption improves welfare because it makes temptation less attractive. To see this, remember that the consumer's actual choice maximizes the sum of the commitment and temptation utilities. Because of the envelope theorem, a small increase (from zero) in the subsidy for period-2

consumption has no effect on the sum of the commitment and temptation utilities. But this increase reduces the maximal level of temptation: the consumer receives a smaller subsidy if he gives in to temptation (because in that case he consumes more today and less tomorrow), but the (lump-sum) tax that the consumer pays to finance the subsidy remains unchanged (because it depends on the consumer's *actual* choice).

We then consider general-equilibrium effects, which are important for two reasons. First, in an endowment economy, tax policy is not useful at all. In this case, the consumption allocation cannot be altered and for it to be supported in equilibrium by a triangle budget set, the slope of this set, net of taxes, must be unaffected by policy. Thus, pre-tax prices adjust to undo fully the tax wedge. Second, we show that if the production technology takes the standard neoclassical form (constant returns to scale and diminishing marginal products), government policy again has a role to play by altering equilibrium investment. In particular, using a representative-agent equilibrium model, we show that the partial-equilibrium result remains intact: it is optimal to subsidize investment. The intuition underlying this result is the same as in the partial-equilibrium model, although general-equilibrium effects on prices reduce the size of the optimal subsidy (but do not, as in the endowment economy, eliminate any role for a subsidy). The contrast between the general-equilibrium economies with and without intertemporal production makes it clear that the government must be able to influence the slope of the budget constraint, and thereby aggregate savings, if it is to improve welfare outcomes in the presence of temptation and self-control problems.

Are our results special to the two-period model? In a setting with standard preferences and a choice between distorting either investment or labor supply, for example, Chamley (1986) and Judd (1985) showed that, in the long run, the government should not distort investment. In our model, labor supply is inelastic, but, nonetheless, is it possible that in the long run, investment should not be distorted/subsidized? We show, with a simple example, that the answer is, in general, no. We extend the two-period model to a T-period model in a way guided by the applied macroeconomics literature, which uses time-additive, stationary utility. Thus, we let commitment utility take the standard form and allow temptation utility simply to have a different current (or short-run) discount factor than commitment utility, reflecting impatience. This means that temptation utility reflects "quasigeometric" discounting of the future, which amounts to the assumption that nothing can be tempting other than changing current consumption relative to future consumption. We also introduce a parameter, γ, which regulates how strongly temptation utility influences consumption choices; $\gamma = 0$ delivers standard utility, where consumers act without self-control problems.

Quasigeometric temptation nests two cases of special interest: the first is the time-inconsistent preferences considered by Laibson (1997) (the case $\gamma \to \infty$, where the consumer succumbs completely to temptation) and the second is

the special case in Gul and Pesendorfer (2004) in which temptation utility puts zero weight on the future (so that the consumer is tempted to consume his entire wealth today). For the Laibson case, in particular, we show that the consumer discounts utility using the "long-run" discount rate, thereby resolving the problem of which self to use when evaluating welfare in the multiple-selves model.[2]

Specializing further to logarithmic period utility, we solve fully for the laissez-faire and optimal outcomes. We find that optimal investment subsidy rates increase over time in any finite-horizon model. More importantly, as $T \to \infty$, the optimum calls for a constant subsidy rate on investment, in contrast to the Chamley–Judd result. Finally, in the Laibson case in which consumers succumb completely to temptation, we show, for any period utility function featuring constant relative risk aversion, that linear taxes are, in fact, not restrictive, but instead deliver first-best welfare outcomes (i.e., the welfare outcome associated with the command outcome) and that, as in the logarithmic case, the optimum calls for a subsidy to savings as $T \to \infty$.

Section 2 looks at the two-period model and Section 3 looks at the T-period model. Section 4 concludes. Proofs of Propositions 6 and 8 are gathered in the Appendix. All other proofs are provided in the Supplemental material (Krusell, Kuruşçu, and Smith (2010)).

2. THE TWO-PERIOD MODEL

For illustrating temptation and self-control problems in the savings context, a two-period model captures much of the essence, and in this section we provide some general results for this setting. In Section 3 we then examine some aspects of the further dynamics that appear in models with more periods.

2.1. *Preferences*

A typical consumer in the economy values consumption today (c_1) and tomorrow (c_2). Specifically, the consumer has Gul–Pesendorfer preferences represented by two functions $u(c_1, c_2)$ and $v(c_1, c_2)$, where u is commitment utility and v is temptation utility. The decision problem of a typical consumer then is

$$\max_{c_1, c_2}\{u(c_1, c_2) + v(c_1, c_2)\} - \max_{\tilde{c}_1, \tilde{c}_2} v(\tilde{c}_1, \tilde{c}_2)$$

subject to a budget constraint that we specify below. The consumer's actual choice maximizes the sum $u(c_1, c_2) + v(c_1, c_2)$ of the commitment and temptation utilities, and for any choice bundle (c_1, c_2), the cost of self-control is $\max v(\tilde{c}_1, \tilde{c}_2) - v(c_1, c_2)$. We make three assumptions.

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[2]See Proposition 6.

ASSUMPTION 1: $u(c_1, c_2)$ and $v(c_1, c_2)$ are twice continuously differentiable.

ASSUMPTION 2: $\frac{u_1(c_1,c_2)}{u_2(c_1,c_2)} < \frac{v_1(c_1,c_2)}{v_2(c_1,c_2)}$ for all c_1 and c_2.

ASSUMPTION 3: $u_1, u_2, v_1, v_2 > 0$, $u_{11}, u_{22}, v_{11}, v_{22} < 0$, and $u_{12}, v_{12} \geq 0$.

Assumption 2 specializes to the case where temptation utility is tilted toward current consumption more than is commitment utility.

2.2. Budget Constraints

Each consumer is endowed with k_1 units of capital at the beginning of the first period and with one unit of labor in each period. Consumers rent these factors at given prices. Let r_1 (r_2) and w_1 (w_2) be the gross return on savings and wage rate in the first (second) period, respectively, and let P be the price vector defined as $P = (r_1, r_2, w_1, w_2)$. We will specify the determination of prices in the following subsections. Given these prices, the consumer's budget set is

$$B(k_1, P) \equiv \{(c_1, c_2): \exists k_2: c_1 = r_1 k_1 + w_1 - k_2 \text{ and } c_2 = r_2 k_2 + w_2\},$$

where k_2 is the consumer's asset holding at the beginning of period 2 (i.e., his savings in period 1).

Inserting the definitions of the functions u and v into the consumer's objective function and combining terms, a typical consumer's decision problem is

$$\max_{(c_1,c_2)\in B(k_1,P)} \{u(c_1, c_2) + v(c_1, c_2)\} - \max_{(\tilde{c}_1,\tilde{c}_2)\in B(k_1,P)} v(\tilde{c}_1, \tilde{c}_2).$$

In this two-period problem, the temptation part of the problem (i.e., the second maximization problem in the objective function) plays no role in determining the consumer's actions in period 1. The temptation part of the problem does, however, affect the consumer's welfare, as we discuss below in Section 2.4.

Letting $\bar{u} = u + v$, the consumer's intertemporal first-order condition is

$$\frac{\bar{u}_1(c_1, c_2)}{\bar{u}_2(c_1, c_2)} = \frac{u_1(c_1, c_2) + v_1(c_1, c_2)}{u_2(c_1, c_2) + v_2(c_1, c_2)} = r_2.$$

It is straightforward to see that the intertemporal consumption allocation (which, in effect, maximizes $u + v$) represents a compromise between maximizing u and maximizing v. In contrast, the allocation that maximizes the temptation utility satisfies $v_1(\tilde{c}_1, \tilde{c}_2)/v_2(\tilde{c}_1, \tilde{c}_2) = r_2$. Assumptions 2 and 3 imply that $\tilde{c}_1 > c_1$ and $\tilde{c}_2 < c_2$.

2.3. *Government Policy*

We examine the effects of proportional taxes and subsidies. Thus, let there be a lump-sum transfer s and a proportional tax τ_i on investment in the first period. The consumer's budget set then is

$$B_\tau(k_1, P) \equiv \{(c_1, c_2) : \exists k_2 : c_1 = r_1 k_1 + w_1 + s - (1 + \tau_i) k_2 \text{ and}$$

$$c_2 = r_2 k_2 + w_2\},$$

where k_2 is the consumer's asset holding at the beginning of period 2 (i.e., his savings in period 1). We assume that the government balances its budget in each period. Since the government has no exogenous expenditures to finance, its budget constraint reads $s = \tau_i \bar{k}_2$, where $\bar{k}_2$ is the representative agent's savings in period 1.

The restriction to linear schemes is motivated primarily by practical concerns: most real-world tax schemes for savings are linear or close to linear and it is interesting to know whether, without using a more sophisticated instrument, it is possible to improve welfare if consumers suffer from temptation and self-control costs. We thus simply answer the broad question, "Can a subsidy to savings improve welfare?," a question which presumes linearity. In our environment, it is entirely nontrivial what the answer is: short of "forcing the consumer to consume what she likes," is it possible for the government to improve matters? We show below that the answer is yes in all cases except one (see Section 2.5.1), and also that in one case of particular interest (Section 3.4), linearity is actually not restrictive at all: it can achieve the full optimum. These results and the reasons for them are, we think, quite illuminating. Of course, it is an open question why linear or near-linear tax schedules are used so often. Linear schemes are simple and likely easier to use than many nonlinear ones, although it would be quite challenging to make this point formally.[3] It would indeed be very interesting to extend the tax setting used here to allow limited forms of nonlinearity, for example, cases with savings floors, particularly in a version of the model where nonlinearity is particularly "costly." For example, if consumers have different discount rates, a savings floor which is uniform—say, because discount rates are private information—may be quite costly if some consumers have much stronger discounting than others; for an analysis along these lines, see Amador, Werning, and Angeletos (2006).

However interesting, it is beyond the scope of the present paper to pursue these ideas further. In what follows, we will use the command policy—where the government is fully unrestricted and thus can give the consumer a singleton

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[3]For example, when there is consumer heterogeneity, implementing a nonlinear taxation scheme—such as the command policy—might impose large transactions costs on the government, requiring it to know each consumer's preferences and resources in every state of the world.

choice set—as a natural benchmark against which to evaluate the efficacy of linear taxation.[4]

The government's objective, then, is to choose the tax rate and transfer so that an individual's welfare is maximized (subject to the government's budget constraint). With a change in taxes, individuals are induced to behave differently; in addition, temptation changes because taxation changes the shape of the budget sets. It is thus not a priori clear how taxes influence equilibrium utility.

2.4. *Partial Equilibrium*

In this section, we examine the effects of proportional taxes and subsidies for a fixed price vector $P = (r_1, r_2, w_1, w_2)$.[5] Proposition 1 states that it is optimal to subsidize savings in this case.

PROPOSITION 1: *In the partial-equilibrium two-period model, the optimal investment tax is negative.*

As becomes clear in the proof of the proposition, the optimal investment subsidy is positive if the representative consumer's actual saving is greater than what would have been chosen had he succumbed to temptation. This can be explained intuitively. Consider increasing the investment subsidy from $\tau_i = 0$. The marginal effect of this increase on $u + v$, evaluated at the consumption bundle actually chosen by the consumer, (c_1, c_2), is zero in the two-period model since the consumer is choosing his saving optimally. However, the marginal effect of this increase on the maximal level of temptation, $v(\tilde{c}_1, \tilde{c}_2)$, is negative. The government sets tax rates so as to balance the budget based on equilibrium behavior: in equilibrium, the consumer pays a lump-sum tax equal to the amount of the investment subsidy received. When the investment subsidy is positive, a consumer who deviates to save less would thus not receive as large an investment subsidy, while paying the same tax. Therefore, succumbing to temptation is now less attractive: increasing the investment subsidy reduces the maximal level of temptation.

With optimal taxation, thus, the consumer is induced to save more, so that his intertemporal consumption allocation is tilted more toward the future than in the absence of taxation. At the same time, the change in the slope of the consumer's budget constraint reduces (other things equal) the temptation faced by the consumer. The net result is to increase the consumer's welfare.

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[4]When consumers are tempted to consume "too much," the command policy is equivalent to the imposition of a savings floor, with the floor chosen to replicate the command allocation.

[5]The partial-equilibrium economy could also be viewed as one in which there are two linear production technologies, one using capital (with fixed marginal returns r_1 and r_2) and one using labor (with fixed marginal returns w_1 and w_2).

2.5. *General Equilibrium*

In this section, we study economies in which prices are no longer exogenous but instead adjust to clear perfectly competitive markets. There are important differences between the cases with and without intertemporal production. In an endowment economy, tax policy cannot influence aggregate savings and, consequently, has no influence on equilibrium welfare. In an economy with production, by contrast, tax policy can influence aggregate savings and welfare, though to a smaller extent than in partial equilibrium because of equilibrium effects on prices.

2.5.1. *An Endowment Economy*

This section considers the role of taxation in an endowment economy. Proposition 2 states that a (linear) investment tax cannot improve welfare in an endowment economy.

PROPOSITION 2: *In an endowment economy, an investment tax has no effect on equilibrium welfare.*

In an endowment economy, prices adjust so that consumers choose to hold the endowment at all points in time. The proportional tax τ_i, therefore, cannot influence (realized) consumption. Furthermore, taxes are not useful for decreasing the disutility of self-control either. In equilibrium, the slope of the budget line at the endowment point is given from preferences (where commitment utility and temptation utility both matter). Because equilibrium consumption cannot change in response to taxes, this slope does not change: the slope is determined by the net-of-tax return on savings, and any change in tax rates simply changes the before-tax return. Thus, taxes do not influence the choice set of consumers: whatever temptations consumers face, they cannot be influenced by a proportional tax.[6]

2.5.2. *An Economy With Intertemporal Production*

In this section, we examine the effects of proportional taxes in a general-equilibrium economy with production. Let f be a standard neoclassical aggregate production function and let there be standard geometric depreciation at rate d. The main difference between the endowment economy and the economy with intertemporal production is that prices (wages and interest rates) are

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[6]Nonlinear taxes, of course, would change the consumer's equilibrium choice set and would, therefore, affect the disutility of self-control. In addition, we study an economy with a representative consumer; in an economy with heterogeneous consumers who differ in, say, their short-run discount rates, it is conceivable that linear taxation could affect individual allocations, and hence welfare, even if the aggregate allocation is fixed. We leave the examination of this possibility to future research.

determined by the aggregate savings behavior of consumers according to the usual marginal-product conditions. Specifically,

$$r_1 = r(\bar{k}_1) = 1 + f'(\bar{k}_1) - d \quad \text{and} \quad w_1 = w(\bar{k}_1) = f(\bar{k}_1) - f'(\bar{k}_1)\bar{k}_1$$

and

$$r_2 = r(\bar{k}_2) = 1 + f'(\bar{k}_2) - d \quad \text{and} \quad w_2 = w(\bar{k}_2) = f(\bar{k}_2) - f'(\bar{k}_2)\bar{k}_2,$$

so that tax policy can influence prices through an impact on investment. Proposition 3 states that the government can improve an individual's welfare by imposing a negative tax (i.e., a subsidy) on investment.

PROPOSITION 3: *In the two-period production economy, the optimal investment tax is negative.*

Propositions 1 and 2 establish that investment subsidies can improve welfare when the government can influence aggregate saving outcomes, but they are silent on the sizes of both the optimal subsidies and the welfare improvements that accompany them. A proper quantitative analysis, however, requires both extending the model to a long (infinite) time horizon and finding a reasonable way to calibrate its key parameters (especially the parameters governing preferences). Krusell, Kuruşçu, and Smith (2010) tackled these tasks.

Nonetheless, there is a qualitative question of theoretical interest: how does optimal policy and, in particular, the implied saving behavior, compare to that dictated by commitment utility (that is, that which would be chosen if the consumer—or the government, via general nonlinear taxation—had access to commitment)? Actual choices in this model are informed by both commitment and temptation utility, that is, they end up in between what commitment utility and temptation utility would dictate. Here we specialize utility functions to show that there is no presumption that, in general, optimal saving (i.e., saving under the optimal tax rate) lies in between also. In particular, optimal saving can actually prescribe less consumption today than would commitment utility.

We specialize by making functional-form assumptions that are typical in the applied macroeconomics literature. Specifically, we assume that preferences are additively separable across time and that the period utility function features constant relative risk aversion,

$$u(c_1, c_2) = \frac{c_1^{1-\sigma}}{1-\sigma} + \delta \frac{c_2^{1-\sigma}}{1-\sigma} \quad \text{and}$$

$$v(c_1, c_2) = \gamma \left\{ \frac{c_1^{1-\sigma}}{1-\sigma} + \delta\beta \frac{c_2^{1-\sigma}}{1-\sigma} \right\},$$

so that $\beta < 1$ regulates temptation impatience relative to commitment impatience. In this case, the consumer's problem can be written as

$$\max_{(c_1,c_2)\in B_\tau(k_1,P)} (1+\gamma)\frac{c_1^{1-\sigma}}{1-\sigma} + \delta(1+\beta\gamma)\frac{c_2^{1-\sigma}}{1-\sigma}$$

$$- \gamma \max_{(\tilde{c}_1,\tilde{c}_2)\in B_\tau(k_1,P)} \left\{ \frac{\tilde{c}_1^{1-\sigma}}{1-\sigma} + \delta\beta\frac{\tilde{c}_2^{1-\sigma}}{1-\sigma} \right\}.$$

In the expression above, $\gamma\{\frac{\tilde{c}_1^{1-\sigma}}{1-\sigma} + \delta\beta\frac{\tilde{c}_2^{1-\sigma}}{1-\sigma} - [\frac{c_1^{1-\sigma}}{1-\sigma} + \delta\beta\frac{c_2^{1-\sigma}}{1-\sigma}]\}$ is the cost of self-control. As we show in Section 3, where we extend the model to T periods, under these functional-form assumptions, the Gul–Pesendorfer model of temptation and self-control nests the multiple-selves model. In addition, the tractability provided by these assumptions allows us to obtain some additional analytical results.

We consider first the case of logarithmic (period) utility. The following proposition gives explicit solutions for both laissez-faire and optimal saving.

PROPOSITION 4: *In the two-period model with logarithmic utility, laissez-faire savings are a fraction* $\frac{\delta(1+\beta\gamma)}{1+\gamma+\delta(1+\beta\gamma)}$ *of present-value wealth. The optimal investment tax is* $\tau_i^* = \frac{\gamma(\beta-1)}{1+\gamma} < 0$ *and the associated savings fraction is* $\frac{\delta}{1+\delta}$.

The optimal subsidy depends only on preference parameters and not on the specification of technology. Specifically, it increases as temptation grows larger: it decreases as β increases (thereby reducing the gap between the "long-run" discount rate δ and the "short-run" discount rate $\beta\delta$) and it increases in the strength, γ, of the temptation.

To obtain the savings rate under commitment, set $\gamma = 0$ (or $\beta = 1$) in the expression for the laissez-faire savings rate; to obtain the savings rate when succumbing to temptation, let $\gamma \to \infty$. The laissez-faire savings rate then lies in between these two extremes. Moreover, the optimal savings rate is identical to the laissez-faire savings rate under commitment. This result holds because, in the special case of logarithmic utility, the ratio of temptation consumption to actual consumption is a constant that depends only on preference parameters. This fact implies, in turn, that the cost of self-control depends only on preference parameters and, in particular, does not depend either on prices or taxes. Changes in the subsidy, consequently, leave the cost of self-control unchanged when utility is logarithmic, and the government in effect chooses the optimal subsidy rate simply to maximize commitment utility.

With logarithmic utility, the competitive equilibrium allocation with optimal taxation coincides with the command (or commitment) outcome, that is, the allocation that obtains when the government chooses *for* the consumer by restricting his consumption set to a singleton (or when the consumer does

not suffer from self-control problems). Specifically, the command (or commitment) allocation maximizes welfare using the commitment utility function with $\beta = 1$. But welfare is higher under the command outcome than under the competitive equilibrium allocation with optimal taxation because the consumer does not incur a self-control cost when his choice set is a singleton.

Proposition 4 shows that optimal policy under logarithmic utility dictates more than a marginal distortion: the prescription is to distort so that the equilibrium allocation is the same one that would obtain under commitment. Is this case a bound on the size of the distortion or does optimal policy sometimes prescribe a distortion that is strong enough to go beyond the commitment allocation? The following proposition answers this question affirmatively when the elasticity of intertemporal substitution, σ^{-1}, is close to 1 (i.e., when utility is close to logarithmic).

PROPOSITION 5: *Given σ, let $\tau_i(\sigma)$ be the optimal investment subsidy, and let $c_1(\tau_i(\sigma))$ and $c_2(\tau_i(\sigma))$ be the associated equilibrium consumption allocation. Then $\frac{d\tau_i(\sigma)}{d\sigma}\big|_{\sigma=1} < 0$ and $\frac{d}{d\sigma}\left(\frac{c_2(\tau_i(\sigma))}{c_1(\tau_i(\sigma))}\right)\big|_{\sigma=1} > 0$.*

Thus, near $\sigma = 1$, the optimal subsidy is larger (smaller) than the optimal subsidy under logarithmic utility when $\sigma > (<)$ 1. Moreover, when $\sigma > (<)$ 1, the competitive equilibrium allocation under optimal taxation is tilted *more* (*less*) toward future consumption than is the commitment allocation.

3. THE *T*-PERIOD MODEL

Does the prescription that investment should be subsidized extend to a longer horizon model? To answer this question, we extend the general-equilibrium model with production analyzed above to have T periods. This extension requires us to specialize preferences, again along the lines of what seems useful for applied macroeconomic modeling and for comparisons with the well known Chamley–Judd result that the optimal tax on investment is zero in the long run. In particular, we use "quasigeometric temptation," which we show nests the Laibson model (the case $\gamma \to \infty$) for constant relative risk aversion (CRRA) preferences. Our demonstration that investment should be subsidized in the long run uses a further specialization of preferences, first to logarithmic utility and then to CRRA utility when $\gamma \to \infty$ (i.e., the Laibson case), since these assumptions permit an explicit solution for both laissez-faire outcomes and optimal outcomes.

3.1. *Quasigeometric Temptation*

Consider a T-period (periods $0, 1, \ldots, T$) production economy where taxes and transfers are allowed to be different across periods. The agent makes his decision by taking as given the aggregate prices as functions of the aggregate

capital $\bar{k}$, the law of motion for aggregate capital $\bar{k}' = G_t(\bar{k})$, and the sequence of transfers and taxes. The problem of the price-taking agent in period t, using recursive notation (a prime symbolizes next-period values), is given by

$$U_t(k, \bar{k}) = \max_{c,k'} u(c) + \delta U_{t+1}(k', G_t(\bar{k}))$$

$$+ V_t(k', G_t(\bar{k})) - \max_{\tilde{c}, \bar{k}'} V_t(\tilde{k}', G_t(\bar{k})),$$

where the temptation function is *quasigeometric*,

$$V_t(k, \bar{k}) = \gamma \big[u(c) + \beta \delta U_{t+1}(k', G_t(\bar{k})) \big],$$

with a budget constraint (which applies for both actual and temptation choices) given by

$$c + (1 + \tau_{it})k' = r(\bar{k})k + w(\bar{k}) + s_t.$$

The investment subsidy τ_{it} is allowed to depend on time, and the lump-sum transfer s_t varies with τ_{it} and $\bar{k}$ so as to ensure that the government's budget balances. The consumer's actual savings are determined by a "realized" decision rule $k' = g_t(k, \bar{k})$; similarly, savings when succumbing to temptation are determined by a "temptation" decision rule $\tilde{k}' = \tilde{g}_t(k, \bar{k})$.

DEFINITION 1: A time-t recursive competitive equilibrium for this economy consists of a pair of decision rules $g_t(k, \bar{k})$ and $\tilde{g}_t(k, \bar{k})$, a pair of value functions $U_t(k, \bar{k})$ and $V_t(k, \bar{k})$, pricing functions $r(\bar{k})$ and $w(\bar{k})$, and a law of motion for aggregate capital $G_t(\bar{k})$, such that (i) given $U_t(k, \bar{k})$ and $V_t(k, \bar{k})$, $g_t(k, \bar{k})$ solves the maximization problem above and $\tilde{g}_t(k, \bar{k})$ maximizes $V_t(k, \bar{k})$, (ii) prices are given by $r(\bar{k}) = 1 - d + f'(\bar{k})$ and $w(\bar{k}) = f(\bar{k}) - f'(\bar{k})\bar{k}$, (iii) the law of motion for aggregate capital is consistent with the individual decision rule, that is, $g_t(\bar{k}, \bar{k}) = G_t(\bar{k})$, and (iv) the government budget balances in each period: $s_t = \tau_{it} G_t(\bar{k})$.

We require the government to run a balanced budget in this definition, but this requirement is not restrictive, because a Ricardian-equivalence result obtains straightforwardly in this environment (i.e., given the sequence of investment subsidies and accompanying balanced-budget lump-sum taxes, government deficits and/or surpluses financed by incremental lump-sum taxes/subsidies would have no effect on equilibrium allocations).[7]

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[7]With borrowing constraints, Ricardian equivalence might fail to hold in the model of temptation and self-control even if the timing of taxes does not influence actual consumption choices or equilibrium interest rates. In particular, borrowing constraints could still affect welfare if they restrict the temptation choice but not the actual choice.

3.2. *Generalized Euler Equations*

Solving for equilibrium requires finding two decision rules: one for actual savings decisions and one for temptation savings decisions. It is straightforward to derive a pair of generalized Euler equations (GEEs) that determine these two decision rules. These GEEs will prove useful for interpreting the policy results in the T-period model. The GEE for the actual choice is

$$u'(c_t) = \delta \frac{1 + \beta\gamma}{1 + \gamma} \frac{r(\bar{k}_{t+1})}{1 + \tau_{it}} \{(1 + \gamma)u'(c_{t+1}) - \gamma u'(\tilde{c}_{t+1})\},$$

where c_t and c_{t+1} are the actual consumption levels in periods t and $t + 1$, and $\tilde{c}_{t+1}$ is temptation consumption in period $t + 1$. The GEE for the temptation choice is

$$u'(\tilde{c}_t) = \delta\beta \frac{r(\bar{k}_{t+1})}{1 + \tau_{it}} \{(1 + \gamma)u'(c_{t+1}^s) - \gamma u'(\tilde{c}_{t+1}^s)\},$$

where $\tilde{c}_t$ is the consumption level in period t in the hypothetical case that the consumer succumbs to temptation today and c_{t+1}^s and $\tilde{c}_{t+1}^s$ are the actual and temptation consumption levels in period $t + 1$ given that the consumer succumbs today.

The GEEs differ from standard Euler equations in two ways. First, the discount factors are smaller than the discount factor for commitment utility, δ (the discount factor in the GEE for actual consumption is between δ and the discount factor for temptation utility, $\beta\delta$). Second, there is an additional term $\gamma(u'_{t+1} - \tilde{u}'_{t+1})$ on the right-hand side of the GEEs. This term is positive because utility is strictly concave and temptation consumption exceeds actual consumption (assuming impatience). Thus, relative to the standard consumption–savings model, there is an additional benefit to saving here.

3.3. *Characterization*

In this section we specialize preferences to cases that are of particular interest from the perspective of the macroeconomics literature. These will then be used in the subsequent section, where we study optimal policy in the T-period model.

We look first at (period) utility functions with a constant elasticity of intertemporal substitution, that is, $u(c) = c^{1-\sigma}/(1 - \sigma)$ for $\sigma > 0$ (or logarithmic utility if $\sigma = 1$). For this case, our model nests the Laibson formulation. In particular, Proposition 6 shows that (given prices) as $\gamma \to \infty$, the consumer's value function converges to the function under commitment utility but evaluated at temptation consumption.

PROPOSITION 6: *Given a law of motion for aggregate capital*, $\bar{k}' = G_t(\bar{k})$, *and a sequence of taxes and transfers, as $\gamma \to \infty$, the Gul–Pesendorfer (GP) model*

converges to the Laibson model, that is, the value functions and consumption choices of the consumer in the GP setting are given by

$$U_t(k, \bar{k}) = \frac{c^{1-\sigma}}{1 - \sigma} + \delta U_{t+1}(k', G_t(\bar{k})),$$

where

$$(c, k') = \arg\max \frac{c^{1-\sigma}}{1 - \sigma} + \delta \beta U_{t+1}(k', G_t(\bar{k}))$$

$$s.t. \quad c + (1 + \tau_{it})k' = r(\bar{k})k + w(\bar{k}) + s_t.$$

This limit offers a resolution to the problem of which of the consumer's selves to use when assessing welfare in the multiple-selves model. Specifically, in this limit the consumer succumbs completely to temptation, but he evaluates welfare by discounting using the discount factors in commitment utility. For the case of logarithmic utility, we obtain a similar result regardless of the extent to which the consumer succumbs to temptation (i.e., for *any* value of γ).

PROPOSITION 7: *Given a law of motion for aggregate capital, $\bar{k}' = G_t(\bar{k})$, and a sequence of taxes and transfers, when $u(c) = \log(c)$, the value function and consumption choices of the agent are given by*

$$U_t(k, \bar{k}) = \log(c) + \delta U_{t+1}(k', G_t(\bar{k})) + \Omega,$$

where

$$(c, k') = \arg\max(1 + \gamma)\log(c) + \delta(1 + \beta\gamma)U_{t+1}(k', G_t(\bar{k}))$$

$$s.t. \quad c + (1 + \tau_{it})k' = r(\bar{k})k + w(\bar{k}) + s_t,$$

where Ω is a constant that depends only on preference parameters.

This result holds because, as in the two-period model with logarithmic utility, both actual and temptation consumption are proportional (at any point in time) to lifetime income, with the constant of proportionality depending only on preference parameters (and not on prices or taxes). Thus, the ratio of actual to temptation consumption depends only on preference parameters at any point in time. As in the two-period model, the self-control cost (the constant Ω in Proposition 7) depends only on preference parameters and does not vary either with prices or with policy.

3.4. *Optimal Policy*

In this section, we study optimal policy in the T-period model under the assumption that the government can commit to a sequence of tax and/or subsidy rates. In Proposition 8, we analyze the case of logarithmic preferences for any values of β and γ. In Proposition 9, we analyze the case of CRRA preferences when $\gamma \to \infty$ (for any value of β). We therefore nest the Laibson multiple-selves formulation (which appears in the limit as $\gamma \to \infty$).

As in the previous sections, the government's objective is to maximize time-0 lifetime utility of the representative agent. Proposition 7 shows that for logarithmic utility, the welfare of the representative agent at time 0 is

$$U_0(\bar{k}_0, \bar{k}_0) = \text{a constant} + \sum_{t=0}^{T} \delta^t u(c_t).$$

The government's goal is to maximize this welfare function subject to the aggregate resource constraint $c_t + \bar{k}_{t+1} - (1-d)\bar{k}_t = f(\bar{k}_t)$. The welfare-maximizing consumption allocation, therefore, must satisfy the first-order condition $u'(c_t)/u'(c_{t+1}) = \delta r(\bar{k}_{t+1})$ at every point in time. As in the two-period model with logarithmic utility, the government's optimal policy replicates the commitment allocation.

To find the tax policy that generates the commitment allocation as a competitive equilibrium outcome, it is straightforward to use the optimality conditions of a typical (competitive) consumer to find the sequence of tax rates that induces him to choose it (see the proof of Proposition 8 for details). Proposition 8 gives the optimal sequence of subsidies to investment.[8]

PROPOSITION 8: *Under logarithmic utility, the optimal tax at time t is given by*

$$\tau_{it} = \begin{cases} \dfrac{\gamma(\beta - 1)}{1 + \gamma} & \text{for } t = T - 1, \\[2ex] \dfrac{\gamma(\beta - 1)}{1 + \gamma + \delta(1 + \delta + \cdots + \delta^{T-2-t})(1 + \beta\gamma)} & \text{for } t < T - 1. \end{cases}$$

The optimal investment subsidies are all positive, because when the self-control cost is independent of prices and policies (as it is under logarithmic utility), the government's objective reduces to maximizing the commitment utility function. Thus, the optimal government policy is to replicate the commitment savings rate, and since the savings rate is lower in competitive equilibrium than under commitment, the optimal policy is to subsidize savings.

[8]In a multiple-selves consumption–savings model, Laibson (1996) also argued that optimal policy requires subsidizing savings.

Under logarithmic utility, the optimal subsidies depend only on preference parameters. This result follows from the fact that the income and substitution effects of changes in interest rates exactly offset each other when utility is logarithmic, so that the ratio of consumption to income at any point in time is a constant that depends only on preference parameters. Furthermore, at each time t, the optimal subsidies are decreasing in β (provided $\beta < 1$) and increasing in γ: that is, they increase as temptation grows stronger.

Finally, the optimal subsidies increase as an individual comes closer to period T: $\tau_{i,T-2} < \cdots < \tau_{i1} < \tau_{i0} < 0$. To see why, examine the GEE at time t:

$$\frac{c_{t+1}}{c_t} = \frac{\delta(1+\beta\gamma)}{1+\gamma} \frac{r(\bar{k}_{+1})}{1+\tau_{it}} \left[1 + \gamma\left(1 - \frac{c_{t+1}}{\tilde{c}_{t+1}}\right)\right].$$

The ratio of actual consumption to temptation consumption grows larger as t increases, so the term in square brackets on the right-hand side of the (rearranged) GEE is larger at earlier than at later dates.[9] As a result, the right-hand side of the Euler equation is closer to the right-hand side of the commitment Euler equation at early dates. Replicating the right-hand side of the commitment Euler equation, therefore, requires a smaller subsidy at earlier dates.

An immediate implication of Proposition 8 follows.

COROLLARY 1: *Under logarithmic utility, as $T \to \infty$, the optimal tax at any fixed t converges to*

$$\tau_i = \frac{\gamma(\beta - 1)}{1 + \gamma + \dfrac{\delta}{1-\delta}(1+\beta\gamma)}.$$

Thus, the celebrated Chamley–Judd result that investment (alternatively, capital income) should be undistorted in the long run does not apply in this

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[9]To see why the ratio of actual to temptation consumption decreases with age, note that consumption at time t is given by

$$c_t = \frac{1}{1 + \dfrac{\delta(1+\beta\gamma)}{1+\gamma}m_t} Y_t$$

and temptation consumption is given by

$$\tilde{c}_t = \frac{1}{1 + \delta\beta m_t} Y_t,$$

where Y_t is lifetime income at time t and $m_t = 1 + \delta + \cdots + \delta^{T-t-2}$. The m_t's decrease over time, so $c_t/\tilde{c}_t$ increases over time.

model.[10] For any finite horizon, the optimal subsidy rate will in fact increase over time, and for the infinite-horizon case, the optimal subsidy rate is time-invariant.

We turn now to the determination of optimal policy under CRRA utility for the limiting case in which $\gamma \to \infty$. Proposition 6 shows that the objective of the government is the same as in the logarithmic case, except that the constant term in the objective function (which captures the cost of self-control in the logarithmic case) is equal to zero. The optimal policy, therefore, is to replicate the commitment allocation, as stated formally in Proposition 9.

PROPOSITION 9: *Under CRRA utility, in the limiting case $\gamma \to \infty$, the optimal sequence of investment taxes implements the commitment allocation and generates the first-best welfare outcome for the consumer.*

When the consumer succumbs completely to temptation, therefore, restricting the set of tax instruments to a linear class does not prevent the government from achieving the first-best welfare outcome. Moreover, we can again show that as $T \to \infty$, the optimum calls for a subsidy to savings, in contrast to the Chamley–Judd result.

4. CONCLUSION AND REMARKS

This paper makes clear that when consumers suffer from temptation and self-control problems, linear tax schedules can improve consumers' welfare, even though such schedules are not very powerful tools for restricting consumers' choice sets. The direction of the change is the expected one: when temptation is characterized by "excessive" impatience, optimal policy is to subsidize savings. Moreover, in the special case in which consumers succumb completely to temptation (i.e., the multiple-selves model), linear taxes deliver first-best welfare outcomes.

As discussed in Section 2.3, it would be very interesting to extend the present analysis to nonlinear taxation, especially when there is consumer heterogeneity and private information about types makes it costly to use nonlinear (and linear) schemes. It would also be interesting to consider political-economy constraints on taxes; in practice, we observe a range of tax policy outcomes that do not appear to line up with theoretical prescriptions. For example, we suggest (Section 3.4) that the Chamley–Judd prescription that "optimal taxes on capi-

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[10]The usual setting for the Chamley–Judd result is an infinite-horizon economy. Here, we obtain results for the infinite-horizon economy by studying the limit of a sequence of finite-horizon economies as the horizon grows long.

tal income should be zero in the long run" could be sharpened to "... should be negative...," but in reality these taxes are positive, and in some places large, and it is highly likely that these outcomes have political-economy underpinnings.[11] Integrating such constraints is an important topic that we hope to address in future work.

APPENDIX

PROOF OF PROPOSITION 6: Let Y_t be the lifetime income from period t on which is given by $r_t k_t + w_t + \frac{w_{t+1}}{r_{t+1}} + \cdots$ for a given price sequence $\{(r_t, w_t)\}_{t=0}^T$. (For simplicity, we assume that taxes and transfers are zero in the proof of the proposition, but it is straightforward to adapt the proof to allow for nonzero taxes.) The budget constraint of the agent at time t in terms of consumption in that period and the lifetime income from next period on is given by

$$c_t + \frac{Y_{t+1}}{r_{t+1}} = Y_t.$$

To prove this proposition, we show that the optimization problem of the consumer in period t takes the form

$$U_t(Y_t) = \max_{Y_{t+1}} \frac{1+\gamma}{1-\sigma}\left(Y_t - \frac{Y_{t+1}}{r_{t+1}}\right)^{1-\sigma} + \delta(1+\beta\gamma)U_t(Y_{t+1})$$
$$- \gamma\left\{\max_{\widetilde{Y}_{t+1}} \frac{1}{1-\sigma}\left(Y_t - \frac{\widetilde{Y}_{t+1}}{r_{t+1}}\right)^{1-\sigma} + \delta\beta U_t(\widetilde{Y}_{t+1})\right\},$$

where $U_t(Y_t)$ is given by

$$U_t(Y_t) = b_t \frac{Y_t^{1-\sigma}}{1-\sigma}.$$

b_t is a constant that depends on utility parameters, prices, and time period t. In addition, starting from the last period, we show that, as $\gamma \to \infty$, $Y_{t+1} \to \widetilde{Y}_{t+1}$ and that the value function of the consumer for each t is given by

$$U_t(Y_t) = \frac{1}{1-\sigma}\left(Y_t - \frac{\widetilde{Y}_{t+1}}{r_{t+1}}\right)^{1-\sigma} + \delta U_t(\widetilde{Y}_{t+1}).$$

---

[11]See, for example, Acemoglu, Golosov, and Tsyvinski (2008).

The optimal decision rules for the actual and temptation solutions, respectively, are given by

$$c_t = \frac{1}{1 + \left(\dfrac{\delta(1+\beta\gamma)b_{t+1}}{1+\gamma}\right)^{1/\sigma} r_{t+1}^{(1-\sigma)/\sigma}} Y_t \quad \text{and}$$

$$Y_{t+1} = \frac{\left(\dfrac{\delta(1+\beta\gamma)b_{t+1}}{1+\gamma}\right)^{1/\sigma} r_{t+1}^{1/\sigma}}{1 + \left(\dfrac{\delta(1+\beta\gamma)b_{t+1}}{1+\gamma}\right)^{1/\sigma} r_{t+1}^{(1-\sigma)/\sigma}} Y_t$$

and

$$\widetilde{c}_t = \frac{1}{1 + (\delta\beta b_{t+1})^{1/\sigma} r_{t+1}^{(1-\sigma)/\sigma}} Y_t \quad \text{and}$$

$$\widetilde{Y}_{t+1} = \frac{(\delta\beta b_{t+1})^{1/\sigma} r_{t+1}^{1/\sigma}}{1 + (\delta\beta b_{t+1})^{1/\sigma} r_{t+1}^{(1-\sigma)/\sigma}} Y_t.$$

We start with period $T-1$ and continue backward. Note that $b_T = 1$. From the expressions above, it should be clear that $c_{T-1} \to \widetilde{c}_{T-1}$ and $Y_T \to \widetilde{Y}_T$. Next we show that

$$U_{T-1}(Y_{T-1}) = \frac{\widetilde{c}_{T-1}^{1-\sigma}}{1-\sigma} + \delta \frac{\widetilde{Y}_T^{1-\sigma}}{1-\sigma}.$$

To show this, we need to show that $\lim_{\gamma\to\infty} \gamma(c_{T-1}^{1-\sigma} + \delta\beta Y_T^{1-\sigma} - \widetilde{c}_{T-1}^{1-\sigma} - \delta\beta \widetilde{Y}_T^{1-\sigma})$. Inserting the decision rules into $\gamma(c_{T-1}^{1-\sigma} + \delta\beta Y_T^{1-\sigma} - \widetilde{c}_{T-1}^{1-\sigma} - \delta\beta \widetilde{Y}_T^{1-\sigma})$, we obtain

$$
(1) \qquad \lim_{\gamma\to\infty} \gamma \left(\frac{1 + \delta\beta \left(\dfrac{\delta(1+\beta\gamma)}{1+\gamma}\right)^{(1-\sigma)/\sigma} r_T^{(1-\sigma)/\sigma}}{\left(1 + \left(\dfrac{\delta(1+\beta\gamma)}{1+\gamma}\right)^{1/\sigma} r_T^{(1-\sigma)/\sigma}\right)^{1-\sigma}} \right.
$$

$$
\left. - \frac{1 + \delta\beta(\delta\beta)^{(1-\sigma)/\sigma} r_T^{(1-\sigma)/\sigma}}{(1 + \delta\beta(\delta\beta)^{(1-\sigma)/\sigma} r_T^{(1-\sigma)/\sigma})^{1-\sigma}} \right).
$$

Applying l'Hôpital's rule by letting $\widetilde{\gamma} = 1/\gamma$ and $\widetilde{\gamma} \to 0$, it is easy to show that the limit above converges to zero. Thus,

$$\lim_{\gamma \to \infty} U_{T-1}(Y_{T-1}) = \frac{\widetilde{c}_{T-1}^{1-\sigma}}{1 - \sigma} + \delta \frac{\widetilde{Y}_T^{1-\sigma}}{1 - \sigma}.$$

In period t, expression (1) contains b_{t+1} as

$$(2) \qquad \lim_{\gamma \to \infty} \gamma \left(\frac{1 + \delta\beta b_{t+1} \left(\dfrac{\delta(1 + \beta\gamma)b_{t+1}}{1 + \gamma} \right)^{(1-\sigma)/\sigma} r_T^{(1-\sigma)/\sigma}}{\left(1 + \left(\dfrac{\delta(1 + \beta\gamma)b_{t+1}}{1 + \gamma} \right)^{1/\sigma} r_T^{(1-\sigma)/\sigma} \right)^{1-\sigma}} \right.$$

$$\left. - \frac{1 + (\delta\beta b_{t+1})^{1/\sigma} r_T^{(1-\sigma)/\sigma}}{(1 + (\delta\beta b_{t+1})^{1/\sigma} r_T^{(1-\sigma)/\sigma})^{1-\sigma}} \right),$$

where $b_T = 1$ and b_t is given recursively as

$$b_t = (1 + \gamma) \left(1 + \left(\frac{\delta(1 + \beta\gamma)b_{t+1}}{1 + \gamma} \right)^{1/\sigma} r_{t+1}^{(1-\sigma)/\sigma} \right)^{\sigma}$$

$$- \gamma \left(1 + (\delta\beta b_{t+1})^{1/\sigma} r_{t+1}^{(1-\sigma)/\sigma} \right)^{\sigma}.$$

Using this equation and the fact that $b_T = 1$, we can show that $\lim_{\widetilde{\gamma} \to 0} \frac{db_{t+1}}{d\widetilde{\gamma}} = 0$ for all t, which implies that the expression in (2) converges to zero as $\widetilde{\gamma} \to 0$. Thus,

$$U_t(Y_t) = \frac{\widetilde{c}_t^{1-\sigma}}{1 - \sigma} + \delta b_{t+1} \frac{\widetilde{Y}_{t+1}^{1-\sigma}}{1 - \sigma} = \frac{\widetilde{c}_t^{1-\sigma}}{1 - \sigma} + \delta U_{t+1}(\widetilde{Y}_{t+1}). \qquad Q.E.D.$$

PROOF OF PROPOSITION 8: Proposition 7 provides the value function for the consumer evaluated at the competitive equilibrium allocation, which is also the objective function for the government. The government maximizes the objective function by choosing consumption allocations subject to the economy's resource constraint at each point in time. Thus, setting $k_t = \bar{k}_t$, the government's problem reduces to

$$U_t(\bar{k}_t, \bar{k}_t, \tau) = \max_{c_t, \bar{k}_{t+1}} \log(c_t) + \delta U_t(\bar{k}_{t+1}, \bar{k}_{t+1}, \tau)$$

subject to economy's resource constraint

$$c_t + \bar{k}_{t+1} = (1 - d)\bar{k}_t + f(\bar{k}_t).$$

The optimal allocation must satisfy the Euler equation

$$\frac{1}{c_t} = \delta r(\bar{k}_{t+1}) \frac{1}{c_{t+1}}.$$

The government implements this allocation by choosing tax rates such that the Euler equation for the consumer is equivalent to the government's Euler equation above. The proof of Proposition 7 shows that the competitive equilibrium allocation satisfies the Euler equation

$$\frac{1}{c_t} = M_{t+1} \frac{r(\bar{k}_{t+1})}{1 + \tau_{i,t}} \frac{1}{c_{t+1}},$$

where $M_T = \frac{\delta(1+\beta\gamma)}{1+\gamma}$, $M_{T-1} = \frac{\delta(1+\delta)(1+\beta\gamma)}{1+\gamma+\delta(1+\beta\gamma)}$, $\ldots$, $M_{t+1} = \frac{\delta(1+\delta+\cdots+\delta^{T-t-1})(1+\beta\gamma)}{1+\gamma+\delta(1+\delta+\cdots+\delta^{T-t-2})(1+\beta\gamma)}$. Thus, the government chooses $\tau_{i,t}$ such that $\frac{M_{t+1}}{1+\tau_{i,t}} = \delta$, which delivers $\tau_{i,T-1} = \frac{\gamma(\beta-1)}{1+\gamma}$, $\tau_{i,T-2} = \frac{\gamma(\beta-1)}{1+\gamma+\delta(1+\beta\gamma)}$, $\ldots$, $\tau_{i,t} = \frac{\gamma(\beta-1)}{1+\gamma+\delta(1+\delta+\cdots+\delta^{T-2-t})(1+\beta\gamma)}$. $\qquad$ *Q.E.D.*

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Institute for International Studies, Stockholm University, 106 91 Stockholm, Sweden; per.krusell@iies.su.se,

Economics Dept., University of Toronto, 150 St. George St., Toronto, Ontario M5S 3G7, Canada; burhan.kuruscu@utoronto.ca,
and
Economics Dept., Yale University, 28 Hillhouse Avenue, New Haven, CT 06520, U.S.A.; tony.smith@yale.edu.